Session 1

What Does It Mean To Measure?

Key Terms in This Session

New in This Session

- measurable properties
- measurement
- precision
- surface area
- unit
- volume
- weight

Introduction

In this session, you will begin to explore the questions "What can be measured?" and "What does it mean to measure something?" You will identify measurable properties of objects such as weight, surface area, and volume, and discuss which metric units are appropriate for measuring these properties. You will also learn that measurement is, by its nature, approximate. Finally, you will consider how to make measurements using nonstandard units.

For the list of materials that are required and/or optional in this session, **see Note 1.**

Learning Objectives

In this session, you will do the following:

- Begin exploring why measurement is always approximate
- Learn how to identify measurable properties as well as the metric units that are most appropriate for measuring these properties
- Use and learn about different methods for measuring, such as displacement
- Explore the advantages of using standard units when measuring

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Note 1. Materials Needed:

- A clean rock roughly the size of an egg
- Tinfoil
- Two-pan balance and three-arm balance scale (optional)
- Beakers marked with milliliters
- Fine sand or rice (optional)
- Calculator (optional)

Part A: Comparing Rocks (15 min.)

Measurement is used in all aspects of daily life, as well as in such fields as engineering, architecture, and medicine. We measure things every day. This morning you may have weighed yourself, poured two cups of water into the coffeemaker, checked the temperature outside to help you decide what to wear, cut enough gift wrap off the roll to wrap a present, decided on the size of a storage container for some leftover food, noted on your car's odometer how far you'd driven, monitored both your car's traveling speed and its gas gauge, and kept an eye on the time so that you wouldn't be late.

All of the situations above are easily identifiable as measurement situations. Yet what is at the heart of all of these comparisons? In other words, in order to measure, what must we consider, and then what steps must we take? **[See Note 2]**

To begin thinking about measurement, you will use, of all things, a rock.

Problem A1. Make a list of attributes that could be used to describe the rock.

Problem A2. Some of these attributes might be measurable, and some might not. How do we determine what we can measure? **[See Tip A2, page 24]**

Problem A3. If you were to compare different rocks using each of the measurable attributes you listed in Problem A1, what units would you use?

Problem A4. How could you measure these properties? **[See Tip A4, page 24]**

Note 2. Though we all use measurement daily, most adults have not considered the properties of an object that make it measurable in some way. This first activity is designed to focus attention on the many attributes that are used to compare objects (in this case, rocks) and to sort the attributes into two categories—those that are measurable and those that are not.

Part A adapted from Chapin, Suzanne & Johnson, A. *Math Matters: Understanding the Math You Teach, Grades K–6*. p. 177.

Part B: Which Rock Is the Largest? (65 min.)

Measuring Surface Area

You can measure a number of attributes of rocks—for example, surface area, volume, and weight. Which of these attributes should you use to determine the largest rock? Let's collect measurements of each type, looking at surface area first. **[See Note 3]**

The area enclosing a three-dimensional or solid object is referred to as the surface area. Imagine that a thin skin covers all the surfaces of your rock. How would you determine the size of this skin?

For this activity, you'll need your rock, a sheet of tinfoil large enough to wrap around the rock, and pieces of grid paper with units of 1 cm^2, 0.5 cm^2, and 0.25 cm^2. You can find the grid paper on pages 19–21.

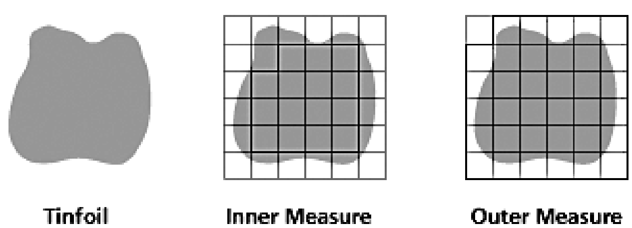
Before you begin measuring, estimate the surface area of your rock. (Later, you can compare your estimate with the approximate surface area you've measured.)

Problem B1. How could you use the tinfoil to find the surface area of the rock? Why would you use this technique?

Problem B2. What unit will you use? Is there more than one choice? Explain.

By first estimating the number of units in a measure, we are forced to consider the size of the object in relation to a standard unit of measurement. If our estimates and approximations are far apart, then we have to reevaluate the size of the unit we chose. When we repeatedly estimate and measure, we improve our ability to measure accurately. It's this "measurement sense" that allows us to establish benchmarks for particular measures (e.g., "I know that 2 L is about the size of a common soda bottle, so I'd say that this container holds around 3 L").

Now take the tinfoil and wrap it around your rock, covering the surface area as best you can. Then superimpose the tinfoil that represents the surface area of your rock on the 1 cm^2 grid paper and trace around it. Count the number of squares that are completely covered by the tinfoil. This is your inner measure. Then count the number of squares that are both completely and partially covered by the tinfoil. This is your outer measure. Find the average of the inner and outer measures. You can use that average as the approximate surface area of your rock.



Problem B3. How exact is your measurement? How might using a different unit give you a closer approximation? What else might you do to get a closer approximation?

Measure your rock again, this time using sheets of grid paper with units of 0.5 cm^2 and 0.25 cm^2. Again, superimpose the tinfoil representing the surface area of your rock on each of the grids, trace around it on each sheet of grid paper, and calculate the surface area.

