Session 6

Area

Key Terms in This Session

Previously Introduced

- scale factor

New in This Session

- area
- midline
- similar figures

Introduction

Area is a measure of how much surface is covered by a particular object or figure. Units of measure for area involve shapes that cover the plane, such as rectangles or squares. Common units in the U.S. customary system are square inches, square feet, square yards, and square miles; the standard unit in the metric system is the square meter.

For information on required and/or optional materials for this session, **see Note 1.**

Learning Objectives

In this session, you will do the following:

- Learn that area is a measure of how much surface is covered and that some shapes cover the surface of a plane more completely than other shapes
- Examine how the size of the unit used to indicate the amount of surface covered determines the number of units
- Find the area of irregular shapes
- Learn how to approximate the area of shapes more accurately
- See how the process of counting can be shortened by using formulas
- Gain a deeper understanding of the standard formulas for the area of triangles, trapezoids, and parallelograms

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Note 1. Materials Needed:

- Geoboard (optional)
- Rubber bands (optional)
- Power Polygons (optional)

Geoboards and Power Polygons can be purchased from:

ETA/Cuisenaire, 500 Greenview Court, Vernon Hills, IL 60061; Phone: 800-445-5985/800-816-5050 (Customer service); Fax: 800-875-9643/847-816-5066; http://www.etacuisenaire.com

Part A: Measuring Area (35 min.)

Measuring a Surface

Think of some situations that involve the measurement of area. How would you determine these areas? For example, in order to determine how much paint to buy for your living room, how would you find the area of each surface you plan to paint—your walls, ceiling, etc.?

Units of measurement for area involve shapes that cover a plane. Some shapes, such as rectangles and squares, cover the plane more completely than other shapes, such as circles. As mentioned above, squares are the most common unit of measurement in the U.S. customary system (square inches, square feet, square yards, square miles). The standard unit in the metric system is the square meter, with square centimeters and square hectares (equivalent to 10,000 square meters) used for smaller and larger surfaces.

Most people are familiar with methods for finding the area of familiar shapes, such as rectangles and squares. But is it possible to determine the area of an irregularly shaped object? How would you determine such an area without a formula? Let's explore.

Areas of Irregular Shapes

Trace your hand on a piece of paper. Think about how you might determine the area of your handprint.

Problem A1. Will the amount of area covered differ if you trace your hand with your fingers close together or spread apart? Explain. **[See Tip A1, page 132]**

Units for measuring area must have the following properties:

- The unit itself must be the interior of a simple closed shape.
- The unit, when repeated, must completely cover the object of interest, with no holes or gaps (like a tessellation). Many polygons (e.g., rectangles, rhombuses, and trapezoids) and irregular shapes (e.g., L shapes) have this property and can thus be used as units of measurement.

Problem A2.

a. What units might you use to determine the area of your handprint?

b. Why can't you use a small circle as the unit of measurement?

Problem A3.

a. One method for finding the area of an irregular shape is to count unit squares. Use centimeter grid paper from pages 127–129 to determine the area of your handprint. What are the disadvantages of this method?

b. Another method is to subdivide your handprint into sections for which you can easily calculate the area. Find the area of your handprint using this method. Does using the two methods result in the same area?

Up until now, you have been approximating the area of your handprint. In other words, your measurements were not exact.

Problem A4. What can you do to make your approximation more accurate? Explain why this approach will lead to a better approximation.

Another way to approximate the area of a handprint or any other irregular shape is to determine the number of squares that are completely covered and the number of squares that are partially covered. Average these two numbers to get an approximate area in the number of square units.

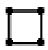
Problem A5. Think about the following statement: If you repeatedly use a smaller and smaller unit to calculate the area of an irregular shape, you will get a closer and closer approximation and eventually find the exact area. What do you think of this line of reasoning? Explain.

Problem A6. The palm of your hand is about one percent of your body's surface area. Doctors sometimes use this piece of information to estimate the percent of the body that is affected in burn victims. Use your data to approximate the amount of skin on your body.

Part B: Exploring Area With a Geoboard (50 min.)

Subdividing Area

Let's further examine the concept of area, using a geoboard. The unit of area on the geoboard is the smallest square that can be made by connecting four nails:



We will refer to this unit as 1 square unit.

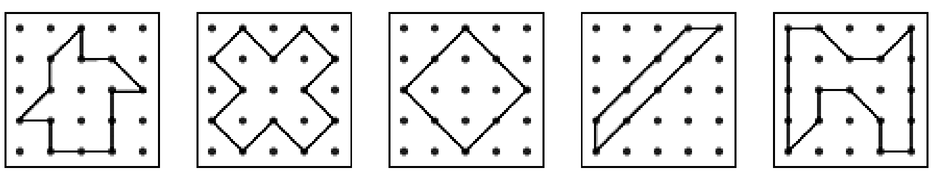
On the geoboard, the unit of length is the vertical or horizontal distance between two nails. Perimeter is the distance around the outside of a shape and is measured with a unit of length.

For a non-interactive version, use an actual geoboard and rubber bands, or use the dot paper worksheet on page 130. Use a space enclosed by five dots both vertically and horizontally to represent a single geoboard.

Try It Online! **www.learner.org**

These problems can be explored as an Interactive Activity. Go to the *Measurement* Web site at www.learner.org/learningmath and find Session 6, Part B.

