Session 7

Circles and Pi (π)

Key Terms in This Session

Previously Introduced

- accuracy
- area
- precision
- scale factor
- similar figures

New in This Session

- circumference
- diameter
- irrational number
- perimeter
- pi (π)

Introduction

In this session, we will explore the common measures that involve circles—circumference and area—and work on activities that help us understand the formulas for these measures. We will also revisit accuracy, precision, and scale in relation to circles. Finally, we will explore how properties of the irrational number pi (π) affect calculations of circumference and area.

For information on required and/or optional materials for this session, **see Note 1.**

Learning Objectives

In this session, you will do the following:

- Investigate circumference and area of a circle
- Understand the formulas for these measures
- Learn how features of the irrational number π affect both circumference and area

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Note 1. Materials Needed:

- Variety of circular objects such as lids, CDs, buttons, Frisbees, bottles, and cans
- Bicycle wheel (If you do not have a bicycle wheel, use another circular object such as a large bowl or can.)
- Measuring tape
- Compass (optional)
- String
- Scissors (optional)
- Graphing calculator (To use a free graphing calculator online, go to http://www.coolmath.com/graphit/index.html.)

Part A: Circles and Circumference (60 min.)

Circumference

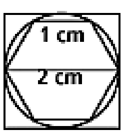
Perimeter—or distance around—is a measurable property of simple, closed curves and shapes. When the figure is a circle, we use the term circumference instead of perimeter. Because the perimeter of an object is a length, we need to measure using units of length such as centimeters, decimeters, meters, inches, feet, etc.

The circumferences of circular objects can be difficult to estimate. Let's use a bicycle wheel as an example. How long is its circumference? Use masking tape to make a mark on the floor or table to indicate the starting point. Estimate the distance of one rotation of the wheel or bowl rim, namely the circumference, by placing another piece of tape on the floor or table. **[See Note 2]**

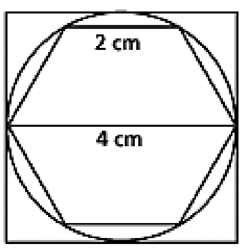
Roll the wheel to find the actual circumference. Was your estimate too short or too long?

There is a relationship between the circumference and diameter of a circle, which we will explore here in a number of ways. A diameter is a chord—a line segment joining two points on the arc of a circle—that passes through the center of the circle. Diameter also refers to the distance between two points on the circle, measured through the center. Let's first look for patterns in the measurements of circles.

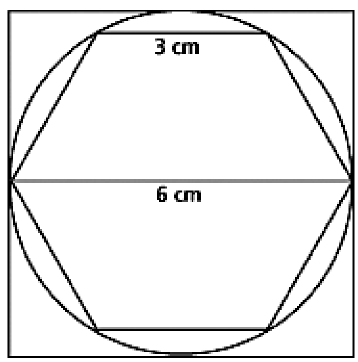
The three designs below show a circle between a regular hexagon and a square.



Design 1



Design 2



Design 3

Use the designs on page 150 to work on Problems A1-A3.

Problem A1. Use the designs to fill in the table at right. For the circle, use string to approximate the circumference. **[See Note 3]**

Problem A2. Look closely at the three designs. What patterns do you see in their measurements?

	Design 1	**Design 2**	**Design 3**
Diameter of the Circle			
Perimeter of the Hexagon			
Perimeter of the Square			
Approximate Circumference of the Circle			

Note 2. If you do not have a bicycle wheel available, you can start this section by estimating the circumference of different circular objects, such as the rim of a large bowl or can. Cut a piece of string the length of the estimate and then compare the estimate to the actual circumference. Most people grossly underestimate circumference.

Note 3. It is important to measure as accurately as possible to avoid measurement errors.

Part A, cont'd.

Problem A3.

a. For each design, how does the diameter of the circle compare to the perimeters of the square and the hexagon?

b. For each design, how does the approximate circumference compare to the perimeters of the square and the hexagon?

c. The circumference of any of these circles is about how many times more than its diameter? If a circle had a diameter of 7 cm, what prediction would you make for the length of its circumference? Why? **[See Note 4]**

Ratio of Circumference and Diameter

You can measure circular objects to verify the pattern you've seen. Choose four or five different circular objects to measure.

Problem A4.

a. For each object, estimate the circumference. Then measure the circumference and the diameter in centimeters to the nearest tenth (e.g., millimeters). Use string or a tape measure. Record your data in the table.

