Session 10, Grades K–2

Classroom Case Studies

This is the final session of the *Measurement* course! In this session, we will examine how measurement concepts from the previous nine sessions might look when applied to situations in your own classroom. This session is customized for three grade levels. Select the grade level most relevant to your teaching.

The session for grades K–2 begins below. Go to page 205 for grades 3–5 and page 217 for grades 6–8.

Key Terms in This Session

Previously Introduced

- area

Introduction

In the previous sessions, we explored many different topics related to measurement. You put yourself in the position of a mathematics learner, both to analyze your individual approach to solving problems and to get some insights into your own understanding of measurement topics. It may have been difficult to separate your thinking as a mathematics learner from your thinking as a mathematics teacher. Not surprisingly, this is often the case! In this session, however, we will shift the focus to your own classroom and to the approaches your students might take with mathematical tasks involving measurement. **[See Note 1]**

Learning Objectives

In this session, you will do the following:

- Explore the development of the concept of area in the early grades
- Investigate instructional tasks on area that are developmentally appropriate for young learners
- Examine young children's understanding of area
- Explore how you might teach related measurement topics

Note 1. This session uses classroom case studies to examine how children in grades K–2 think about and work with measurement concepts. If you are taking this course on your own, you may want to share your observations on Channel Talk or ask some of your colleagues for their input. Using the classrooms of fellow teachers as well as your own as case studies will allow you to make additional observations.

Part A: The Concept of Area (25 min.)

To begin exploring what the teaching of measurement might look like in the classroom, participants in the *Measurement* course first reexamined the big ideas around one topic: area. They considered how children make sense of these ideas and discussed ways to present these concepts to young students.

Video Segment (approximate time: 1:15-2:58): You can find this segment on the session video 1 minute and 15 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, Dr. Chapin reviews four big ideas of area. To plan lessons that will help students understand area concepts, it is essential that teachers consider these and other big mathematical ideas when designing instructional sequences.

Problem A1. Answer the questions based on what you saw in the video:

a. What is one of the fundamental concepts of area? Why?

b. What vocabulary must students understand to make sense of area?

c. Are there related concepts or skills that will affect whether or not students can make sense of these ideas?

d. Thinking back to the big ideas of this course, what are some other ideas that young students should encounter to help extend and deepen their understanding of the topic?

Problem A2. Choose one of the concepts that you listed for Problem A1 and describe an instructional activity that you might use to help students grasp that concept.

Video Segment (approximate time: 3:30-6:20): You can find this segment on the session video 3 minutes and 30 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, kindergarten, first-, and second-grade teachers discuss some of the important ideas about area. One teacher mentions that the conservation of area has to be considered when teaching young children.

Problem A3. What types of experiences engage students in thinking about the conservation of area?

Join the Discussion! **www.learner.org**

Post your answer to Problem A3 on an email discussion list; then read and respond to answers posted by others. Go to the *Measurement* Web site at www.learner.org/learningmath and find Channel Talk.

Part B: Reasoning About Measurement (40 min.)

Exploring Standards

The National Council of Teachers of Mathematics (NCTM, 2000) has identified measurement as a strand in its *Principles and Standards for School Mathematics*. In grades pre-K–12, instructional programs should enable all students to do the following:

- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements

In grades pre-K–2 classrooms, students are expected to do the following:

- Recognize the attributes of length, volume, weight, area, and time
- Compare and order objects according to these attributes
- Understand how to measure using nonstandard and standard units
- Select an appropriate unit and tool for the attribute being measured
- Measure with multiple copies of units of the same size, such as paper clips laid end to end
- Use repetition of a single unit to measure something larger than the unit; for instance, measuring the length of a room with a single meterstick
- Use tools to measure
- Develop common referents for measures to make comparisons and estimates

The NCTM (2000) Measurement Standards suggest that students will begin to understand attributes of area by "looking at, touching, or directly comparing objects." Instruction that provides students with opportunities to measure and explain their findings will help teachers further this goal. Furthermore, "although for many measurement tasks students will use nonstandard units, it is appropriate for them to experiment with and use standard measures such as centimeters and meters and inches and feet by the end of grade 2" (NCTM, 2000, p. 105).

Analyzing a Case Study

To begin the exploration of what measurement topics look like in a classroom at your grade level, you will watch a video segment of a teacher who took the *Measurement* course and then adapted the mathematics to her own teaching situation. We will begin by looking at some of the content addressed in the videotaped lesson. **[See Note 2]**

Note 2. The purpose of the video segments is not to reflect on the teaching style of the teacher portrayed. Instead, look closely at the methods the teacher uses to bring out the ideas of measurement while engaging her students in activities.