Now compare your three approximations. In order to do this, you'll need to use the same units, so convert the first two approximations to units of 0.25 cm^2. (Be careful here—look at the grid papers and notice how many small squares equal a larger square.) **[See Note 4]**

Problem B4. What do you notice about the approximate surface areas using different grids? What conclusions can you draw?

Note 3. Prior to measuring these attributes, it is important to consider what type of unit should be used. Children frequently have a difficult time choosing appropriate units of measure. For example, they often try to measure area using linear units (centimeters), or volume using two-dimensional units (square centimeters). Reflect on your own knowledge of metric units of measure. Does your knowledge of and familiarity with metric units have anything to do with your ability to choose an appropriate unit?

Note 4. The following equivalencies may be helpful: $1\ cm^2 = 4\ (0.5\ cm^2)$ and $1\ cm^2 = 16\ (0.25\ cm^2)$. In order to visualize these relationships (e.g., that four 0.5 cm squares cover the same amount of space as 1 cm^2), draw the 0.5 cm squares on 1 cm^2. Reflect on or discuss why the average of the inner and outer measures is the approximate surface area and not the exact surface area.

Part B, cont'd.

Video Segment (approximate time: 8:12–10:48): You can find this segment on the session video approximately 8 minutes and 12 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, Laura and David measure the surface area of a rock. They wrap tinfoil around it and then approximate its area using grid paper. Watch this segment after you've completed Problems B1–B4.

What are some of the difficulties they came across? Did you experience similar or different problems?

Measuring Volume

One method for determining the volume of irregular objects like rocks uses a technique called displacement. Archimedes (287–212 B.C.E.) is credited with discovering volume relationships. Here's how the story goes: Once there was a king who suspected that his crown might not be made of pure gold. He brought his problem to Archimedes, a "wise man" of the day. Archimedes pondered the question but didn't have an immediate solution. Later, as he was taking his bath, he noticed that the displacement of water in the tub was equal to the immersed part of his body. Archimedes leaped from the tub and ran naked through the streets, shouting, "Eureka!" His observation showed him how to solve the king's problem: Using the displacement-of-water method, he could easily calculate the volume of the crown. By comparing the weight of the crown to a lump of pure gold of the same volume, he found that the crown weighed less—indeed, it was not made of pure gold. As the king suspected, the crown was composed mostly of cheaper metals. Through measurement, Archimedes was able to expose the jeweler's dishonesty.

Before you begin measuring, estimate the volume of the rock. (Later, you can compare your estimate with the approximate volume you've measured.)

Take a graduated beaker marked in milliliters (or a measuring cup similarly marked) that is large enough to hold your rock. Fill the container halfway and record the water level.

Note that to measure the volume by displacement, you will need to fully submerge the rock in the water. Displacement will be equal to the amount of space taken up by the rock.

Problem B5. What units are you using to measure the water? Can you use this unit to measure the volume of a solid? **[See Note 5] [See Tip B5, page 24]**

Carefully place the rock in the water, and again note the height of the water. Determine the difference in water heights. **[See Note 6]**

Problem B6. If you found the volume of your rock using both displacement of water and displacement of rice, will the measurements be the same? Why or why not? **[See Tip B6, page 24]**

Note 5. The relationship between cubic centimeters and milliliters (1 cm^3 = 1 mL) and, accordingly, between cubic decimeters and liters (1 dm^3 = 1 L) will be explored further in Session 3.

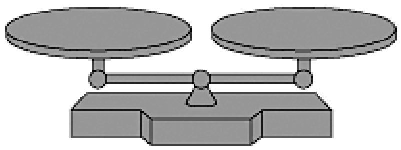
Note 6. This method works best with a beaker marked in milliliters. Other containers may not have adequate markings for you to determine the volume of water displaced when you submerge the rock. If you don't have such a beaker, you could fill the container to the top, measure the amount of water that overflows when you submerge the rock, and then pour the overflow into more precisely marked measuring devices.

Since spilled water can be messy, you might try using a solid material instead, such as fine sand or rice. Fill a container to the top with sand, place your rock in the container, collect the overflowing sand, and then measure the amount that overflowed.

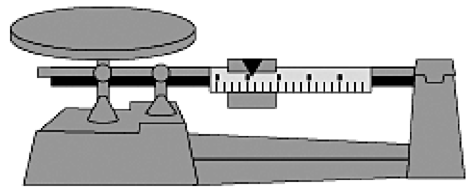
Part B, cont'd.

Measuring Weight

Some people might suggest that we examine the weight of rocks to determine which is the biggest. To measure the weight of your rock, you will need a two-pan balance and a three-arm balance. **[See Note 7]**



Two-Pan Balance



Three-Arm Balance

Estimate the weight of your rock. (Later, you can compare your estimate with the approximate weight you've measured.)

Problem B7. What information can you gather by using a two-pan balance? Can you determine the exact weight of your rock with this balance? **[See Note 8]**

If you have access to a two-pan balance, use it to determine the weight of your rock.

Problem B8. How does a three-arm balance scale work? Can you determine the exact weight of your rock with this balance?

If you have access to a three-arm balance, use it to determine the weight of your rock.

Problem B9. In science, a distinction is made between mass and weight. What do you know about these two terms?

Problem B10. How precise are your rock's measurements? What might affect the precision of this measurement?