Object	**Diameter (*d*) in cm**	**Circumference (C) in cm**	**Ratio of C to *d* (C/*d*)**

b. Examine the table. What do you notice about the ratio of C to *d*? Based on these data, what is the relationship between the diameter and circumference of a circle?

Problem A5.

a. Enter the values from the table for diameter and circumference into a graphing program in your computer or into a table in your graphing calculator to make a scatter plot. Use the horizontal axis (x) for diameter and the vertical axis (y) for circumference. Graph the points. What patterns do you see in the graphical data?

b. What information does a graph of these data provide? **[See Note 5]**

[See Tip A5, page 153]

Note 4. You can again measure the diameter of the bicycle wheel and then use that to estimate the circumference based on observations you make in this problem.

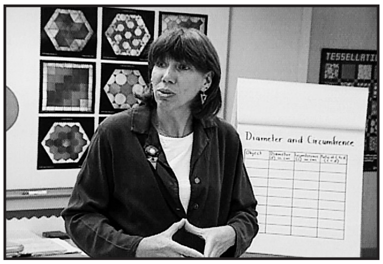
Note 5. Entering coordinates (diameter and circumference) for more than five objects might show a better approximation of a line. You can also find the line of best fit to see the patterns.

To learn more about scatter plots and the line of best fit, go to the *Data Analysis, Statistics, and Probability Web* site at www.learner.org/learningmath and find Session 7, Part A and Part D respectively.

Part A, cont'd.

Problem A6. Find the mean of the data in the C/*d* column. Why find the mean? Does the mean approximate π? **[See Tip A6, page 153]**

By now you have seen that all circles have one trait in common: The ratio of circumference to diameter is a constant value, π, which is a little more than 3. Pi is an irrational number that is represented by the symbol π. **[See Note 6]** Its decimal part continues forever without repeating. As of 1997, π had been extended to 51 billion decimal places (using a computer)! Your calculator has a special key for π, but this is only an approximate value.



Video Segment (approximate time: 9:16-11:10): You can find this segment on the session video approximately 9 minutes and 16 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, participants investigate the relationship of circumference and diameter using different circular objects. They collect the data by measuring, and then make observations about their findings.

Were you surprised to find out that π, which is an irrational number, can be expressed as a constant ratio of two numbers, namely the diameter and circumference of any circle?

Pi (π)

Problem A7. The symbol *r* represents the radius of a circle. Explain why $C = \pi \cdot 2r$ is a valid formula for the circumference of a circle.

Problem A8. An irrational number cannot be written as a quotient of any two whole numbers. Yet we sometimes see π written as 22/7 or 3.14. Explain what the reason for this may be.

Problem A9. Since π is an irrational number, can both the circumference and the diameter be rational numbers? Can one of them be rational? Explain using examples.

Problem A10. When mathematicians are asked to determine the circumference of a circle, say with a diameter of 4 cm, they often write the following:

$$C = \pi \cdot d = \pi \cdot 4$$

In other words, the circumference of the circle is 4π cm. Why do you think they record the answer in this manner? Why not use the π key on the calculator to find a numerical value for the circumference? **[See Tip A10, page 153]**

Note that we have worked with two forms of the standard equation that shows the relationship between circumference and diameter:

$$C = \pi \cdot d$$

$$\pi = C/d$$

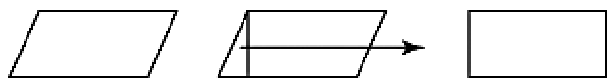
Since π is an irrational number, the exact circumference can only be expressed using the symbol for π. Sometimes, however, we want to solve a real problem and find an approximate value for a circumference. In that case, we must use one of the approximations for π. Inexactness may also occur when determining a numerical value for circumference (or diameter) because of measurement error.

Note 6. To learn more about irrational numbers, go to the *Number and Operations* Web site at www.learner.org/learningmath, and find Session 1, Part C, and Session 2.

Part B: Area of a Circle (60 min.)

Transforming a Circle

In Session 6, we found the areas of different polygons (parallelograms, triangles) by dissecting the polygons and rearranging the pieces into a recognizable simpler shape.

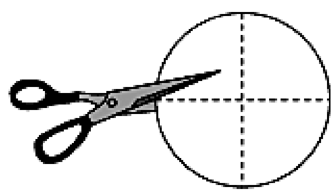


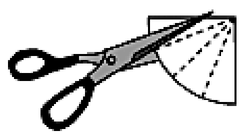
In this case, we transformed a parallelogram into a rectangle by slicing a triangle off one end and sliding it along to fit into the other end. In doing so, we established that the area of the parallelogram was the same as the area of the equivalent rectangle (its base multiplied by the perpendicular height). Can we use the same technique and transform a circle into a rectangular shape?