Part B, cont'd.

Video Segment (approximate time: 11:42-16:35): You can find this segment on the session video 11 minutes and 42 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

Watch this video segment from Ms. Guerino's class and think about how both the lesson and the teacher are assisting students in making sense of some of the measurement concepts described in Part A.

Problem B1. Prior to this lesson, students worked with nonstandard units to measure area. What might students learn from measuring objects using units like circular discs and paper clips?

Problem B2. Before her students determine which rectangle has a larger area, Ms. Guerino asks them to predict which one is larger. Where in the lesson are students using estimating techniques to decide which rectangle is larger? Why is it important to have students make estimates before they actually begin measuring?

Problem B3. Why do you think Ms. Guerino asked students to measure the two shapes using both square inches and square centimeters? How might discussion among students and the teacher contribute to students' understanding of conservation of area? What other measurement concept was Ms. Guerino exploring when she asked students to cover the shapes with different-sized units?

Problem B4. Is it important to have misconceptions surface in a lesson? If misconceptions do occur, how can they be addressed?

Join the Discussion! **www.learner.org**

Post your answer to Problem B4 on an email discussion list; then read and respond to answers posted by others. Go to the *Measurement* Web site at www.learner.org/learningmath and find Channel Talk.

Part C: Activities That Illustrate Measurement Reasoning (55 min.)

In this part, you'll look at several short activities that are appropriate for students in grades K–2. As you read through the activities, answer the following questions:

a. What is the measurement content in the activity? What are the big ideas that you want students to consider and understand?

b. What prior knowledge is required? What content does the activity prepare students for later?

c. How does the content in this activity relate to the mathematics ideas in this course?

d. What other questions might extend students' thinking about the activity?

e. What other instructional activities might you use in conjunction with this one to further your content goals?

Problem C1.

EXPLORING CAPACITY

Activity Summary

Students recognize situations that involve capacity and compare capacities of different containers.

Materials Needed:

- Containers of different sizes and shapes (including measuring spoons)
- Water, rice, or sand
- Pictures that illustrate capacity situations

Show students a variety of containers and ask them what types of things we might use to fill the containers. What might we measure using these containers? Next, show students pictures that illustrate capacity situations, such as a bottle of milk, a box, a sack of rice, a fish tank, and a swimming pool. For each picture, ask the students to describe what they could fill the object with. Be sure to show the students objects or pictures of objects that cannot be filled—a square, a rock, a piece of string. You may want to start using the term capacity, which refers to the available space inside a container, in your discussion. But don't expect your students to become comfortable with this term following just one lesson.

Work with a small group of students at a time, either at the sink or at the sand table. Provide them with a number of containers (use more containers with older students). Then ask them to predict which container holds the most and which holds the least, but do not expect students to be able to determine the greatest capacity merely by looking at the containers. Most students will need to pour materials from one container to another before they can make any sort of prediction.

Following experimentation with many containers, choose three or four containers that are different in height and diameter of base. For example, try to find three or four cans: a short, squat can; a tall, skinny can; and cans that are somewhere in between. Or use three rectangular prisms that differ in height. Have students predict which container holds the most and which holds the least, and then have them use filling (rice, sand, etc.) to put the containers in order from largest to smallest.

After all the groups have had an opportunity to work on the task, conduct a discussion about the results. Ask students to share what they discovered. Which can held the most, and which held the least? Ask the students how they arrived at their conclusions. Did tall cans or prisms always hold the most? What types of containers hold a lot of a particular filling, and what types hold very little? Continue to use the word capacity, and encourage students to talk about the capacity of the cans.

Part C, cont'd.

Problem C2.

COMPARING DISTANCES

Activity Summary

Students indirectly compare distances traveled by toy cars. Older students use nonstandard and standard units to measure the distances.

Materials Needed:

- Toy cars and ramp
- Paper tape and scissors
- Links, multilink cubes, or some other object to use as a nonstandard unit
- Inch rulers and yardsticks

[See Note 3]

Explain that today the students are going to compare the distances different toy vehicles travel. Have students work with a partner, and have each pair choose a small toy car to use. Each pair of students will release their toy vehicle from the starting line at the top of a ramp and then use a piece of paper tape to measure the distance the car traveled.