Problem B11. Now that you've experimented with several different types of measures, which would you use to determine the largest rock in a group of rocks? Should you use a combination of measures? **[See Note 9]**

Take It Further

Problem B12. There are very interesting relationships among metric measures involving water. One cubic centimeter of water is equivalent to 1 mL of water. In addition, 1 mL of water (or 1 cm^3 of water) weighs 1 g. You may then conclude that the amount of water your rock displaced should be equivalent to the weight of your rock. What is faulty about this line of reasoning? **[See Tip B12, page 24]**

Problem B13. Are there any other measurements you could use to determine the largest rock in a group of rocks?

Note 7. In this session, we are using the common term "weight," even though we are technically finding the mass of the rock. The difference between the terms is discussed in detail in Session 3.

Also, many teachers have not had the opportunity to study different types of scales, chiefly because scales are not standard equipment in classrooms. To do a hands-on version of this activity, you'll need a two-pan balance and a three-arm balance. Using the two-pan balance, you can compare an object on one pan to a set number of weights on the other pan, adjusting the weights until the pans are level (or balanced). The three-arm balance has weights built into the instrument. You may be able to borrow scales from colleagues in middle or high school science departments. Try to use the best scales your school system has available. Be sure that the scales have been "balanced" prior to using them.

Note 8. Some people may consider the weight of the rock to be an exact amount, perhaps because it is more difficult to think about using smaller and smaller units to measure weight. But is weight ever exact? Reflect on or discuss this problem as a group.

Note 9. If you and your colleagues are working in several small groups, try to decide which group has the largest rock. Unless the rocks are very different, though, this might not be a simple task. The term "largest" is not an absolute and has many meanings, depending on the circumstances and the judgement of those involved with the decision-making process. Ultimately, you may choose to use a combination of measures, such as those that are discussed in the homework.

Part C: Nonstandard Units (30 min.)

We will now use tangram shapes to explore some measurement concepts.

Problem C1. Take out page 22 and cut out the two small triangles and one medium triangle in the last row. Using all three triangles (you must use all three each time), build the polygons listed below. (The polygons are also traced on page 22.) **[See Note 10]**

- Square
- Rectangle that is not a square
- Parallelogram that is not a rectangle
- Triangle
- Trapezoid

Try It Online! **www.learner.org**

Problems C1–C5 can be explored online as an Interactive Activity. Go to the *Measurement* Web site at www.learner.org/learningmath and find Session 1, Part C.

Problem C2. Without using measuring tools, can you determine which of the polygons has the greatest area? Explain. **[See Note 11]**

Video Segment (approximate time: 17:36–18:47): You can find this segment on the session video approximately 17 minutes and 36 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

Watch this video segment to see how Rosalie and Jonathan, knowing that they used the same pieces to build each shape, reasoned through the problem of whether the areas were the same.

Did you come up with any other reasons why the areas must be the same?

Problem C3. Without using measuring tools, place the polygons in order from least to greatest by the length of their perimeters. How did you determine which shape has the smallest perimeter? **[See Note 12] [See Tip C3, page 24]**

Problem C4. If we can measure without using standard units, why do we need standard units?

Note 10. If you are working in a group, first try working individually to make each of the five polygons (using all three triangles each time). Provide hints to colleagues who are having difficulty making a particular shape by starting the puzzle for them; put two of the three triangles in place and let them figure out where the third triangle belongs. Resist simply showing someone a finished puzzle. Once the five polygons have been completed and traced, discuss Problems C2 and C3.

Note 11. It may appear to some people that the areas of the five shapes are different. Others will argue that the areas must be the same since each polygon is constructed from the same identical pieces. Reflect on this question, or discuss it if you are working in a group.

Note 12. There are different ways to order the polygons by perimeter, all of which require us to first establish a nonstandard unit of measure (e.g., the side of one of the triangles; the length of an eraser) and then to take some measurements in terms of the nonstandard unit. If you are working in a group, share your approaches. Be sure that everyone agrees on the order in which to place the polygons (in terms of perimeter), and that individuals justify their conclusions.

Part C, cont'd.

Video Segment (approximate time: 22:29–24:20): You can find this segment on the session video approximately 22 minutes and 29 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, Lori demonstrates the method her group used to figure out the perimeter of the five different figures, using a nonstandard unit of measure. As you watch the segment, think about the possible difficulties one may encounter when using nonstandard units of measure.

Would Lori's answer be different if she had chosen a different side or a different triangle to measure with?

Take It Further

Problem C5. Use the Pythagorean theorem to determine the lengths of the sides of the small and medium triangles. Recall that the Pythagorean theorem states that $a^2 + b^2 = c^2$ for the legs (a and b) and hypotenuse (c) of a right triangle. If the length of a leg of one of the small triangles is assigned a value of 1 unit, determine the length of the hypotenuse of the small triangle and the lengths of the legs and hypotenuse of the medium triangle. Use this information to determine the perimeters of the polygons in Problem C1.

Part D: Summing It Up (10 min.)

What is measurement?

Measurement is the process of quantifying the properties of an object by expressing them in terms of a standard unit. Measurements are made to answer such questions as, How heavy is my parcel? How tall is my daughter? How much chlorine is in this water?

How do we measure?

The process of measuring consists of three main steps. First, you need to select an attribute of the thing you wish to measure. Second, you need to choose an appropriate unit of measurement for that attribute. Third, you need to determine the number of units.

What procedures are used to determine the number of units?

Some measurements require only simple procedures and little equipment—measuring the length of a table with a meterstick, for example. Others—for example, scientific measurements—can require elaborate equipment and complicated techniques.

Is it possible to measure objects without using standard units?