For a non-interactive version of the activity, use a compass and draw a large circle. Fold the circle in half horizontally and vertically. Cut the circle into four wedges on the fold lines. Then fold each wedge into quarters. Cut each wedge on the fold lines. You will have 16 wedges.

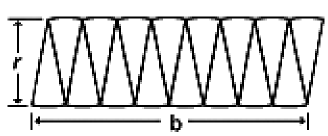
Try It Online! www.learner.org

This activity can be explored online as an Interactive Activity. Go to the *Measurement* Web site at www.learner.org/learningmath and find session 7, Part B.





Tape the wedges to a piece of paper to form the following figure:



Notice that we have a crude parallelogram with a height equal to the radius of the original circle.

Problem B1. How does the area of the figure compare with the area of the circle?

Problem B2. The scalloped base of the figure is formed by arcs of the circle. Write an expression relating the length of the base b to the circumference C of the circle. **[See Tip B2, page 153]**

Problem B3. Write an expression for the length of the base b in terms of the radius r of the circle. **[See Tip B3, page 153]**

Problem B4. If you increase the number of wedges, the figure you create becomes an increasingly improved approximation of a parallelogram with base b and height r. Write an expression for the area of the rectangle in terms of r.

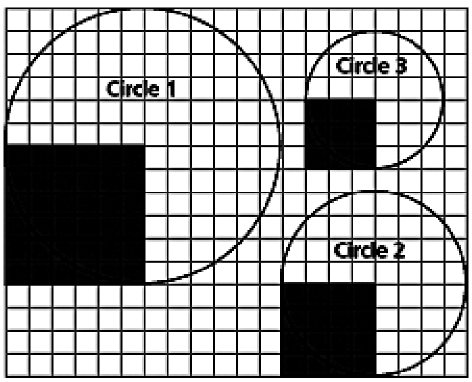
Think about how the activity involving wedges helps explain the area formula of a circle, $A = \pi \cdot r^2$.

Problems B1-B4 adapted from Bass, L.; Hall, B.; Johnson, A.; and Wood, D. *Geometry: Tools for a Changing World.*

Part B, cont'd.

Examining the Formula

Let's further examine the formula for area of a circle, $A = \pi \cdot r^2$. How do we interpret the symbols r^2? If r is the radius of a circle, then r^2 is a square with sides of length r. Examine the circles below. A portion of each circle is covered by a shaded square. We can call each of these squares a radius square.



Problem B5. Use the circles from page 151 to work on this problem. For each circle, cut out several copies of the radius square from a separate sheet of centimeter grid paper (page 152). Determine the number of radius squares it takes to cover each circle. You may cut the radius squares into parts if you need to. Record your data in the table below.

Circle	Radius of Circle	Area of Radius Square	Area of Circle	Number of Radius Squares Needed
1				
2				
3				

Problem B6.

a. What patterns do you observe in your data?

b. If you were to estimate the area of any circle in radius squares, what would you report as the best estimate?

Problem B7. Does the activity of determining the number of radius squares it takes to cover a circle provide any insights into the formula for the area of a circle?

Part B, cont'd.

Video Segment (approximate time: 17:13-20:29): You can find this segment on the session video approximately 17 minutes and 13 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

Watch this segment to see how Janet and David use this method to make sense of the formula for the area of a circle. They cover a circle with four squares with sides equal to the radius of the circle. Then they cut up the squares to find out how many of the four squares fit into the circle. Watch this segment after you've completed Problems B5-B7.

Did you come up with similar observations?

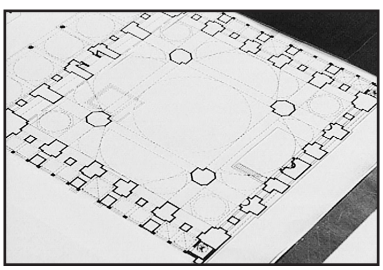
Problem B8. When you enlarge a circle so that the radius is twice as long (a scale factor of 2), what do you think happens to the circumference and the area? Do they double? Experiment by enlarging circles with different radii and analyzing the data.

Take It Further

Problem B9. Experiment by enlarging a circle by a scale factor of 3, by a scale factor of 2/3, and by a scale factor of *k*. Generalize your findings.