Problem B1. Make the following figures and find the number of square units in the area of each: **[See Note 2]**



a. b. c. d. e.

[See Tip B1, page 132]

Problem B2. Make the following figures:

a. A square with an area of 4 square units
b. An isosceles triangle with an area of 4 square units
c. A square with an area of 2 square units (this is not a trick question!)

The Rectangle Method

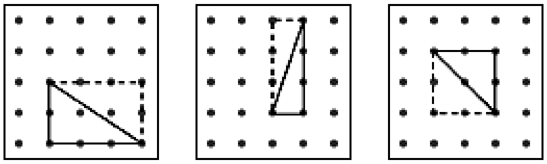
One standard approach to finding the area of a shape is to divide the shape into subshapes, determine the area of each subshape, and then add the areas together. You have used this approach to answer Problems B1 and B2.

A second approach for finding area is to surround the shape in question with another shape, such as a rectangle. For this approach, you first determine the areas of both the rectangle and the pieces of the rectangle that are outside the original shape, and then you subtract those areas to determine the area of the original shape.

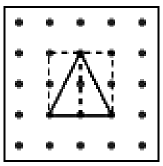
Note 2. If you are working in a group, take this opportunity to learn from one another by sharing your approaches and solution strategies to Problems B1-B4.

Part B, cont'd.

Here are three examples of how to surround a right triangle with a rectangle:

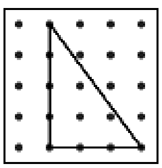


You can also divide a triangle into right triangles, form rectangles around each triangle, and then calculate the areas of the rectangles:

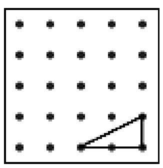


In each case, the area of the triangle is half the area of the rectangle that surrounds it.

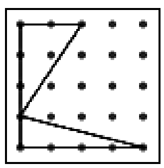
Problem B3. Use the rectangle method to find the area of each figure:



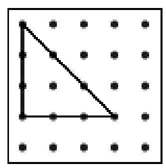
a.



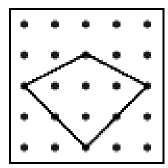
b.



c.

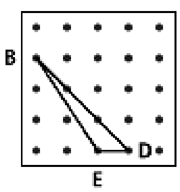


d.

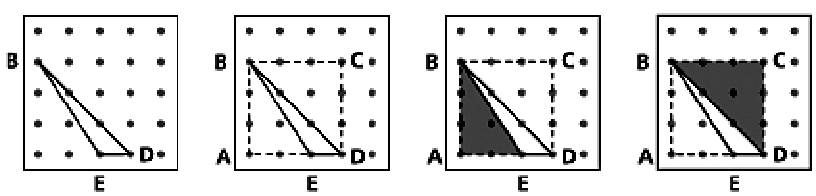


e.

Does this method work for non-right triangles? For example, how might you find the area of a triangle like ΔBDE below?



Here's how to do it: First, form rectangle ABCD around ΔBDE. Determine the area of rectangle ABCD and then subtract the areas of ΔABE and ΔBCD. (Use the rectangle method to determine the areas of these two triangles.) This will give you the area of ΔBDE:



Area of ABCD = 9 square units

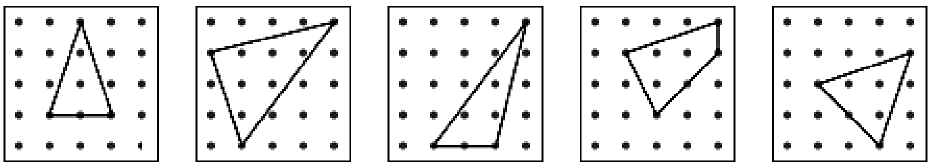
Area of ΔABE = 3 square units

Area of ΔBCD = 4.5 square units

Area of ΔBDE = ABCD - ΔABE - ΔBCD = 9 - 3 - 4.5 = 1.5 square units

Part B, cont'd.

Problem B4. Use this method to find the area of each of the following:



a. b. c. d. e.

[See Tip B4, page 132]

Video Segment (approximate time: 8:22-9:48): You can find this segment on the session video approximately 8 minutes and 22 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this segment, Rosalie demonstrates how to use the rectangle method to find the area of a triangle. Watch this segment after you've completed Problems B3 and B4.

For what kinds of figures on the geoboard might this method be particularly useful?

Take It Further

Problem B5. Construct the following shapes:

a. A triangle with an area of 3 square units
b. A triangle and a square with equal areas (which one has the smaller perimeter?)
c. Triangles with areas of 5, 6, and 7 square units, respectively

[See Tip B5, page 132]

Video Segment (approximate time: 9:49-11:43): You can find this segment on the session video approximately 9 minutes and 49 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this segment, Professor Chapin and Neuza explore what happens to the area of a triangle when its shape is changed, though the height and base lengths remain the same.

Did you come up with a similar conjecture? Explain in your own words why you think this happens.

Part B, cont'd.

Formulas

Most of us use formulas to determine the area of common polygons, such as triangles and rectangles. The formula for the area of a rectangle is A = $l \cdot w$, where l represents the length of the rectangle and w represents the width. The formula for the area of a triangle is, $A = \frac{b \cdot h}{2}$

where b represents the length of the base of the triangle and h represents the height of the triangle. (Height is the length of the segment from a vertex perpendicular to the opposite side.)

