Oil: Black Gold

Introduction

Large oil spills consistently make international headlines. According to the April 13, 1992, *U.S. News & World Report*, "Some 920,000 barrels of oil—roughly 1 out of every 10 barrels produced [in Russia]—are spilled every day in Russia."

The barrel is the international standard for measuring crude oil. Since 1 barrel of oil equals 42 U.S. gallons, 920,000 barrels of oil represents about 39 million gallons.

Discussion 1

a. Describe the number of gallons mentioned above in more familiar terms. For example, how many times would this much oil fill up your classroom?

b. How is the environment affected when oil is spilled into a body of water?

c. What methods are used to clean up oil spills?

Exploration

What happens when crude oil hits water? What shape does the spill make? How large an area will the spill cover? In the following exploration, you simulate the effects of an oil spill on water.

a. Pour water into a shallow container to a depth of approximately 2 cm.

b. Use a medicine dropper to add one drop of oil to the water. Observe how the oil and water interact.

c. Place a second drop of oil on the center of the first drop and note the changes that occur in the oil slick.

d. Quickly add several more drops of oil to the center of the existing slick. Record the number of drops added and note any changes that occur.

Discussion 2

a. In Part **b** of the exploration, how is the volume of oil in the drop related to the volume of oil in the slick?

b. Describe the geometric properties of the oil slick after the additional drops of oil were added.

c. How thick do you think the oil slick is?

d. How could you estimate the area covered by the surface of the oil slick?

e. When an oil spill occurs in the real world, what natural factors might affect the shape of the slick?

Activity 1

In order to reduce oil's harmful effects on the environment, clean-up efforts typically begin as soon as possible after a spill. To help plan their work, clean-up crews need to know both the area of the surface covered by the spill and the volume of oil. Since precise answers are seldom available, working approximations are used.

Mathematics Note

A **simple closed curve** in a plane is a curve with no endpoints that does not intersect itself. Figure **1** shows some examples of simple closed curves.

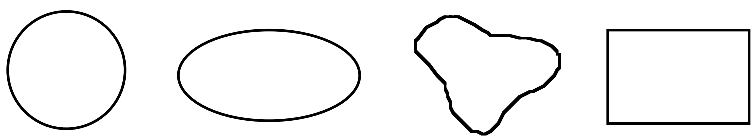


Figure 1: Four simple closed curves

Exploration

Figure **2** shows the shapes of two oil slicks. In the following exploration, you investigate two different methods for estimating the area and volume of these slicks.

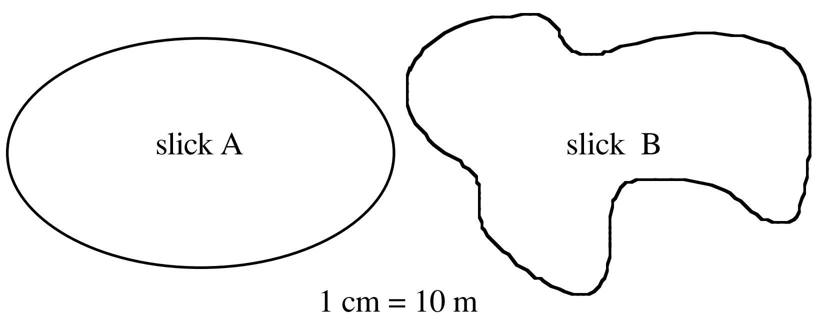


Figure 2: Two oil slicks

a. In the module "What Will We Do When the Well Runs Dry," you estimated the area of irregular shapes by counting squares on a sheet of graph paper.

1. Use this method and a centimeter grid to estimate the area of each slick in Figure **2**.
2. The oil in each slick is 1 mm deep. Use the formula for the volume of a cylinder to determine the volume of each slick.

Mathematics Note

A **cylinder** is a three-dimensional solid with bases that are congruent, simple closed curves (nonpolygons) in parallel planes. For example, Figure **3** shows three different cylinders.