Take It Further

Problem B10. If a circle has a radius of 5 cm and the margin of error in measurement is 0.2 cm, what is a reasonable approximation for the area of the circle?



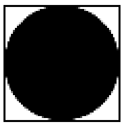
Video Segment (approximate time: 22:27-23:52): You can find this segment on the session video approximately 22 minutes and 27 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

Circles have been widely used throughout the history of humankind in many different applications and human endeavors. In this segment, Afshan Bokhari explains the significance of circles in Islamic tradition, particularly in design and architecture. She shows us how the circle has been used as both an aesthetic element and a structural element in the building of mosques.

Do you know of any other historical examples of the use of circles?

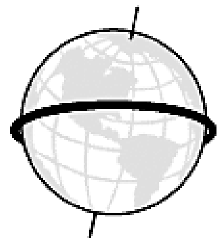
Homework

Problem H1. A circle is inscribed in a square. What percentage of the area of the square is inside the circle?



Take It Further

Problem H2. Imagine that a giant hula hoop is fitted snugly around the Earth's equator. The diameter of the hula hoop is 12,800 km. Next, imagine that the hula hoop is cut and its circumference is increased by 10 m. The hula hoop is adjusted around the equator so that every part of the hula hoop lies the same distance above the surface of the Earth. Would you be able to crawl under it? Walk under it standing upright? Drive a moving truck under it? Determine the new diameter of the hoop, and find out the distance between the Earth and the hula hoop.



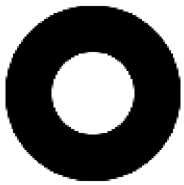
Problem H3. A new car boasts a turning radius of 15 ft. This means that it can make a complete circle with a radius of 15 ft. and return to its original spot. The radius is measured from the center of the circle to the outside wheel. If the two front tires are 4.5 ft. apart, how much further do the outside tires have to travel than the inside tires to complete the circle?

Take It Further

Problem H4. Your dog is chained to a corner of the toolshed in your backyard. The chain measures 10 ft. in length. The toolshed is rectangular, with dimensions 6 ft. by 12 ft. Draw the picture showing the area the dog can reach while attached to the chain. Compute this area. **[See Tip H4, page 153]**

Homework, cont'd.

Problem H5. An annulus is the region bounded by two concentric circles.

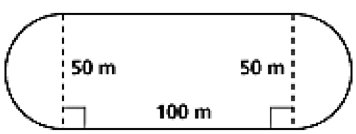


a. If the radius of the small circle is 10 cm and the radius of the large circle is 20 cm, what is the area of the annulus?

b. A dartboard has four annular rings surrounding a bull's-eye. The circles have radii 10, 20, 30, 40, and 50 cm. How do the areas of the annular rings compare? Suppose a dart is equally likely to hit any point on the board. Is the dart more likely to hit in the outermost ring or inside the region containing the bull's-eye and the two innermost rings? Explain.

[See Tip H5, page 153]

Problem H6. An oval track is made by erecting semicircles on each end of a 50 m-by-100 m rectangle. What is the length of the track? What is the area of the region enclosed by the track?