Problem B6. Explain how the formulas below relate to using the geoboard to find an area:

a. Rectangle: A = $l \cdot w$

b. Right triangle: $A = \frac{b \cdot h}{2}$ **[See Tip B6, page 132]**

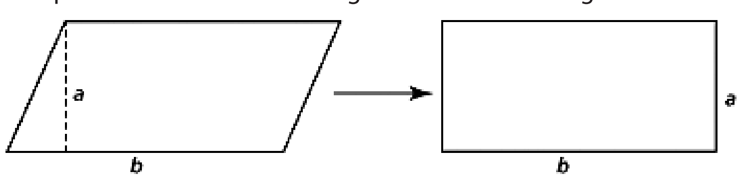
The Triangle Formula

Why does the formula $A = \frac{b \cdot h}{2}$ work for triangles other than right triangles?

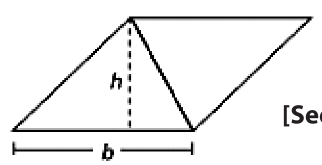
To answer this question, let's look at parallelograms, since we'll derive the triangle formula from the formula for the area of a parallelogram.

Problem B7.

a. The area of a parallelogram with height a and base b is found using the formula A = $a \cdot b$. How does it compare to the area of a rectangle with the same height and base?



b. Examine the figure at right in which two congruent triangles are placed together to form a parallelogram. Using this figure, explain how the formula for the area of a triangle relates to the formula for the area of a parallelogram.



[See Tip B7, page 132]

Take It Further

Problem B8. Make a parallelogram out of two trapezoids as follows:

- Fold a piece of paper in half.
- Draw any trapezoid on the piece of paper.
- Label the top base b_1, the bottom base b_2, and the height h.
- Cut out two identical copies of your trapezoid, and arrange them to form a single parallelogram.

What is the area of this parallelogram?

How does the area compare to the area of a single trapezoid? What is the area of one trapezoid?

Part C: Scaling the Area (35 min.)

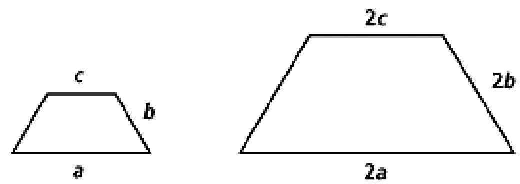
Similar Figures

What happens to the area of a figure if we scale it up or down (i.e., enlarge or reduce it)? In Part C, we review the concept of similarity and examine the relationship between a scale factor and the resulting area of the similar figure. Previously we only explored similar triangles, but in this section we will use a variety of shapes.

When we enlarge or reduce a figure, we are using an important mathematical idea: similarity. Similar figures have the same shape but are not necessarily the same size. More formally, we state that two figures are similar if and only if two things are true: (1) The corresponding angles have the same measure, and (2) the corresponding segments are in proportion. Enlarging or reducing a figure produces two figures that are similar. **[See Note 3]**

The second attribute means that when we are building a similar figure, we must increase or decrease the sides multiplicatively by the scale factor. What happens to the length of each side when we enlarge a figure, say, by a scale factor of 2? Well, since in similar figures the corresponding sides are in proportion, each of the sides of the enlarged similar figure is twice as long as the corresponding side of the original figure.

So, for example, in the enlargement of the trapezoid shown on the right, the enlarged trapezoid is similar to the small trapezoid because the angles are congruent and each of the sides is proportionally larger (twice as long):



Building similar figures, however, is not always so straightforward! For example, the trapezoid below is not similar to the original trapezoid. The angles are congruent, but the corresponding sides are not proportional—some of the sides have been "stretched" more than others:

Scaling Polygons

Make several copies of the sample polygons on page 131 to use in the problems that follow. You'll need to cut out each of the polygons in order to manipulate them. (If you have access to Power Polygons, you can use them for these problems.)

Problem C1.

a. In this problem, you will build similar figures by enlarging Triangle N. Use multiple copies of Triangle N to build the enlarged triangles. First use a scale factor of 2, then a scale factor of 3, and then a scale factor of 4. Check to make sure that all corresponding sides are proportionally larger than the original polygon. Sketch your enlargements.

b. What happens to the area of a figure that you enlarge by a scale factor of 2? Is the area of the enlarged figure twice that of the original?

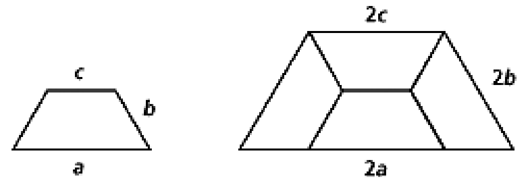
Note 3. To further explore the notion of similarity, go to www.learner.org/learningmath and find *Geometry*, Session 8.

Part C, cont'd.

Problem C2. Use Rectangle C, Parallelogram M, and Trapezoid K to build similar figures with scale factors of 2, 3, and 4, respectively. Then calculate the area of each enlargement in terms of the original polygon, and record your results in the table below. **[See Note 4]**

For example, it takes four copies of the original trapezoid to make a similar shape with a scale factor of 2, so the enlarged trapezoid has an area that is four times greater than the original (at right):

You do not have to build the enlargements of each shape using only that type of polygon, but you will need to determine the area of each enlargement in terms of the original polygon.



