

Annenberg/CPB Course Guide

Learning Math

# Measurement

A 10-part video- and Web-based course for K–8 teachers

**Produced by WGBH Educational Foundation**

## ***Learning Math: Measurement***

is produced by  
WGBH Educational Foundation

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# Table of Contents

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<b>Introduction</b> .....	1
<b>Session 1. What Does It Mean To Measure?</b> .....	9
Tips.....	24
Solutions .....	25
<b>Session 2. Fundamentals of Measurement</b> .....	31
Tips.....	46
Solutions .....	47
<b>Session 3. The Metric System</b> .....	53
Tips.....	63
Solutions .....	64
<b>Session 4. Angle Measurement</b> .....	69
Tips.....	86
Solutions .....	87
<b>Session 5. Indirect Measurement and Trigonometry</b> .....	93
Tips.....	109
Solutions .....	110
<b>Session 6. Area</b> .....	117
Tips.....	132
Solutions .....	133
<b>Session 7. Circles and Pi (<math>\pi</math>)</b> .....	141
Tips.....	153
Solutions .....	154
<b>Session 8. Volume</b> .....	159
Tips.....	170
Solutions .....	171
<b>Session 9. Measurement Relationships</b> .....	175
Tips.....	187
Solutions .....	188

# Table of Contents, cont'd.

---

## Select Your Grade Band:

<b>Session 10, Grades K–2.</b> Classroom Case Studies.....	195
Solutions .....	202
<b>Session 10, Grades 3–5.</b> Classroom Case Studies .....	205
Solutions .....	212
<b>Session 10, Grades 6–8.</b> Classroom Case Studies .....	217
Solutions .....	223
<b>Appendix</b> .....	A-1
Glossary .....	A-2
Credits.....	A-5

# About This Course

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## The *Learning Math* Courses

When teachers have a deep conceptual understanding of mathematics, it can help their students develop strong mathematical skills and knowledge. *Learning Math* is a series of five multimedia, college-level courses designed to teach mathematics content to elementary and middle school teachers. Organized around the content standards developed by the National Council of Teachers of Mathematics (NCTM), the courses cover Number and Operations; Patterns, Functions, and Algebra; Geometry; Measurement; and Data Analysis, Statistics, and Probability.

### Overall Objectives

- To help teachers better understand mathematics content
- To provide engaging explorations of mathematics using video, interactive activities, and problem solving
- To encourage teachers to view mathematics as more than rote sets of rules and procedures

## Overview: *Measurement*

***Learning Math: Measurement***, a video- and Web-based course for elementary and middle school teachers, examines some of the major ideas in measurement. You will explore procedures for measuring and learn about standard units in the metric and customary systems, the relationships among units, and the approximate nature of measurement. You will also examine how measurement can illuminate mathematical concepts such as irrational numbers, properties of circles, or area and volume formulas, and discover how other mathematical concepts can inform measurement tasks such as indirect measurement.

The course consists of 10 sessions, each with a half hour of video programming, problem-solving activities provided online and in a print guide, and interactive activities and demonstrations on the Web. Although each session includes suggested times for how long it may take to complete all of the required activities, these times are approximate. Some activities may take longer. You should allow at least two and a half hours for each session. The 10th session (choose video program 10, 11, or 12, depending on your grade level) explores ways to apply the concepts of measurement you've learned in your own classroom. You should complete the sessions sequentially.

### Session 1. What Does It Mean To Measure?

Explore what can be measured and what it means to measure. Identify measurable properties such as weight, surface area, and volume, and discuss which metric units are more appropriate for measuring these properties. Refine your use of precision instruments, and learn about alternate methods such as displacement. Explore approximation techniques, and reason about how to make better approximations.

### Session 2. Measurement Fundamentals

Investigate the difference between a count and a measure, and examine essential ideas such as unit iteration, partitioning, and the compensatory principle. Learn about the many uses of ratio in measurement and how scale models help us understand relative sizes. Investigate the constant of proportionality in isosceles right triangles, and learn about precision and accuracy in measurement.

### Session 3. The Metric System

Learn about the relationships between units in the metric system and how to represent quantities using different units. Estimate and measure quantities of length, mass, and capacity, and solve measurement problems.