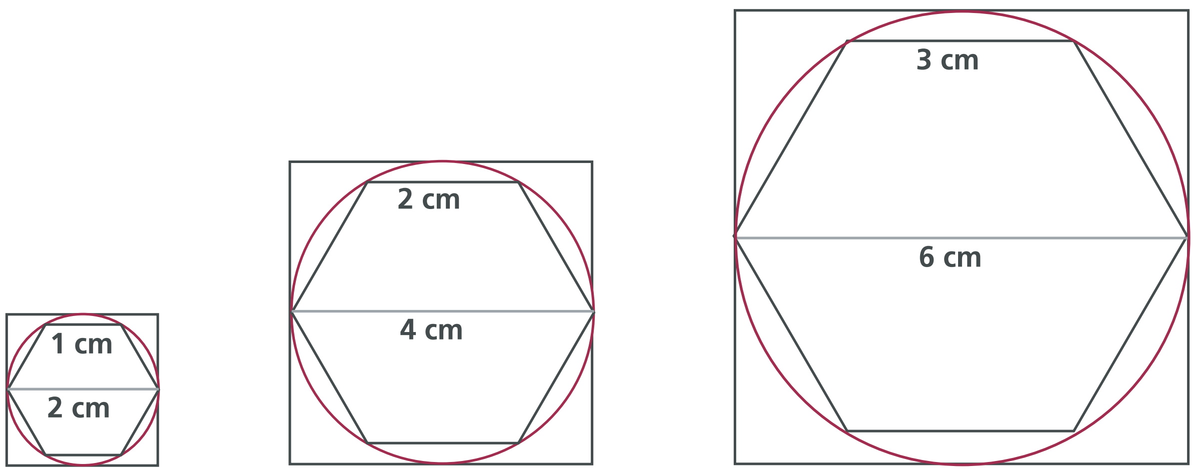


Suggested Reading

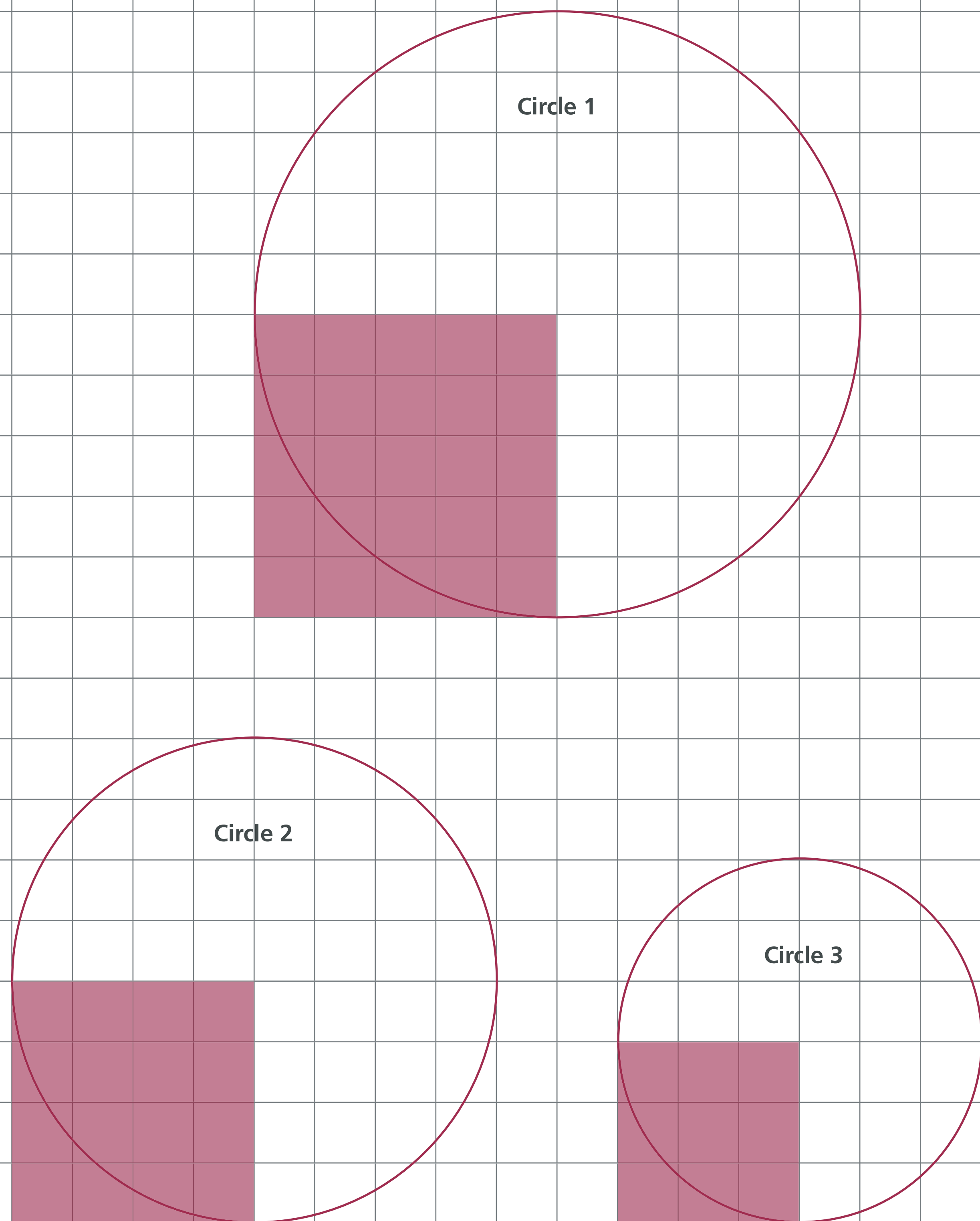
This reading is available as downloadable PDF file on the *Measurement* Web site. Go to www.learner.org/learning-math.