Polygon	Scale Factor of 2—Area of the Enlargement in Terms of the Original Shape	Scale Factor of 3—Area of the Enlargement in Terms of the Original Shape	Scale Factor of 4—Area of the Enlargement in Terms of the Original Shape
Rectangle C			
Parallelogram M			
Trapezoid K			

Problem C3. Examine your enlargements. What is the relationship between the scale factor and the number of copies of the original shape needed to make a larger similar shape?

Problem C4. What is the relationship between the scale factor and the area of the enlarged figure?

Problem C5. If the area of a polygon were 8 cm^2, what would the area of an enlargement with a scale factor of 3 be? **[See Tip C5, page 132]**

Problem C6. If the scale factor of an enlargement is k, explain why the enlarged area is k^2 times greater than the original area.

Video Segment (approximate time: 16:24-18:09): You can find this segment on the session video approximately 16 minutes and 24 seconds after the Annenberg/CPB logo. Use the video image to identify where to begin viewing.

If you increase the size of a shape by a certain scale factor, what does that do to its area? Watch this segment to see how Michelle and David use different polygons to explore this question.

What happens to the area of a shape if you decrease its size by a certain scale factor? Does the same rule apply?

Take It Further

Problem C7. A rep-tile is a shape whose copies can be put together to make a larger similar shape. Look at Polygons B, C, F, L, and M. Which of these are rep-tiles? What do you notice about all of the rep-tile shapes?

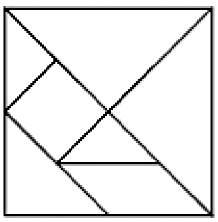
Note 4. If you are working in a group, you can divide the tasks in this problem. You may also want to try scaling up Polygons B, F, H, and L.

In this problem, you use the polygons to create similar figures. You may use more than one type of shape to build the similar figures, but you do not have to. Be sure that the figures you create satisfy both of the requirements for similarity—you may need to measure the sides and angles of the figures to check.

Polygon H, the hexagon, poses a challenge. You can use hexagons, trapezoids, and equilateral triangles to build the similar hexagons in this problem.

Homework

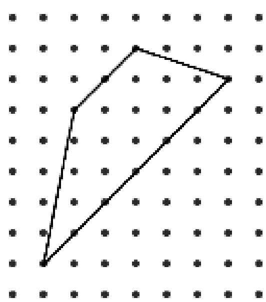
Problem H1. If the area of each of the smallest two triangles in the tangram below is equal to 1 unit, find the area of each of the other pieces, and then find the area of the entire tangram:



Problem H2. The formula for the area of a trapezoid is $A = \frac{(b_1 + b_2) \cdot h}{2}$, where b_1 and b_2 represent the top and bottom base of the trapezoid and h represents the height.

Draw a trapezoid on a sheet of paper, and connect either of the two opposite vertices. Into what shapes has the trapezoid been divided? What are the height and base of each shape? Find the area of each shape and add them together. How does this area compare to the total area of the trapezoid?

Problem H3. Examine the trapezoid below:

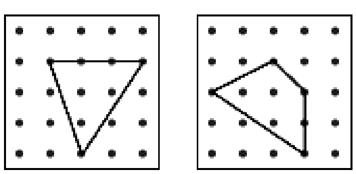


Find the area of this trapezoid using the formula $A = \frac{(b_1 + b_2) \cdot h}{2}$.

Then find the area in a different way. How do the two areas compare?

Take It Further

Problem H4. It is possible to find a formula for the area of geoboard polygons as a function of boundary dots and interior dots. For example, the two polygons below each have five boundary dots and three interior dots:



a. What is the area of each polygon?

Let's gather data to help us find what's known as Pick's formula, which is used for determining the area of a simple closed curve (in our case, the areas of the polygons on a geoboard).

For problems (b)-(d) (next page), build figures on the geoboard or draw figures on dot paper that have the indicated number of boundary dots (b) and interior dots (I). Find the area of each figure, and record your results in the tables that follow (page 126).

Homework, cont'd.

Problem H4, cont'd.

b. If I = 0, calculate the area of each figure:

Number of Boundary Dots	Area (in Square Units)
3	
4	
5	
6	
7	
b	

c. If I = 1, calculate the area of each figure:

Number of Boundary Dots	Area (in Square Units)
3	
4	
5	
6	
7	
b	

d. If I = 2, calculate the area of each figure:

Number of Boundary Dots	Area (in Square Units)
3	
4	
5	
6	
7	
b	

e. What patterns do you notice in these tables? Each time you add a boundary dot, how does it change the area?

f. Find a formula for the area of a geoboard figure if it has *b* boundary dots and I interior dots.

Suggested Reading

This reading is available as a downloadable PDF file on the *Measurement* Web site.
Go to **www.learner.org/learningmath**.

Fan, C. Kenneth (January, 1997). Areas and Brownies. *Mathematics Teaching in the Middle School, (2)* 3, 148-160.