Yes. Nonstandard units (i.e., units that are not agreed upon by large numbers of people) can be used to make comparisons and order objects. But because the units are nonstandard, there is limited value in using them to convey information.

How precise are measurements?

Measurement, by its very nature, is approximate. The precision of the measuring device tells us how finely a particular measurement was made. Measurements made using small units, such as square millimeters, are more precise than measurements made using larger units, such as square centimeters. The accuracy of a measure is determined by how correctly a measurement has been made. Accuracy can be affected by the person making the measurement and/or by the measurement tool. Precision and accuracy, and how to determine them, will be covered in later sessions.

Okay, then—how large is my rock?

It all depends on how you define the word *large*. Your answer will be based on the attributes you decide to consider, such as weight, volume, surface area, and height.

Homework

How Large Is It?

This table shows the height to the shoulder and the weight of several species of buffalo. Which is the largest (i.e., one that is both tall and heavy)? The guar is the tallest, and the water buffalo and the American bison are the heaviest, but how might we determine which is the largest?

Name (Country/Continent)	Height (cm)	Weight (kg)
Water Buffalo (Asia)	165	1,000
African Buffalo	135	560
Yak (Tibet)	200	550
Guar (Asia)	220	850
American Bison (North America)	180	1,000
European Bison	200	900

Problem H1. Maria decides to add the height and the weight as a measure of the total size. The animal with the greatest sum is the largest.

Which three animals, in order from largest to smallest, are the largest, according to Maria's criterion?

Problem H2. Jacob decides to rank the animals from 1 to 6 for both height and weight. The smallest in each category is given a rank of 1, the largest a rank of 6. If two animals have the same height or weight, each receives the average of the ranks they would have received. He then adds the height and weight ranks. The largest sum indicates the largest animal.

Which three animals, in order from largest to smallest, are largest according to Jacob's system?

Problem H3. Quentin decides to find the product of height and weight as a measure of the total size.

Which three animals, in order from largest to smallest, does Quentin think are the largest?

Problem H4. Is Quentin's method a good measure of how large an animal is? Do you think it is better or worse than Jacob's method?

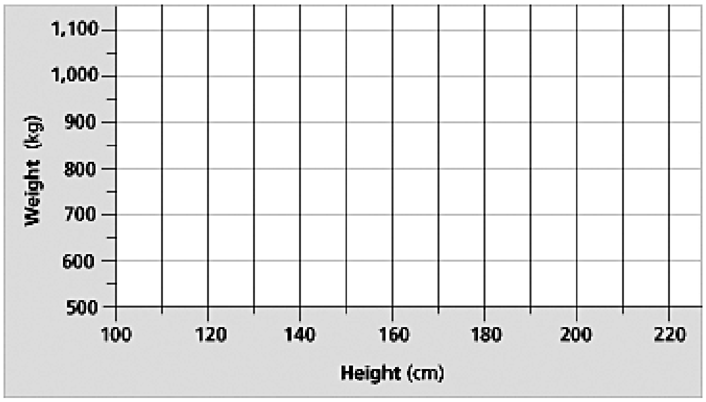
Problem H5. Greg argues that the question would be easier to decide if one of the buffalo were 220 cm tall to the shoulder and weighed 1,000 kg. That animal would clearly be the largest, because it would be both the tallest and the heaviest.

a. Plot the height and weight of each buffalo as an ordered pair. You may want to use the graph on page 23 to plot the points.

Homework, cont'd.

Problem H5, cont'd.

Heights and Weights of Buffalo



b. How far from the point (220,1000) does each buffalo fall? Which point is the shortest distance away? Complete this table:

Name (Country/Continent)	Coordinates	Distance from (220,1000)
Water Buffalo (Asia)	(165,1000)	
African Buffalo	(135,560)	
Yak (Tibet)	(200,550)	
Guar (Asia)	(220,850)	
American Bison (North America)	(180,1000)	
European Bison	(200,900)	

[See Tip H5, page 24]

Problem H6. Which three species, in order from largest to smallest, are closest to the point (220,1000)?

Problem H7. Marcy alters Greg's argument somewhat. She argues that the smallest animal would be (0,0), or no animal at all. Since largest is the opposite of smallest, the largest animal is the one farthest from (0,0).

Which three buffalo, in order from largest to smallest, are largest by Marcy's rule? Complete the table below.

Name (Country/Continent)	Coordinates	Distance from (0,0)
Water Buffalo (Asia)	(165,1000)	
African Buffalo	(135,560)	
Yak (Tibet)	(200,550)	
Guar (Asia)	(220,850)	
American Bison (North America)	(180,1000)	
European Bison	(200,900)	

Homework, cont'd.

In all the criteria used so far, the students equate 1 cm in height to 1 kg in weight. It is not always reasonable to treat all units of measure as being equally important. For example, Maurice thinks that a 1 cm increase in height is more noticeable than a 1 kg increase in weight. In fact, he believes that a 1 cm change in height is twice as important as a 1 kg change in weight, so he uses the value of the expression 2(height) + weight as his measure. According to Maurice's rule, the largest animal is the American bison, followed by the water buffalo and the European bison.

Problem H8. How would you have solved this problem? Can you think of an additional rule that can be used to determine which three animals are the largest?

Problem H9. Which species of buffalo would you say is the smallest? Why?

Suggested Reading

This reading is available as a downloadable PDF file on the *Measurement* course Web site. Go to www.learner.org/learningmath.