If you are working with younger students, you may wish to have them write their names on the end of the lengths. These lengths can then be taped to a bulletin board to make a bar graph. Conduct a discussion about the graph. In particular, ask students to compare the distance their cars traveled. Whose cars went the farthest? Whose cars went the shortest distance? How can we tell which cars traveled farther than Anita's (pick a distance in the middle of the group) by looking at the graph? Can we tell how much farther one car went than another? Depending on the toy vehicles used, you may find that the heavier cars traveled the greatest distance.

Older students, or those who are ready to use numbers, can determine the distances the cars traveled by using nonstandard units, standard units, or both. Students write the number of units on the tape prior to making the bar graph. When discussing the graph, they can use either the lengths of the tapes and/or the number of units to determine which car went the farthest. Furthermore, if both nonstandard and standard units were used to measure the distances, this is a great time to discuss why the number of units is not the same (e.g., why the car went 65 inches but not 65 links) for both measures.

When students are measuring with units, notice how they approach the task. Do they place units end to end? Do they use iteration of one unit, or do they use rulers and yardsticks to measure? If using nonstandard units, it is easier to use units such as links or multilink cubes that can be connected together. During the measurement process is the perfect time to give students individualized instruction on how to measure accurately and precisely.

Note 3. Multilink cubes are cubic units that can be connected together (cubic centimeters and 3/4 cubic inches). These can be purchased from:

Delta Education, 80 Northwest Boulevard, P.O. Box 3000, Nashua, NH 03061-3000; Phone: 1-800-442-5444; http://www.delta-education.com or ETA/Cuisenaire, 500 Greenview Court, Vernon Hills, IL 60061; Phone: 800-445-5985/800-816-5050 (Customer service); Fax: 800-875-9643/847-816-5066; http://www.etacuisenaire.com

Homework

Solutions are not provided for these homework problems, since answers will vary depending on individual experiences.

Problem H1. Assume that you need to report back to your grade-level team or to the entire school staff at a faculty meeting about your experiences and learning in this course. What are the main messages about the teaching of measurement you would share with your colleagues? Prepare a one-page handout or an overhead or slide that could be distributed or shown at the meeting.

Problem H2. Look at a lesson or activity in your own mathematics program for your grade level that you think has potential for developing students' reasoning about measurement. If you were to use this lesson or activity now, after taking this course, how might you modify or extend it to bring out more of the important concepts about measurement?

Solutions

Part A: The Concept of Area

Problem A1.

a. A fundamental concept of area is that it is the measure of how much surface is covered. For students of this age, understanding what it means to cover a surface completely with a particular unit is central. Some other concepts of area are as follows: a) some shapes cover a surface more completely than other shapes; b) the units associated with area measurement are square units; and c) the smaller the square unit, the more square units are needed to determine the area.

b. To make sense of area, students need to be familiar with vocabulary such as surface, covering, and squares. Vocabulary that students will acquire as they explore area include unit and square unit.

c. Students who can recognize a square and the two-dimensionality of a square and who also have a solid understanding of rectangles are in good shape to tackle the concept of area. Students will also need prior experience covering surfaces with different objects to know that some shapes fit together with no holes or gaps (e.g., rectangles and triangles), while other shapes leave holes (e.g., circles).

d. Some other ideas, in addition to the ones already mentioned, include conservation and transitivity. Students need to learn that the area of a shape will not change if it is moved to a different position, or if it is cut and transformed in a certain way. They also need to understand that when you can't compare two objects directly, you can compare them by means of a third object.

Problem A2. Answers will vary. The following activity can be used to help students start thinking about some of the concepts of area: On a large piece of paper, draw several shapes of different sizes. Then ask students to cover one of these shapes with different pattern blocks. (Pattern blocks are a commercial product found in most primary classrooms that consist of blocks in six shapes—triangles, squares, hexagons, trapezoids, and two rhombuses.) Have students cover one shape at a time and count the number of blocks needed to cover each shape. Students will find that they need more smaller blocks than larger blocks to cover the shape on the paper. This helps students start to internalize the idea that the size of the unit (in this case, pattern blocks) affects the number of units needed to cover a surface. It also adds to children's development of the idea of area as a covering with no holes or gaps.

Problem A3. During their study of area, many students will be challenged by the idea of conservation. Conservation is the principle that an object maintains the same size and shape even if it is repositioned or divided in certain ways. But at the heart of teaching this principle, as one teacher said, is the notion of taking an abstract idea like area and making it concrete or tactile for young children. Students should have many experiences with determining area by tiling shapes or figuring out how much surface a shape takes up. Understanding that a shape will preserve its area regardless of its orientation is also an important first step.