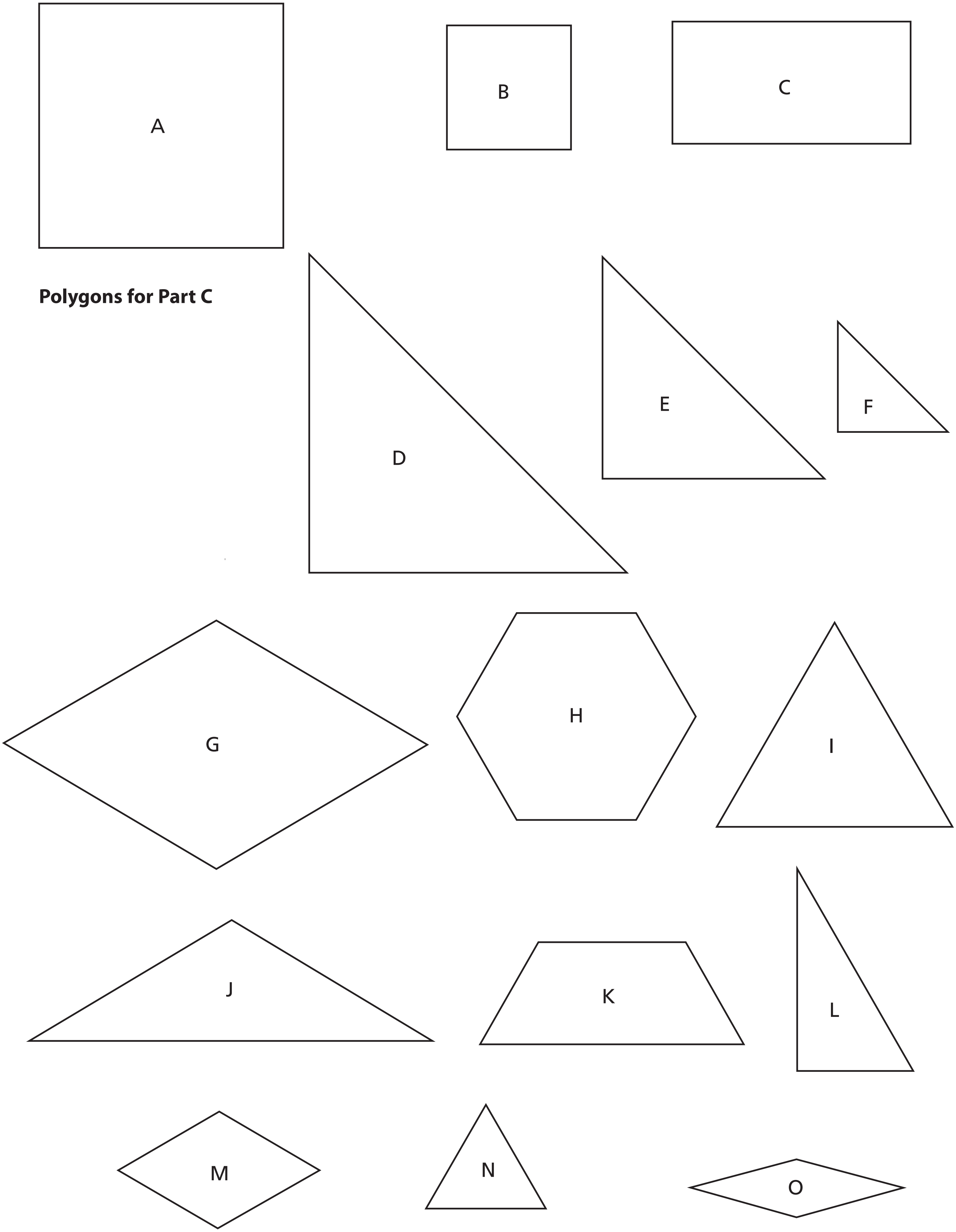
1 cm^2 Grid Paper

0.5 cm² Grid Paper

0.25 cm^2 Grid Paper

Dot Paper Worksheet for Part B

Polygons for Part C

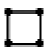


Tips

Part A: Measuring Area

Tip A1. Think about the activity you did with the tangram triangles in Session 1, Part C.

Part B: Exploring Area With a Geoboard

Tip B1.  equals 1 square unit.  equals 0.5 square unit.

Tip B4. Completely surround the figure with a rectangle; the vertices of the figure should touch the sides of the rectangle. Then find the areas of the outside spaces—the parts of the rectangle that are not inside the figure in question. Some people like to cover all of the rectangle except the section they are working on so as not to be distracted by overlapping lines and shapes.

Tip B5. Use a guess-and-check strategy: First make a triangle on the geoboard. Next determine its area, using one of the methods mentioned earlier. Adjust the shape of your triangle as needed (i.e., make it larger or smaller), and repeat the process, refining the size of your triangle as you get closer to the desired area.

Tip B6. Think about the rectangle method you used on the geoboard to find the area of a right triangle.

Tip B7. Note the height of the triangle, *h*, is the length of the line segment perpendicular to the base and adjoining it to the opposite vertex. This is equal to the height of the parallelogram.

Part C: Scaling the Area

Tip C5. You may want to sketch a rectangle that has an area of 8 cm^2 and the enlargement of that rectangle using a scale factor of 3.

Solutions

Part A: Measuring Area

Problem A1. No, the surface area should be identical, since it is the fingers, not their location, that determine the area. The same shapes in different locations will have the same area.