# About This Course, cont'd.

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## **Session 4. Angle Measurement**

Review appropriate notation for angle measurement, and describe angles in terms of the amount of turn. Use reasoning to determine the measures of angles in polygons based on the idea that there are 360 degrees in a complete turn. Learn about the relationships among angles within shapes, and generalize a formula for finding the sum of the angles in any  $n$ -gon. Use activities based on Geo-Logo to explore the differences among interior, exterior, and central angles.

## **Session 5. Indirect Measurement and Trigonometry**

Learn how to use the concept of similarity to measure distance indirectly, using methods involving similar triangles, shadows, and transits. Apply basic right-angle trigonometry to learn about the relationships among steepness, angle of elevation, and height-to-distance ratio. Use trigonometric ratios to solve problems involving right triangles.

## **Session 6. Area**

Learn that area is a measure of how much surface is covered. Explore the relationship between the size of the unit used and the resulting measurement. Find the area of irregular shapes by counting squares or subdividing the figure into sections. Learn how to approximate the area more accurately by using smaller and smaller units. Relate this counting approach to the standard area formulas for triangles, trapezoids, and parallelograms.

## **Session 7. Circles and Pi ( $\pi$ )**

Investigate the circumference and area of a circle. Examine what underlies the formulas for these measures, and learn how the features of the irrational number pi ( $\pi$ ) affect both of these measures.

## **Session 8. Volume**

Explore several methods for finding the volume of objects, using both standard cubic units and non-standard measures. Explore how volume formulas for solid objects such as spheres, cylinders, and cones are derived and related.

## **Session 9. Measurement Relationships**

Examine the relationships between area and perimeter when one measure is fixed. Determine which shapes maximize area while minimizing perimeter, and vice versa. Explore the proportional relationship between surface area and volume. Construct open-box containers, and use graphs to approximate the dimensions of the resulting rectangular prism that holds the maximum volume.

## **Session 10. Classroom Case Studies**

Explore how the concepts developed in this course can be applied at different grade levels. Examine case studies of K–2, 3–5, and 6–8 teachers (former course participants, all of whom have adapted their new knowledge to their classrooms), as well as a set of typical measurement problems for these levels of students. In this session, view video 10 for K–2 teachers, video 11 for grade 3–5 teachers, and video 12 for grade 6–8 teachers.

# About This Course, cont'd.

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## Course Components

Each *Learning Math* course consists of 10 two-and-a-half-hour sessions. The first nine sessions are devoted to mathematics content; the 10th session covers classroom applications. Concepts are developed within and across the sessions. Each session includes reading, problem solving, and group or individual activities that are available on the Web and in print, and a half-hour of video viewing, available on the Web,\* on the Annenberg/CPB Channel,\*\* or on videocassette.\*\*\* There are additional problems and readings to complete for homework.

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\* Broadband access is required to view the video on the Web; see Tech Tips, page 6.

\*\* The schedule for broadcasts on the Annenberg/CPB Channel can be found on the course Web site.

\*\*\* Purchase videocassettes at [www.learner.org](http://www.learner.org) or by calling 1-800-LEARNER.

**Visit the *Measurement* Web site at [www.learner.org/learningmath](http://www.learner.org/learningmath).**

# About This Course, cont'd.

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The following components are in each course:

## Key Terms

Key mathematical terms relevant to each session are listed at the beginning of that session. These terms are divided into two parts: terms that are new in that session and terms that were introduced in a previous session. Definitions for key terms may be found in the glossary in the Appendix of this guide.

## Notes

Notes can be used by facilitators, study groups, or individuals working alone. They provide extended information about the topics presented in the course, including help for dealing with stumbling blocks that may come up and recommendations for different ways to approach the content.

## Problems

Each session contains mathematical problems to be solved individually or by groups. Problems build upon previous concepts as the course progresses.

## Take It Further

The problems marked "Take It Further" are optional and are not counted as part of the two-and-a-half hour time-frame for each session. These problems are designed for individuals who would like to explore a topic in greater depth. They are often more difficult than the other problems in the session, and they may introduce new information or concepts not previously discussed.

## Interactive Activities

Each session in the course includes at least one interactive activity on the course Web site. These activities help you learn new mathematics content or reinforce existing knowledge through hands-on exploration directly on the Web. The interactive activities require the Flash plug-in, which you can download for free from Macromedia's Web site (see Tech Tips, page 6). There are also non-Flash versions of each activity that don't require the Flash plug-in and can be completed offline. If you are working with only the guide, the interactive activities have been adjusted and are included in the guide.