Zebrowski, Ernest (1999). *A History of the Circle* (pp. 48-49). Piscataway, N.J.: Rutgers University Press.

Designs for Problems A1-A3



Circles for Problem B5



Centimeter Grid Paper for Problem B5

Tips

Part A: Circles and Circumference

Tip A5. A straight line indicates that the data are increasing by a constant amount. What is the constant in this case?

Tip A6. A mean is an average, or a sum of all the data values divided by the total number of data values. **[See Note 7]**

Tip A10. Is the value for π given on a calculator an approximation or an exact amount?

Part B: Area of a Circle

Tip B2. Is b equal to the circumference of the circle? You may need to re-form the wedges back into a circle and then back again into a parallelogram.

Tip B3. $C = \pi d$ or $C = 2\pi r$. Substitute one of these expressions for C in your equation in Problem B2.

Homework

Tip H4. Draw a diagram of the shed and the possible areas that the dog could reach on its chain. Then divide the space into different sections and calculate the area of each section.

Tip H5. Express the areas in terms of π and then compare them.

Note 7. To learn more about mean, go to the *Data Analysis, Statistics, and Probability* Web site at www.learner.org/learningmath, and find Session 5.

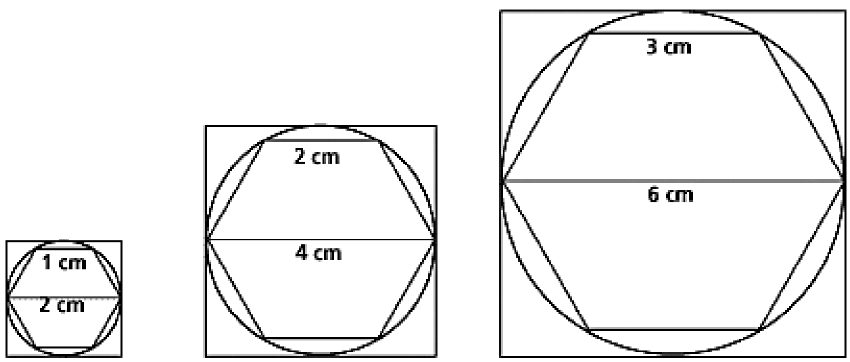
Solutions

Part A: Circles and Circumference

Problem A1. Here is the completed table:

	Design 1	**Design 2**	**Design 3**
Diameter of the Circle	2 cm	4 cm	6 cm
Perimeter of the Hexagon	6 cm	12 cm	18 cm
Perimeter of the Square	8 cm	16 cm	24 cm
Approximate Circumference of the Circle	6.3 cm	12.6 cm	18.9 cm

Problem A2. The measurements stay in scale. In all three, the diagonal of the hexagon is twice the length of the hexagon's side. Also, as we move from one design to the next, the length of each side of the inscribed hexagon increases by 1; the length of each side of the inscribed hexagon is equal to the radius of the circle (as shown by the inscribed equilateral triangles):



The length of each side of the square is the same as the diameter of the circle inscribed within; the ratio of the length of the diameter of the circle to the length of one side of the hexagon is 2/1 for all three designs.

Problem A3.

a. The perimeter of the hexagon is three times the diameter of the circle, and the perimeter of the square is four times the diameter of the circle.

b. The circumference of the circle is between these two values, and closer to the hexagon's perimeter.

c. The circumference appears to be between 3.1 and 3.2 times larger than the diameter. If a circle had a diameter of 7 cm, you might predict its circumference to be somewhere near 22 cm. As we explore this further, we will see that the relationship between the circumference and diameter is a constant value.

Problem A4.

a. Answers will vary.

b. The measured ratio of C/d should be approximately the same for all circular objects. It seems that the relationship between diameter and circumference is linear, and there is some number k so that $C = k \cdot d$ for every circle's circumference and diameter. For now, we can say that this number is just slightly larger than 3.

Solutions, cont'd.

Problem A5.

a. Answers will vary. The points should roughly form a straight line. If you were to place a line of best fit onto your scatter plot, the line would be $y = 3.14 \cdot x$.

   Notice that the ratio C/d is about 3.14 no matter what the size of the circle. This is called a constant ratio since the value is constant, regardless of the circle. Constant change is represented by a straight-line graph and is sometimes referred to as a linear relationship. If the ratio between circumference and diameter differed for every circle, the graph would not be a straight line.

b. This suggests that the diameter and circumference are in direct variation; that is, the circumference is a direct multiple of the diameter. Note that since you measured the circumference and diameter, there are likely to be measurement errors which will affect the graphed data.