Problem A2.

a. A good unit of measurement might be a square centimeter or a square millimeter on grid paper. An important aspect is that the unit should tile well.

b. A circle does not tile well; circles leave holes in between them, and counting the area of the circles would not approximate the total area of the hand.

Problem A3.

a. Answers will vary. Counting squares is subject to several errors, including the possibility of miscounting the squares and the rounding error introduced by trying to count "half squares." Additionally, it may not be the most precise method. It also takes a while to do.

b. Answers will vary. Using two different methods will most likely result in different numerical values, since the area in both cases is an approximation and therefore subject to error.

Problem A4. Answers will vary. One useful method is to use paper with a finer grid, as smaller squares (units) will result in fewer rounding errors.

Problem A5. This is a reasonable method of approximation, although there are still other forms of measurement error that can have an effect on the calculation. We can increase the accuracy of our measurement by making the units smaller, for example, using mm^2 instead of cm^2 grid paper. But because we are physically measuring it, the area will always be an approximation, no matter how small the unit (and there's always a smaller unit!). The measurement process always results in an approximate rather than an exact value.

Problem A6. Answers will vary. You can make this approximation by multiplying your palm's area by 100.

Part B: Exploring Area With a Geoboard

Problem B1.

a. The total area is 7.5 square units.

b. The total area is 10 square units.

c. The total area is 8 square units.

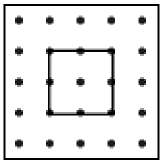
d. The total area is 3.5 square units.

e. The total area is 10 square units.

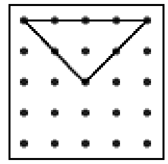
Solutions, cont'd.

Problem B2.

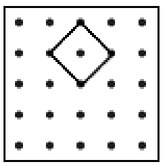
a. A = 4 square units



b. A = 4 square units



c. A = 2 square units



Problem B3.

a. The area of the rectangle formed is 12 square units, so the triangle's area is 6 square units.

b. The area of the rectangle formed is 2 square units, so the triangle's area is 1 square unit.

c. This time, two rectangles must be formed. The one on top has an area of 6 square units, and the one on the bottom has an area of 4 square units, for a total area of 10 square units; therefore, the area of the figure is 5 square units.

d. The area of the rectangle formed is 9 square units, so the triangle's area is 4.5 square units.

e. Dividing the kite into four right triangles (formed by the kite's diagonals) and surrounding them by rectangles gives an area of the rectangles of 12 square units. So the kite's area is 6 square units.

Problem B4.

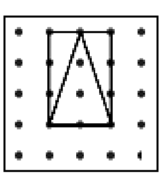
a. The surrounding rectangle has an area of 6 square units, and there are two triangles with a total area of 3 square units to "subtract," so the figure's area is 3 square units.

b. The surrounding rectangle has an area of 16 square units, and there are three triangles with a total area of 9.5 square units to "subtract," so the figure's area is 6.5 square units.

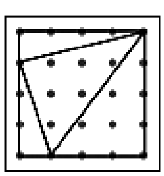
c. The surrounding rectangle has an area of 12 square units, and there are two triangles with a total area of 8 square units to "subtract," so the figure's area is 4 square units.

d. The surrounding rectangle has an area of 9 square units, and there are three triangles with a total area of 4.5 square units to "subtract," so the figure's area is 4.5 square units.

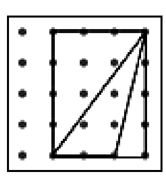
e. The surrounding rectangle has an area of 9 square units, and there are three triangles with a total area of 5 square units to "subtract," so the figure's area is 4 square units.



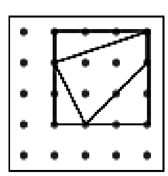
a.



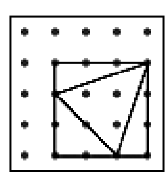
b.



c.



d.

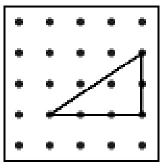


e.

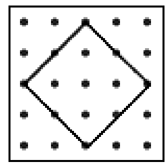
Solutions, cont'd.

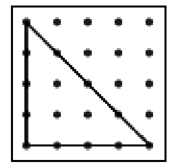
Problem B5.

a. Here is one possible solution:



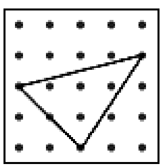
b. Here is one possible solution:



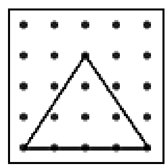


In this example, the area of each is 8. Other solutions are also possible—see, for example, Problem B2 (a) and (b). Regardless of what the area is, though, the square will always have the smaller perimeter.