## Tips

Tips are available for problems you may find more difficult or need help in getting started. Tips may be found at the end of each session in this guide.

## Solutions

A solution is provided for every problem in *Learning Math*, with the exception of a few open-ended questions. When solving a problem with multiple parts, consider writing down your answers to all of the parts on paper first before checking the solution, because the answers to each part of the problem will be visible at once on the solution page. Solutions may be found at the end of each session in this guide.

The following sequence of activities will give you a sense of what you will do as a student using *Learning Math*:

1. Watch the **session video** in its entirety. You can watch the video before you begin the session to become comfortable with the material, or you can view the video after completing the session (so as not to view answers to problems).
2. Do problems sequentially. If you are having difficulty, refer to the **Tip**. If you want a challenge, try a **Take It Further**.
3. Check the **Solutions** at the end of each session.
4. You may want to read **Notes** as you go along to establish a deeper context for the content.
5. Watch **video segments** strategically placed throughout the session, either online by clicking on the "Play Video" button or on videocassette by fast-forwarding to the image and approximate time code that appears in the guide. Zero your VCR clock when the Annenberg/CPB logo appears at the beginning of the program to locate the image using the time code.
6. Do **homework problems** and **readings** (available as PDF files online) at the end of each session to reinforce your learning.



# About This Course, cont'd.

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## Session Videos

Each *Learning Math* session includes viewing a video that is available on the course Web site, on videotape, or on the Annenberg/CPB Channel. The videos feature K-8 teachers working on the *Learning Math* course materials in a workshop with a facilitator. The videos for the nine content sessions show onscreen participants as they are introduced to the concepts, work through the problems, sometimes struggle to reach an understanding, and then reflect on what they have learned. At the end of most videos there is an example of how the content from the session is applied in a “real world” situation. The videos for the 10th session show participants from the videotaped workshops as they apply the content that they have learned back in their own classrooms. You may choose to watch each of these videos before or after you work on the associated course session.

## Video Segments

Each session includes short excerpts from the associated video, which you watch (or review) and reflect on to see how the onscreen participants grapple with the same or similar problems and concepts you encounter in the course. Instructions are given to find the segments on videotape. The segments are also available on the course Web site, if you are watching the complete programs online or on the Annenberg/CPB Channel.

## Homework

Each session includes approximately 45 minutes of homework problems and/or reflective writing assignments that reinforce the session’s content.

## Readings

Readings from journals and books are cited at the end of some sessions. They are available on the *Measurement* Web site as downloadable PDF files. Go to [www.learner.org/learningmath](http://www.learner.org/learningmath). If you do not have access to the Internet, call 1-800-LEARNER to obtain a free set of readings.

## Using the Videos, Guide, and Web Site

Each *Learning Math* course includes sequentially organized problems, video viewing, interactive activities, readings, and homework. The multimedia elements of the course create an exciting environment for probing mathematics content. The course can be taken entirely on the Web, followed in this print guide, or completed using a combination of Web and print. You can watch the videos online, on the Annenberg/CPB Channel, or on videocassette. If you are watching the programs on the Channel, we recommend taping them so you can look at short video segments when prompted in each session. These segments punctuate concepts developed in the course and create a “virtual” community of learners.

## Ways To Take *Learning Math*

*Learning Math* was flexibly designed for a variety of users and situations. You may choose to work through the sessions on your own, in a study group, or as part of a facilitated, face-to-face, graduate-level course for credit.

## Channel-Talk

Join an email discussion group and converse with other teachers taking this course. Go to the course Web site at [www.learner.org/learningmath](http://www.learner.org/learningmath) and select Channel-Talk.

## Registration and Credit

Go to [www.learner.org/4gradcredit](http://www.learner.org/4gradcredit) for details on receiving graduate credit for *Learning Math*.

# About This Course, cont'd.

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## Taking Multiple *Learning Math* Courses

The five *Learning Math* courses are designed to be independent of one another. You can take just one course, several courses, or all five courses in the order that fits your needs or the needs of your group. The courses also complement one another, with some topics discussed in more than one course but approached differently depending on the focus of that course. Taking several of these courses will increase your own conceptual understanding and ultimately that of your students.