Problem A6. Answers will vary, but should be close to 3.14 (an approximation for π). Finding the mean minimizes any measurement errors in the calculations of Problem A5.

Problem A7. We've seen that $\pi = C/d$, where C is the circumference and d is the diameter of a circle. We can multiply both sides of the equation by d to get a new equation: $C = \pi d$. Because the diameter of a circle is always twice its radius, we can write the new equation as $C = \pi \cdot 2r$, which is what we wanted.

Problem A8. These forms make calculations involving π easier by using an approximation. In cases where there may already be measurement error, it doesn't make sense to use an overly accurate version of π. Fractions like 22/7 or decimals like 3.14 do the job nicely in different situations.

In practical terms, it is impossible to buy, for example, a length of fencing that measures 4π. In applications, we often want an approximation that we can measure and work with. In mathematics problems, however, it is almost always preferable to use the symbol π.

Problem A9. One or the other may be rational, but not both. If they were both rational, their ratio (which is π) would also have to be rational, which it is not. A circle may have a diameter of exactly 12 cm with an irrational circumference, or a circumference of exactly 100 m with an irrational diameter.

They can, however, both be irrational.

Problem A10. This answer provides the only way to write the answer exactly. Additionally, it is easier to perform arithmetic with 4π than with a decimal approximation of it. An approximation may be substituted later if needed.

Part B: Area of a Circle

Problem B1. The area of the figure is exactly the area of the circle, since no area has been removed or added, only rearranged.

Problem B2. The length of the base is one-half the circle's circumference, since the entire circumference comprises the scalloped edges that run along the top and bottom of the figure, and exactly half of it appears on each side. The base length is C/2.

Problem B3. Since the circumference is $2 \cdot \pi \cdot r$, where r is the radius, the base is half of this. The base length is $\pi \cdot r$.

Problem B4. As the number of wedges increases, each wedge becomes a nearly vertical piece. The base length becomes closer and closer to a straight line of length $\pi \cdot r$ (or half the circumference), while the height is equal to r. The area of such a rectangle is $\pi \cdot r \cdot r$, or $\pi \cdot r^2$.

Solutions, cont'd.

Problem B5. Here is the completed table:

Circle	Radius of Circle	Area of Radius Square	Area of Circle	Number of Radius Squares Needed
1	6	36	$36 \cdot \pi$	A little more than 3
2	4	16	$16 \cdot \pi$	A little more than 3
3	3	9	$9 \cdot \pi$	A little more than 3

Problem B6.

a. In each case, it takes a little more than three radius squares to form the circle. If using approximations, it should always take around 3.14 of the squares to cover the circle.

b. The best estimate is somewhere between 3.1 and 3.2, which we know is roughly the value of π.

Problem B7. The formula for the area of a circle is $A = \pi \cdot r^2$. The activity helps one understand that a bit more than three times a radius square is needed to cover the circle. Namely, it illustrates why the formula is $\pi \cdot r^2$.

Problem B8. Think about a circle with a radius equal to 1 ($r = 1$). The circumference and the area of this circle are as follows:

$C = 2 \cdot 1 \cdot \pi = 2\pi$

$A = 1^2 \cdot \pi = \pi$

Now double the radius to 2 units ($r = 2$). The circumference and the area of the new circle are as follows:

$C = 2 \cdot 2 \cdot \pi = 4\pi$

$A = 2^2 \cdot \pi = 4\pi$

The circumference of the new circle doubled, but the area is multiplied by a factor of 4 (the square of the scale factor). You can replace the 1 with any other number, or with a variable r, to see that this relationship will always hold.

Problem B9.

Scale factor 3:

$C = 2\pi \cdot (3r) = 6\pi r$

$A = \pi \cdot (3r)^2 = 9\pi r^2$

Scale factor 2/3:

$C = 2\pi \cdot (2/3r) = (4/3)\pi r$

$A = \pi \cdot (2/3r)^2 = (4/9)\pi r^2$

Scale factor k:

$C = 2\pi \cdot (kr) = k(2\pi r)$

$A = \pi \cdot (kr)^2 = k^2\pi r^2$

As with other similar figures, the circumference (or perimeter) of the shape is multiplied by the scale factor, while the area is multiplied by the square of the scale factor. This is also evident in the formulas for each; the circumference formula involves r, while the area formula involves r^2.

Solutions, cont'd.