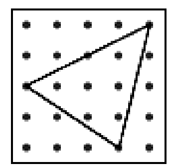
c. These can all be done using the geoboard:



A = 5 square units



A = 6 square units



A= 7 square units

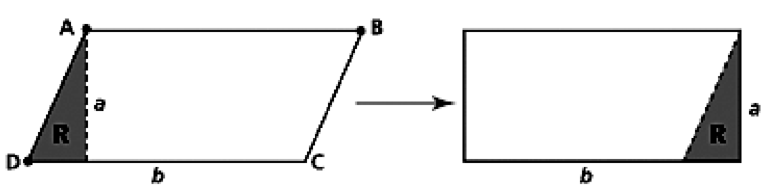
Problem B6.

a. On the geoboard, the area of a rectangle can be found by counting the unit squares or multiplying the length by the width, which is the same as the formula A = $l \cdot w$.

b. It's easy to visualize this in the case of the right triangle. Using the rectangle method to find the area of a right triangle with base b and height h, you enclosed the triangle in a rectangle with an area equal to $b \cdot h$. You then divided the area in two, since one right triangle has half the area of a rectangle (in other words, two right triangles completely fill a rectangle). This is the same as the formula A = $\frac{b \cdot h}{2}$.

Problem B7.

a. The two figures have the same area since they are made up of the same shapes (take the parallelogram and transform it into a rectangle as shown below). The base of the rectangle is made up of the same shapes that form the base of the parallelogram, so it is still the same length (b). The height is the same as well. The area of the rectangle is base multiplied by height, or $a \cdot b$, so the parallelogram's area must also be $a \cdot b$.



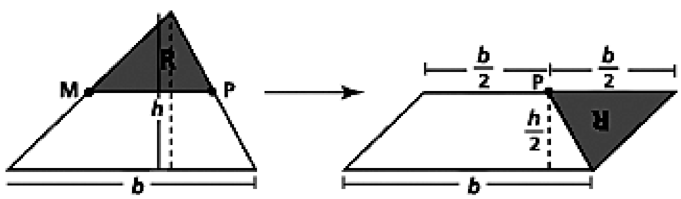
b. Any two identical triangles (isosceles, equilateral, scalene, etc.) can be put together to form a parallelogram. The base and height of the triangle and parallelogram will be equal. Therefore, to find the area of one of the triangles, divide the area of the parallelogram in half:

$$A = \frac{b \cdot h}{2}$$

Solutions, cont'd.

Problem B7(b), cont'd.

Alternatively, you can arrive at the same result using what's known as the midline theorem. As you can see in the picture below, we've transformed a triangle into a parallelogram by cutting along the MP segment (M and P are the midpoints of their respective sides):



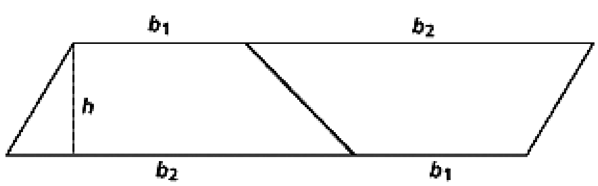
MP, called the midline, is parallel to the base of the triangle and half as long; it divides the height of the triangle in half. The new parallelogram has the same base length as the triangle but half of its height, and, like the triangle, its area can be found with the formula $\frac{b \cdot h}{2}$ —half the product of the base and the height.

To learn more about the midline theorem, go to www.learner.org/learningmath and find *Geometry*, Session 5.

Problem B8.

a. The area of the parallelogram is $(b_1 + b_2) \cdot h$, since both the top and the bottom of the trapezoid comprise the base of the parallelogram.

b. The areas of the two trapezoids add up to $(b_1 + b_2) \cdot h$, so the area of each trapezoid is $\frac{(b_1 + b_2) \cdot h}{2}$.



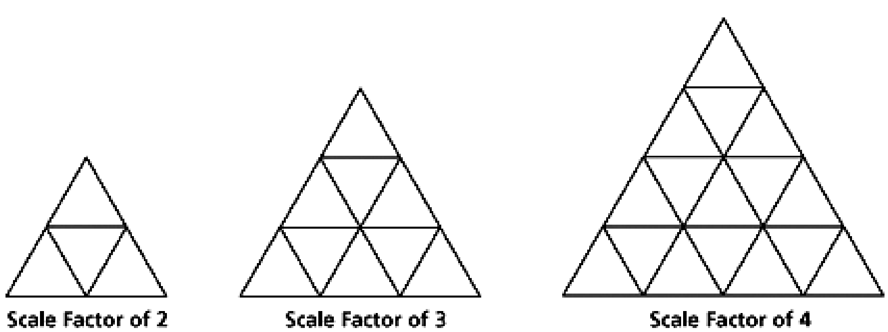
Alternatively, you could find the area of one of the trapezoids by transforming it into a rectangle. The formula will be the same.

To learn more about making such transformations, go to www.learner.org/learningmath and find *Geometry*, Session 5, Part B.

Part C: Scaling the Area

Problem C1.

a. All sketches produce a similar triangle:



Solutions, cont'd.

Problem C1, cont'd.

b. No, it is four times larger. In all cases, the area of a doubled polygon is four times the area of the original. Tripling the side lengths results in a polygon nine times larger in area. Quadrupling the side lengths results in a polygon 16 times larger in area—and so on.

Problem C2. If A_o is the area of the original polygon, then we can write the following:

Polygon	Scale Factor of 2—Area of the Enlargement in Terms of the Original Shape	Scale Factor of 3—Area of the Enlargement in Terms of the Original Shape	Scale Factor of 4—Area of the Enlargement in Terms of the Original Shape
Rectangle C	$4 \cdot A_o$	$9 \cdot A_o$	$16 \cdot A_o$
Parallelogram M	$4 \cdot A_o$	$9 \cdot A_o$	$16 \cdot A_o$
Trapezoid K	$4 \cdot A_o$	$9 \cdot A_o$	$16 \cdot A_o$

Problem C3. The number of copies needed is the square of the scale factor. For example, making a copy that is three times larger in each direction will take nine copies of the original shape.