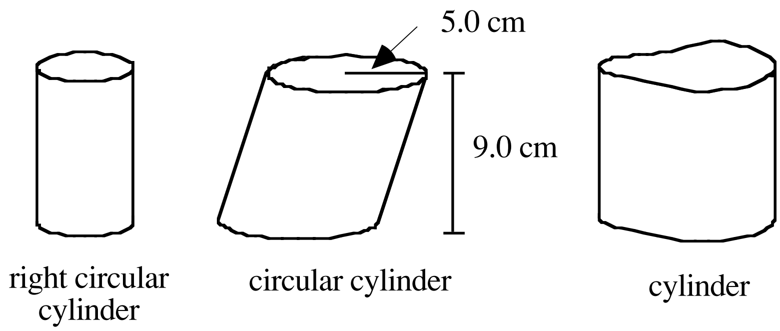


Figure 3: Three cylinders

The volume (V) of a cylinder can be found by multiplying the area of its base (B) by its height (h): $V = B \bullet h$.

For example, to find the volume of the circular cylinder in Figure **3**, you must first calculate the area of the base. Since the base is a circle with a radius of 5.0 cm, its area is $\pi(r^2) = \pi(5 \text{ cm})^2 \approx 79 \text{ cm}^2$. Since the height of the cylinder is 9.0 cm, its volume is $79 \text{ cm}^2(9 \text{ cm}) \approx 710 \text{ cm}^3$.

b. Another method for estimating the area of irregular shapes uses the formula for the area of a circle with a radius r, $A = \pi r^2$. Complete Steps **1–5** below for both slicks in Figure **2**.

1. Locate a point at the approximate center of the slick.
2. Measure the distance from the center to several random points on the edge of the slick.
3. Find the mean of the distances from Step **2**. This mean is the approximate radius of a circle with comparable area.
4. Use the mean radius found in Step **3** and the formula for the area of a circle to estimate the area of the slick.
5. Assuming that the oil is 1 mm deep, determine the volume of the slick.

Discussion

a. When using a grid to estimate area, why do you divide the number of partially covered squares by 2?

b. How could you modify the grid used in Part **a** of the exploration to obtain more accurate estimates of area?

c. How might an environmental engineer find the area covered by the surface of an oil spill?

d. **1.** Compare the two estimates you obtained for the volume of each slick.

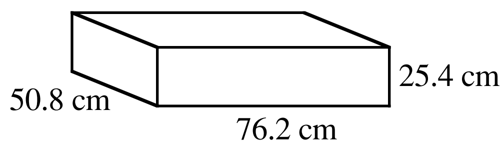
2. If you were in charge of cleaning up slick B, which estimate would you use?

Assignment

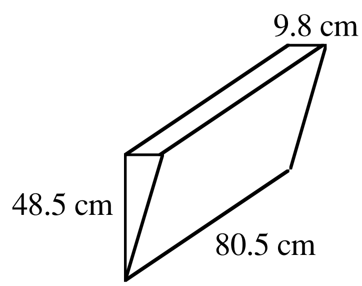
1.1 The formula for the volume of a prism is $V = B \bullet h$, where V is the volume, B is the area of the base, and h is the height.

a. Determine the volume of each fuel tank below in cubic centimeters.

1. This truck fuel tank is a rectangular prism.



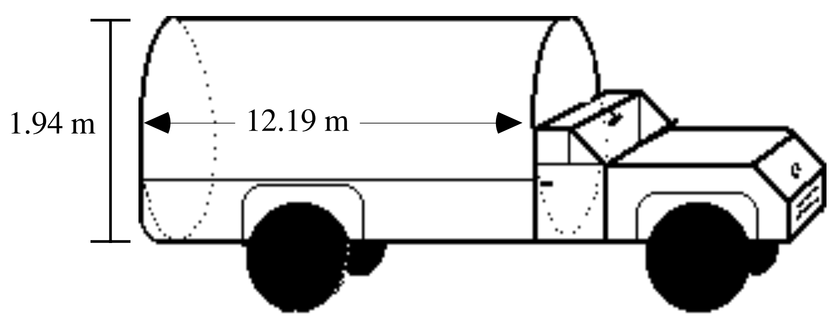
2. This boat fuel tank is a triangular prism in which the bases are right triangles.



b. How many liters of fuel will each tank hold?

1.2 Paper is usually purchased in reams. One ream contains 500 sheets of paper and is about 5.2 cm thick. Find the volume of one sheet of letter-size paper in cubic centimeters.