Part B: Reasoning About Measurement

Problem B1. Measuring with nonstandard units provides students the opportunity to choose the unit they want and to gain a sense of the physical process of a measuring experience. When students use nonstandard units (e.g., paper clips, playing cards, or buttons) to measure area, they notice that they can't cover the surface completely. The other difficulty with using nonstandard measures is that students can't compare the measures; for example, 110 paper clips vs. 12 playing cards.

Problem B2. Students made predictions about whether the rectangle or square was larger at the start of the lesson. Doing so helps them consider which attributes are important as well as visualize how much space the shapes take up. Students may even visualize cutting up one shape to cover the other. In the video, one group of students compares the dimensions of the two shapes. The students in this group notice that the length of the rectangle is longer than that of the square, and then they use finger span to estimate that the heights of both the rectangle and square are the same. They conclude that the rectangle must be larger.

Problem B3. Ms. Guerino asked students to use both square inches and square centimeters to measure the shapes so that she could address the idea of conservation. Students may not realize that the areas will stay the same even if they measure with different-sized units.

To help students understand conservation, ask them to predict which rectangle will have the largest area before they cover the shapes with the new unit. Also, asking students to explain why the larger shape is still larger, and if it will always be larger, can help them reason about conservation.

Another measurement concept that Ms. Guerino explored when she asked students to cover the shapes with different-sized units is the concept that the smaller the unit, the greater the number needed to cover the shapes.

Problem B4. It is not only important, but actually helpful when misconceptions surface in a lesson. Misconceptions often provide the contrast needed for students to understand an idea. One strategy to use when misconceptions arise is to present two opposing viewpoints and ask students which idea they agree with and why. You can also ask students to give a counterexample, or give the students a counterexample yourself and ask them what they think. Another reason why it is important to discuss misconceptions: By confronting a conception that turns out to be incorrect, students become engaged in understanding why it is a misconception.

Part C: Activities That Illustrate Measurement Reasoning

Problem C1.

a. In this lesson, students are learning about volume or capacity, making estimates, gaining a physical "sense" of capacity, and getting the physical experience of measuring. A big idea is that the capacity of a three-dimensional object, such as a cylinder, is the measure of how much "stuff" it holds. This lesson is also likely to bring out the challenge students face in understanding important ideas such as conservation and transitivity.

b. This lesson can build on similar experiences with area and prepare students for future work with two-dimensional and three-dimensional figures, such as understanding the difference between them, as well as looking at more concrete ways of working with and measuring volume (perhaps using unit cubes).

c. In the course, volume was explored extensively to a higher degree of complexity than is typical for students at this level. Transitivity, conservation, and units of measurement were explored as well.

d. If the containers have lids, you can turn them on their sides and have students consider the impact of repositioning on capacity. This is especially useful if students think "taller" cans hold the most.

e. Building on this lesson and reconnecting with lessons where rectangles are covered in tiles to determine area, you could have students fill rectangular prisms (or boxes) with centimeter (or other appropriate-sized) cubes.

Solutions, cont'd.

Problem C2.

a. In this lesson, students are learning about the concept of length, comparing lengths indirectly, and learning how to measure length with standard and nonstandard units. They are also learning about different units and unit iteration (repetition of the same unit needed to measure) as well as transitivity (measuring by making a paper tape and understanding that it's equal to the distance traveled). Finally, they are learning about bar graphs and getting some early exposure to data representation.

b. This lesson builds on students' previous experiences with ordering numbers and deepens their understanding of length or distance. For younger students, experiences such as these prepare them for measuring length with standard tools. This lesson also prepares students for considering dimensional attributes of shapes. Depending on students' previous experiences, the lesson can either build on or prepare students for work in data display.

c. Mathematics ideas from this course include unit iteration, comparison of measurements using different units, conservation, and transitivity related to measuring lengths. The course also looks at measurement as an approximation and at measurement error. Lastly, it focuses on the ability to distinguish measurable properties and the fact that length, unlike area and volume, is one-dimensional.

d. If students are using numbers, you can ask them questions about how they arrived at their distance. Did they use any counting shortcuts or strategies?

e. Students at this age need many opportunities to develop their understanding of the particular attributes they are comparing or measuring, and they need to practice measuring lengths with both standard and nonstandard units. Measuring distances that they create themselves can be very motivating to students. For example, they can measure the length of their stride or how far they can jump. Or they can measure how far they can walk in a set amount of time.