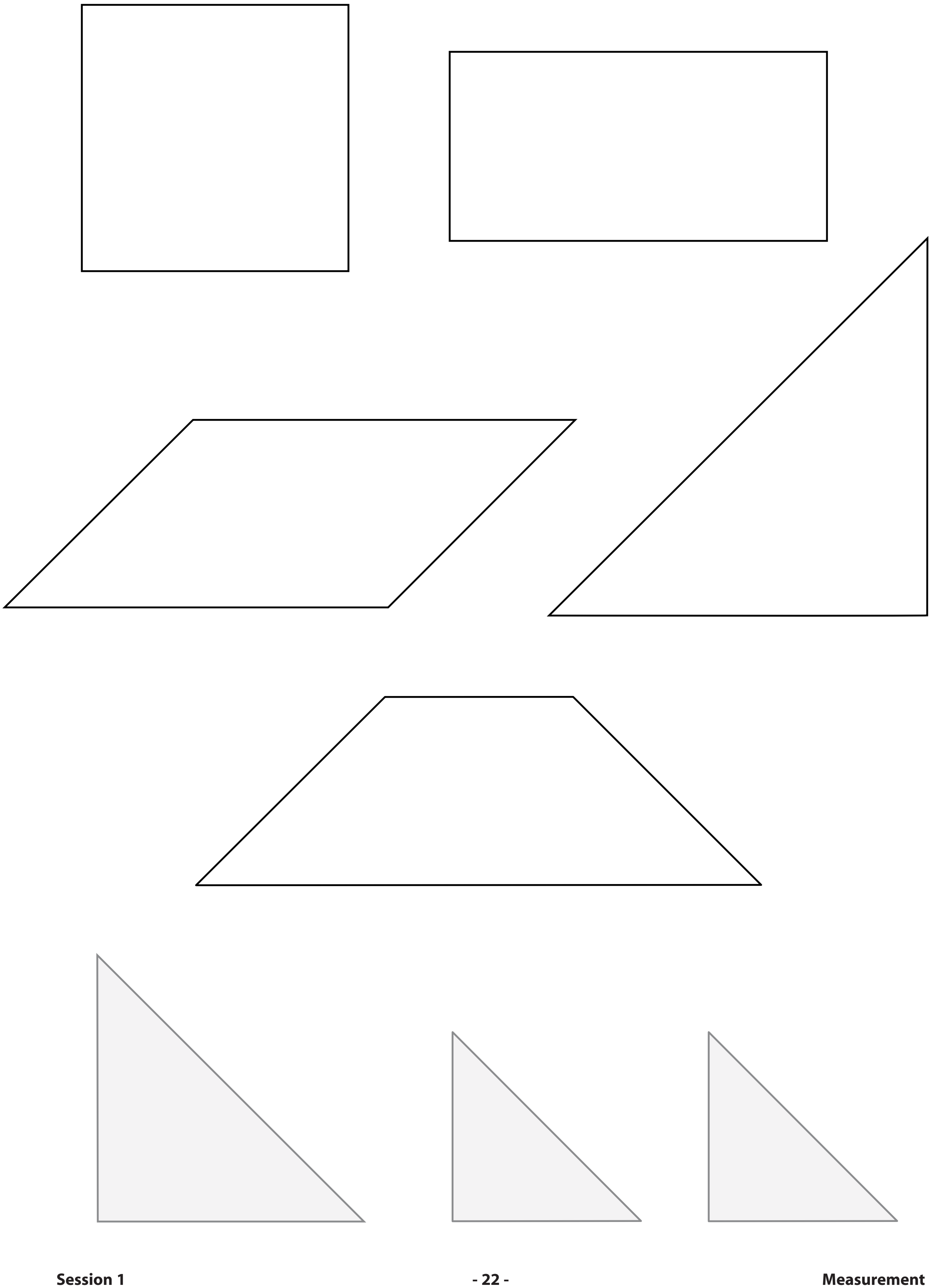
Duke, C. (May, 1998). Tangrams. *Mathematics Teaching in the Middle School*, 485–487.