Problem B10. A reasonable approximation is 25π cm^2, but the margin of error will be larger than 0.2 cm^2. The actual area in square centimeters may be anywhere from $(4.8)^2\pi$ (lower limit) to $(5.2)^2\pi$ (upper limit). Since 4.8^2, or 23.04, and 5.2^2, or 27.04, are each about 2 units away from 5^2, the margin of error for the area is approximately 2π cm^2.

Homework

Problem H1.

Area of circle: $A_C = \pi r^2$

Area of square: $A_S = (2r)^2 = 4r^2$

$A_C/A_S = \pi r^2/4r^2 = \pi/4$

The fractional part of the area is $\pi/4$. This is equal to 0.785, and therefore, expressed as a percentage, is approximately 78.5%.

Problem H2. Since the diameter of the hula hoop is 12,800 km, the circumference is approximately 40,212.386 km. If we cut the hula hoop and add 10 m (0.01 km), the circumference is now 40,212.396 km. The new diameter of the hula hoop is found by dividing 40,212.396 by π; it is 12,800.003 km. The difference between the two diameters is 0.003 km, or 3 m. Dividing this difference in half (since $d = 2r$) results in a 1.5-meter height change between the Earth and the hula hoop. You could easily crawl under it or walk under it in a crouched position, but you could not drive a truck under it!

The interesting fact about this problem is that the distance added to the diameter (and radius) is independent of the original diameter and circumference:

C + addition to C = π (d + addition to diameter)   [new]

C = $\pi \cdot d$ [original]

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addition to C = $\pi \cdot$ (addition to diameter)   [subtract]

or,

(addition to C)/π = addition to diameter

Problem H3. Since the radius of the circle formed by the outside tires is 15 ft., the radius formed by the inside tires is (15 - 4.5) = 10.5 ft. The circumference of the two circles can be calculated and subtracted:

$15 \cdot 2 \cdot \pi - 10.5 \cdot 2 \cdot \pi = 4.5 \cdot 2 \cdot \pi$,

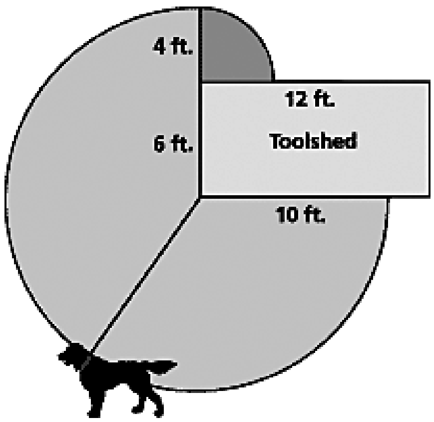
which is approximately 28.3 ft. Note that the radius of the circle formed by the outside tires was not important to the final result (which only used the 4.5-foot difference). This means that the calculation is valid for any car whose wheels are 4.5 ft. apart.

Solutions, cont'd.

Problem H4. The area is a three-quarters circle with radius 10 ft., plus a quarter-circle with radius 4 ft. (The dog can reach this area by stretching along the six-foot wall and then pointing into the exposed area.) The total area is:

$3/4 \cdot \pi \cdot (10)^2 + 1/4 \cdot \pi \cdot (4^2) = 75 \cdot \pi + 4 \cdot \pi = 79 \cdot \pi$,

or approximately 248 ft^2.



Problem H5.

a. The area of the annulus is the difference between the circles' areas. For these circles, the area is 300π cm^2, or approximately 943 cm^2.

b. If the smallest circle has a radius of 10 cm, then the area of that bull's-eye circle is 100π cm^2. The area of the first annular ring is the $400\pi - 100\pi$, or 300π cm^2. Since the second interior circle has a radius of 20 cm, we can find the area of it and then subtract the area of the bull's-eye. Using this line of reasoning, the area of the second annular ring is $900\pi - 400\pi$, or 500π cm^2; the area of the third annular ring is $1{,}600\pi - 900\pi$, or 700π cm^2; and the area of the fourth (outer) annular ring is $2{,}500\pi - 1{,}600\pi$, or 900π cm^2. The probability of a dart thrown at random hitting the outermost ring or a dart hitting the bull's-eye and the two innermost rings is exactly the same; both regions have an area of 900π cm^2.

Problem H6. The length and area can be more easily calculated by isolating the circular sections, which then form a complete circle of radius 25 m.

Length: $2 \cdot \pi \cdot 25 + 2 \cdot 100$, or approximately 357 m

Area: $\pi \cdot 25^2 + 100 \cdot 50$, or approximately 6,963 m^2