## Facilitating the Course

You can prepare for facilitating the course by reading through each session and its "Notes" section prior to meeting with your group. Reading through the material will help you become clear about the activities, plan how much time you need to spend on each one, and pull together necessary materials. The course is designed for use by an individual, but the Notes may suggest ways for groups to work through the sessions.

## Tech Tips

To use all or portions of the *Learning Math* course online, we recommend the following:

### Internet Access

A minimum 56K modem connection is required, but an ISDN or high-speed connection is recommended. The slower your connection, the longer it will take to load larger features, such as the Flash activities.

To view the video programs and video segments online, a broadband connection to the Internet (DSL, cable modem, or LAN connection to a T1 line or greater) is required.

### Web Browser

You will need Netscape 4.6 (or a higher version) or Internet Explorer 5.0 (or a higher version). Javascript should be enabled, if your browser allows you to disable it. Text fonts and colors, and many features, such as hidden Tips, may not be displayed correctly in older browsers.

### Plug-Ins

These plug-ins will allow you to get the most out of the courses. Links to sites where you can download these programs for free are provided on the course Web site.

- Shockwave Flash 5 (or a higher version) for using the interactive activities
- Windows Media Player 7 (or a higher version) for watching videos
- Adobe Acrobat Reader for viewing the Readings in the Homework sections

### Printing Web Pages

If you are having trouble printing some of the course content pages, you may try doing one or more of the following (from the "Print Preview," "Print...," or "Page Setup..." menu):

- Turn on "Shrink to Fit" mode (IE 5 only)
- Print the page in "Landscape" mode
- Reduce the scale of the printer output
- In the *Measurement* course, certain PDF documents are drawn to scale. When printing them, be sure that the Acrobat print dialog box settings for page scaling or shrinking oversized pages are turned off.

# Who's Who

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## Content Developer/Facilitator



### Suzanne Chapin, Ed.D.

Dr. Chapin is a professor of mathematics education at Boston University where she teaches graduate- and undergraduate-level courses in mathematics curriculum, mathematics content and methodology, mathematics for special needs students (gifted and learning disabled), and educational reform. Dr. Chapin has been the principal investigator on a number of projects, including Project Challenge (Jacob K. Javits Gifted & Talented Students Education Program) and Partners in Change Project (U.S. DOE), and has worked with many colleagues as an investigator or consultant on grant-related research. Dr. Chapin is author or co-author of many mathematics textbooks and programs including the Prentice Hall grades 6-8 textbook series, *Middle Grade Mathematics: Tools for Success*; *MEGA Projects - Math Explorations and Group Activities* for students in grades 1-8; *Algebra* and *Advanced Algebra* for students in grades 8-12; and *Classroom Discussions: Using Math Talk To Help Students Learn*. She also is the senior author on *Math Matters: Understanding the Math You Teach, Grades K-6*. Dr. Chapin is a frequent speaker at national meetings of mathematicians, mathematics educators, researchers, and policy makers.

## Onscreen Participants



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Teacher, Grade 6



Jonathan Belber  
Math Teacher, Grade 4



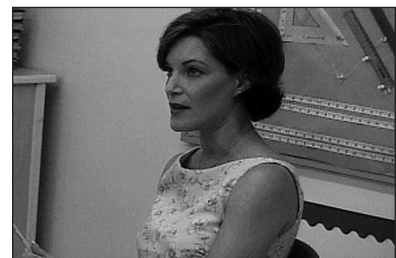
Jayne Byrne  
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Teacher, Grade 1

# Who's Who, cont'd.

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Tracy Feeney  
Teacher, Grade 6



Mary Guerino  
Teacher, Grades 1–2



Celeste Janey  
Special Education Teacher,  
Grades K–2



Katy McGraw  
Math Specialist, Grades 3–4



Lori Monterotti  
Math Specialist, Grade 4



Susan Pagliaro  
Math Teacher, Grade 8



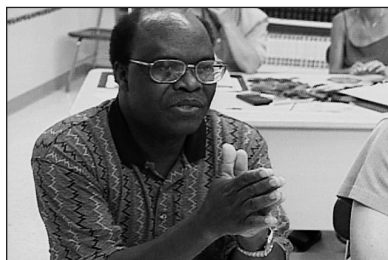
Rosalie Paillard  
Teacher, Grade 1



David Russell  
Teacher, Grades 6–7



Laura Trabucco  
Math Teacher, Grade 8



Lombi Zuendoki  
Math Teacher, Grade 7