0.25 cm^2 Grid Paper

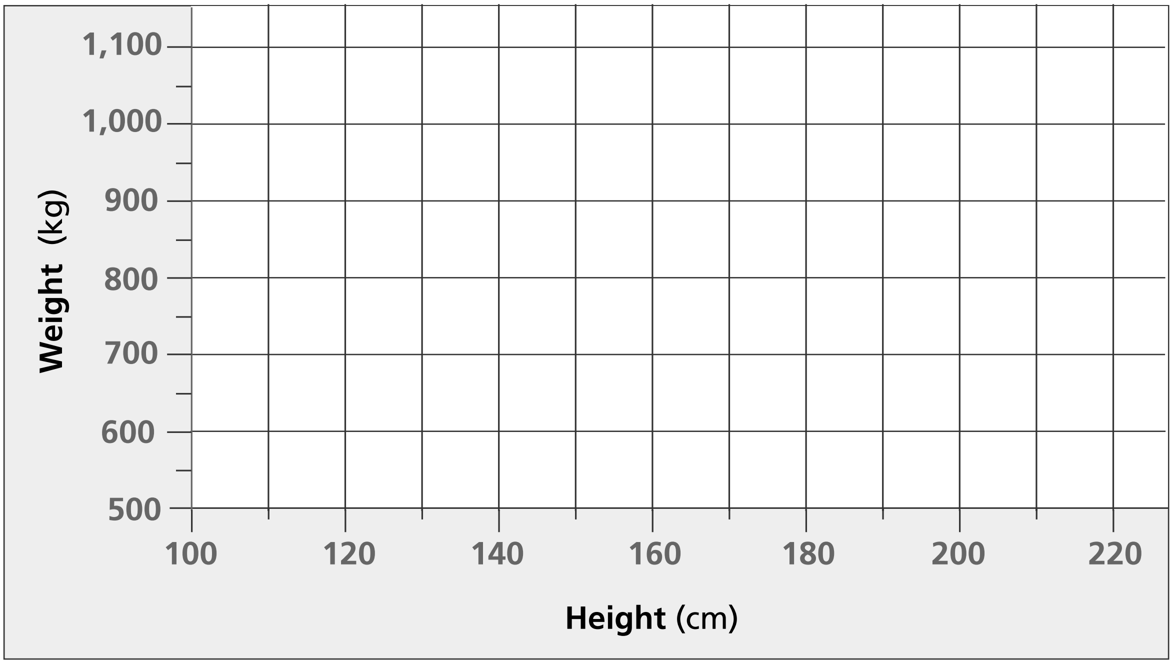
0.5 cm^2 Grid Paper

1 cm^2 Grid Paper

Tangrams for Problems C1-C5



Graph for Homework Problem H5



Tips

Part A: Comparing Rocks

Tip A2. If you are having difficulty sorting the attributes, consider which attributes can be quantified. For example, the texture of a rock (e.g., smooth, bumpy, rough) isn't quantifiable using any of the standard units we know; in contrast, the weight of the rock is quantifiable and can be measured in ounces or grams.

Another suggestion is to see what happens when you combine your object with another, similar object. If the attribute is measurable, then it will increase when the objects are combined. For example, when you combine two rocks, the texture won't increase or change in any way, but the weight certainly will.

Tip A4. Think about what instruments, devices, or methods you might use.

Part B: Which Rock Is the Largest?

Tip B5. One milliliter is 1/1,000 of a liter. There is an interesting relationship in the metric system that can be used to help you here—namely, 1 mL is equivalent to 1 cm^3. We often think of measuring solid objects using cubic centimeters, but because of this special relationship, we can also use milliliters (or liters).

You have probably heard the medical term "cc's." When used this way, 200 cc's is equivalent to 200 cm^3 or 200 mL. The medical term refers to units of length and volume, rather than units of liquid measure.

Tip B6. Do you think that 1 mL of rice equals 1 cm^3 of rice?

Tip B12. Is the rock heavier, lighter, or the same weight as water?

Part C: Nonstandard Units

Tip C3. Notice that the length of the hypotenuse of the small triangle and the length of the legs of the medium triangle are equal. This length can be used as one of your "nonstandard" units. What other nonstandard unit can be established by examining the lengths of the sides? Use these nonstandard units to establish the perimeters of the shapes.

Homework

Tip H5. Use the Pythagorean theorem to find distances between two points. Imagine a right triangle whose hypotenuse lies between the two points and whose legs are parallel to the x- and y-axes (the right angle between them). The shortest distance between the points will be the length of the hypotenuse. How can you use the coordinates of the two points to figure out the lengths of the legs you will need in order to calculate the length of the hypotenuse?

Solutions

Part A: Comparing Rocks

Problem A1. Answers will vary. Some answers might be the rock's length, surface area, volume, weight, color, and texture.

Problem A2. A measurable property is a property that can be quantified using some kind of unit as a basis. For example, length is measurable, since there is a unit of length (an inch, a centimeter, etc.) and we are counting or measuring the number of units in our object. A non-measurable property is one without a standard unit. When we combine objects with a measurable property, the property must increase.

If we wanted to measure some of the properties not commonly measured, we would have to invent a method to do it. For example, to measure texture, we could look at the curvature over the small areas of the object; if the curvature doesn't change much, we could say that the texture of the object was smooth. Some interesting modern research in mathematics focuses on such "nonstandard" measurements.

Problem A3. Answers will vary. For those listed in our solution to Problem A1, we can measure length in centimeters, surface area in square centimeters, volume in cubic centimeters, and weight in grams.

Problem A4. Answers will vary. We could measure the length of the rock using a ruler or tape measure. We could measure the weight using a scale, the volume using a beaker of water (for displacement), and the surface area using tinfoil.

Part B: Which Rock Is the Largest?

Problem B1. You could wrap tinfoil around the rock, just covering it; then unwrap the tinfoil, lay it flat, and measure its area. One reason to use this method is that it is far easier to measure a flat (two-dimensional) area than it is to measure the surface area of a three-dimensional object, and the tinfoil can be laid flat while still representing the three-dimensional surface area. Tinfoil is also quite flexible and can wrap tightly around most irregular surfaces of the rock.

Problem B2. Yes, there is more than one choice. The units for surface area will be square: square inches, square centimeters, square millimeters, and so on. The size of the unit depends entirely on the size of the object being measured. Since the rock you used is relatively small, it is reasonable to use square centimeters or square half-centimeters as the unit of surface area.

Problem B3. It is not very exact. There are several sources of error, including the error of estimating that the tinfoil has exactly enveloped the rock, and the error of rounding our answers. A closer approximation could be obtained by using a finer grid. It is more difficult to overcome the error of estimating using foil, but other materials could be used instead.

Problem B4. Answers will vary, but you will probably find that the approximate areas are different for each type of grid. Again, measurement is not exact and is subject to error, including error introduced by the method of measurement itself.