Problem C4. The area of the enlarged figure is the original area multiplied by the square of the scale factor.

Problem C5. Because the scale factor is 3, the area is nine times larger. Therefore, the area of the enlarged figure is 72 cm^2.

For example, suppose the original figure were a 4-by-2 rectangle (with an area of 8 cm^2). The new shape would then be 12 by 6, with an area of 72 cm^2—nine times the original area. Here's how it breaks down:

$A = 12 \cdot 6$

$A = (4 \cdot 3) \cdot (2 \cdot 3)$

$A = 4 \cdot (3 \cdot 2) \cdot 3$ associative property

$A = 4 \cdot (2 \cdot 3) \cdot 3$ commutative property

$A = (4 \cdot 2) \cdot (3 \cdot 3)$ associative property

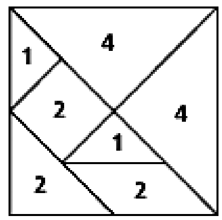
Problem C6. One way to think about it is that enlarging an object will require *k* copies of that object in each direction: *k* copies in one direction, multiplied by *k* copies in the other direction, for a total of k^2.

Problem C7. All of these polygons are rep-tiles. Most rep-tiles have side lengths that have a common factor, but this is not a requirement.

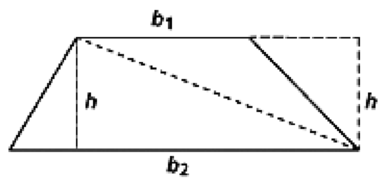
Solutions, cont'd.

Homework

Problem H1. Using the fact that each of the tangram pieces can be divided into some number of small triangles, we get the following areas: The small square, the parallelogram, and the medium triangle each have areas of 2 square units, and the large triangles each have areas of 4 square units. The area of the entire tangram is 16 square units.

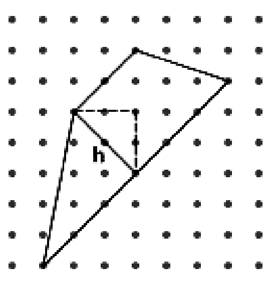


Problem H2. The trapezoid is divided into two triangles, each with height h, the height of the trapezoid. One triangle has base b_1 while the other has base b_2:



Since the area of a triangle is $\frac{b \cdot h}{2}$, the total area is $\frac{b_1 \cdot h}{2} + \frac{b_2 \cdot h}{2}$ or $\frac{(b_1 + b_2) \cdot h}{2}$, which is also the area of the trapezoid.

Problem H3.



Using the Pythagorean theorem, $a^2 + b^2 = c^2$, we can calculate the lengths of the bases and the height. The bases have lengths of $2\sqrt{2}$ (the hypotenuse of the triangle with legs 2 and 2) and $6\sqrt{2}$ (the hypotenuse of the triangle with legs 6 and 6), respectively. Similarly, the height is $2\sqrt{2}$ (it is the length perpendicular to the bases and the hypotenuse of the triangle with legs 2 and 2).

The area is $\frac{(b_1 + b_2) \cdot h}{2} = \frac{(2\sqrt{2} + 6\sqrt{2}) \cdot 2\sqrt{2}}{2} = \frac{8\sqrt{2} \cdot 2\sqrt{2}}{2} = \frac{16(\sqrt{2} \cdot \sqrt{2})}{2} = \frac{16 \cdot 2}{2}$,

which equals 16 square units.

We can also find the area by drawing a rectangle around the trapezoid and then subtracting the smaller areas, or by dividing the trapezoid into two triangles, or by any of several other methods. But any way you slice it, the area is 16 square units.

To learn more about the Pythagorean theorem, go to www.learner.org/learningmath and find *Geometry*, Session 6.

Solutions, cont'd.

Problem H4.

a. Both shapes have areas of 4.5 square units.

b.

Number of Boundary Dots	Area (in Square Units)
3	0.5
4	1
5	1.5
6	2
7	2.5
b	$\frac{b}{2} - 1$

In each case, the area is half the number of boundary dots minus 1. If there are *b* boundary dots and one interior dot, the area is $\frac{b}{2} - 1$.

c.

Number of Boundary Dots	Area (in Square Units)
3	1.5
4	2
5	2.5
6	3
7	3.5
b	$\frac{b}{2}$

In each case, the area is half the number of boundary dots. If there are *b* boundary dots and zero interior dots, the area is $\frac{b}{2}$.

d.

Number of Boundary Dots	Area (in Square Units)
3	2.5
4	3
5	3.5
6	4
7	4.5
b	$\frac{b}{2} + 1$

In each case, the area is half the number of boundary dots plus 1. If there are *b* boundary dots and two interior dots, the area is $\frac{b}{2} + 1$.

Solutions, cont'd.

Problem H4, cont'd.

e. Every time you add a boundary dot, the area goes up by half a unit, regardless of the orientation of the shape.

f. The formula is $A = I + \frac{b}{2} - 1$, and it can be used on many of the figures in Part B of this session.