But more importantly, you will probably notice that the inner and outer measures are closer together on the finer grid paper. This means that your answer will be closer to the actual area. By using smaller units, we can increase the precision of our measurements and get better approximations.

Solutions, cont'd.

Problem B5. The units are in milliliters. Yes, this unit can be used to measure the volume of a solid because of the relationship between a milliliter and a cubic centimeter. The number of milliliters of displacement will be equal to the number of cubic centimeters of volume for the rock.

Problem B6. Probably not. Since rice is solid, it will not completely fill the container, and several measurements of the same amount of rice will likely have different results. Additionally, since we are measuring twice with two different methods, there is very likely to be a measurement error between the two.

Problem B7. You can determine a measure (i.e., an approximation) for the weight of your rock relative to other known weights, but not the exact weight of the rock.

Problem B8. The three-arm balance works on the lever principle, in which moving a weight farther from a balance point produces a greater force on that side of the balance. (This is the same principle used in balancing a seesaw.) We can determine the rock's approximate weight using this type of scale, but not the exact weight.

Problem B9. Mass is a measure of the amount of material making up an object (specifically, its molecules). All objects have mass, but not all have weight, which is the effect of a gravitational field on a body that has mass. For example, a U.S. flag placed on the Moon has the same mass as one placed on the Earth, but it weighs less on the Moon as a result of the Moon's lower gravitational pull. Objects can be weightless, but they can never be without mass.

Problem B10. The precision is based on how fine the measuring instrument is. In a two-pan balance, precision is based on the values of the pan weights being used. The smaller the value of the unit, the more precise the measurement. For example, measurements made using milligrams are more precise than those using grams or kilograms.

Problem B11. The type of measure you use depends on what you're really looking for, since there is no absolute meaning of "largest." In a group of rocks, one may have the greatest surface area, another may have the greatest volume, and a third may have the greatest weight. The meaning of "largest" depends on circumstances and the judgement of those involved in the decision-making process. So, for example, if you decide that the largest rock is the heaviest rock, you would use a scale, rather than the tinfoil-and-grid-paper or water-displacement methods.

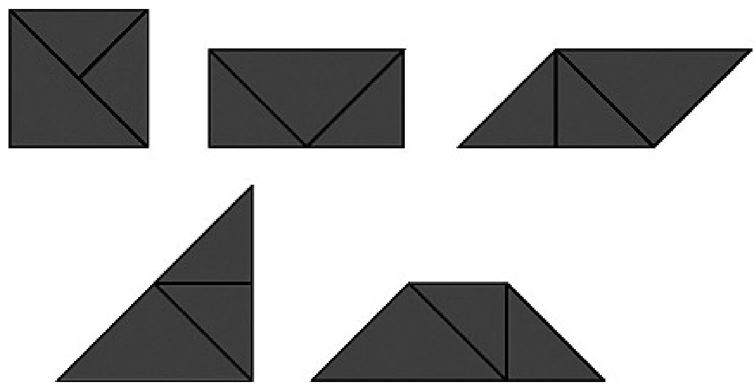
Problem B12. The fault in this line of reasoning is that your rock does not have the same density as water. For most rocks, 1 cm^3 of rock weighs more than 1 cm^3 of water. This can be seen by noting whether the rock sinks or floats when placed in water. If it sinks, it is denser than water and has more weight than the same volume of water.

Problem B13. You can choose to measure length in any direction, once the rock is placed in a particular orientation. Or you might choose to measure the circumference of the rock. As with surface area, volume, and weight, none of these linear measurements could be used by itself to determine if a rock is "largest"—unless that's your only criterion!

Solutions, cont'd.

Part C: Nonstandard Units

Problem C1.



Problem C2. All five polygons have the same area, since they are made up of the same three smaller polygons. Since there is no overlapping, the area of all five is the same as the sum of the area of the three triangles.

Problem C3. First, you need to arbitrarily choose a unit with which to measure the perimeters. One way to do this is to choose the length of a leg of the smaller triangle as your unit. Whatever unit you choose, you will discover that the square has the smallest perimeter. The rectangle's perimeter is slightly larger. The parallelogram, triangle, and trapezoid are tied for the largest perimeters.

Problem C4. Standard units provide a frame of reference that can always be relied on. Although we can say, "This one is three times longer than that one," standard units provide everyone with an equivalent value for "that one." With standard units, anyone measuring the same item could say, "This one is six times longer than 1 cm."

Problem C5. Small triangle: Using the Pythagorean theorem to determine the length of the hypotenuse, we can write the following:

$c^2 = a^2 + b^2 = 1^2 + 1^2 = 2$, or

$c = \sqrt{2}$

The length of the hypotenuse is $\sqrt{2}$, or approximately 1.414 units.

Medium triangle: The length of the legs is equal to the hypotenuse of the smaller triangle, or $\sqrt{2}$. So, to determine the hypotenuse, we can write the following:

$c^2 = a^2 + b^2 = \sqrt{2^2} + \sqrt{2^2} = 4$, or

$c = 2$

So the length c is equal to 2 units.

Solutions, cont'd.

Problem C5, cont'd.

Remember, our unit is the length of one leg of the small triangle. So the perimeters are as follows:

Square:

$4 \cdot \sqrt{2}$, or approximately 5.656 units

Rectangle that is not a square:

$2 \cdot 1 + 2 \cdot 2 = 6$ units

Parallelogram that is not a rectangle:

$2 \cdot \sqrt{2} + 2 \cdot 2$, or approximately 6.828 units

Triangle:

$2 + 2 + (2 \cdot \sqrt{2})$, or approximately 6.828 units

Trapezoid:

$2 \cdot \sqrt{2} + 1 + 3$, or approximately 6.828 units

Homework

Problem H1. According to Maria's criterion, here are the values:

Name (Country/Continent)	Height (cm) + Weight (kg)
Water Buffalo (Asia)	1,165
African Buffalo	695
Yak (Tibet)	750
Guar (Asia)	1,070
American Bison (North America)	1,180
European Bison	1,100

The three largest animals by this criterion are the American bison, the water buffalo, and the European bison.

Note that this method is somewhat problematic since the answers could change drastically if we change the units of measurement. For example, the results would look quite different if we measured weight in tons (1 metric ton = 1,000 kg) instead of in kilograms. Clearly, this can lead to potential errors.

Solutions, cont'd.

Problem H2. Here are the ranks according to Jacob's criterion:

Name (Country/Continent)	Height Rank	Weight Rank	Total
Water Buffalo (Asia)	2	5.5 (tie)	7.5
African Buffalo	1	2	3
Yak (Tibet)	4.5 (tie)	1	5.5
Guar (Asia)	6	3	9
American Bison (North America)	3	5.5 (tie)	8.5
European Bison	4.5 (tie)	4	8.5

The three largest animals by this criterion are the guar, the American bison, and the European bison.

Problem H3. Here are the products for each animal:

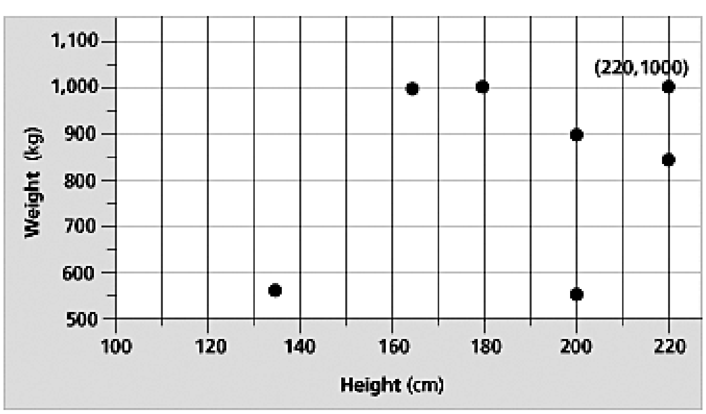
Name (Country/Continent)	Height (cm) • Weight (kg)
Water Buffalo (Asia)	165,000
African Buffalo	75,600
Yak (Tibet)	110,000
Guar (Asia)	187,000
American Bison (North America)	180,000
European Bison	180,000

The three largest animals by Quentin's criterion are the guar, the American bison, and the European bison.

Problem H4. These two methods result in the same ordering of the animals, but some may argue that Quentin's method is more capricious than Jacob's, since there is no natural meaning to multiplying height and weight.

Problem H5.

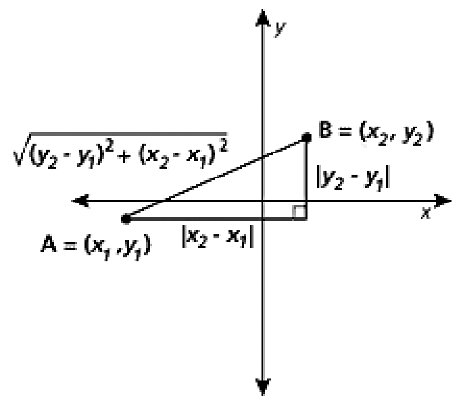
a.



Solutions, cont'd.

Problem H5 cont'd.

b. Using the Pythagorean theorem, we can derive a formula for the distance between two points A (x_1,y_1) and B (x_2,y_2) as
$D = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$.



Next we can calculate the distances. So for the Asian water buffalo, $D = \sqrt{(1{,}000 - 1{,}000)^2 + (220 - 165)^2} = \sqrt{0 + 55^2} = 55$. Here is the completed table:

Name (Country/Continent)	Coordinates	Distance from (220,1000)
Water Buffalo (Asia)	(165,1000)	55
African Buffalo	(135,560)	448.13
Yak (Tibet)	(200,550)	450.44
Guar (Asia)	(220,850)	150
American Bison (North America)	(180,1000)	40
European Bison	(200,900)	101.98

To learn more about the Pythagorean theorem and distance formula, go to www.learner.org/learningmath and find *Geometry,* Session 6.

Problem H6. The three species closest to (220,1000) are the American bison, the water buffalo, and the European bison.

Problem H7. Here is the completed table:

Name (Country/Continent)	Coordinates	Distance from (0,0)
Water Buffalo (Asia)	(165,1000)	1,013.52
African Buffalo	(135,560)	576.04
Yak (Tibet)	(200,550)	585.23
Guar (Asia)	(220,850)	878.01
American Bison (North America)	(180,1000)	1,016.07
European Bison	(200,900)	921.95

According to Marcy's criterion, the three largest animals are the American bison, the water buffalo, and the European bison.

Problem H8. Answers will vary. Another possible method is to use weight as the first determining factor and to use height only in case of a tie.

Problem H9. Answers will vary, but the most likely answer is the African buffalo, which is the shortest and nearly the lightest. It wins easily on Jacob's criterion and is the smallest for many others.