1.3 The tank on the oil truck in the figure below is a circular cylinder. Determine the volume of the tank.



1.4 Imagine that some highly refined oil is spilled into a calm body of water. Under these conditions, the oil can spread to a very thin film, approximately $2.5 \bullet 10^{-3}$ cm thick.

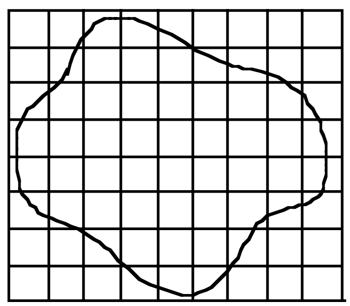
a. What type of three-dimensional figure could be used to describe the oil slick?

b. The mean radius of the slick is 405 m. Determine the volume of oil in the spill in liters.

1.5 **a.** Estimate the area that would be covered by a spill of 1 barrel of highly refined oil in a calm body of water. (There are approximately 3.8 L in 1 gal.) Describe how you determined your estimate.

b. Do you think that an actual spill of 1 barrel of oil will spread as much as you estimated in Part **a**? Why or why not?

1.6 The following diagram shows an aerial view of an oil slick. The grid superimposed on the photograph has squares that measure 1 km on each side.



a. Estimate the area covered by the slick.

b. Determine the volume of oil involved in the spill if the slick is 0.05 cm thick. Record your answer in barrels. (There are 42 gal in 1 bbl.)

1.7 In December of 1989, an explosion on an Iranian supertanker spilled 19 million gallons of crude oil into the Atlantic Ocean. The oil slick covered an area of about 260 km^2.

a. Approximately how thick was this oil slick?

b. How does the thickness of this spill compare to the thickness of a spill of highly refined oil?

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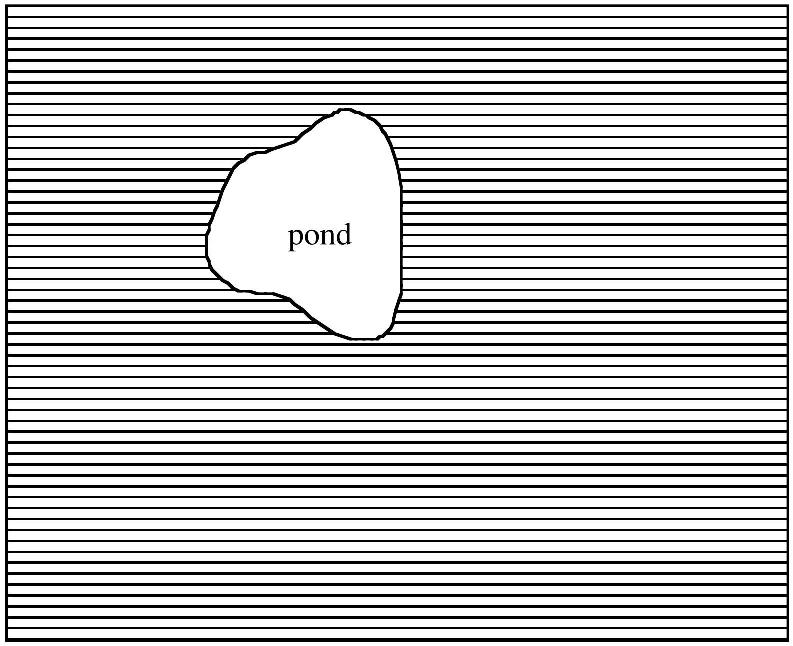
1.8 The federal government assures wheat farmers of a certain base price per bushel if they farm a limited acreage. In order to determine if a farmer is in compliance, regulators take an aerial photograph of the farm. The diagram below shows an aerial view of a wheat field with a pond. This farm has been allotted a maximum of 200 acres of wheat. (One acre is approximately 4047 m^2.) Is this farmer in compliance?



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1.9 Use an ordinary soft-drink can to complete Parts **a–c** below.

a. Measure the height and diameter of the can.

b. Use these measurements to find the volume of the can in cubic centimeters.

c. Convert the volume of the can in cubic centimeters to milliliters. How does this value compare to the volume printed on the can?

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Activity 2

A **simulation** of a real-world event involves creating a similar, but more simplified, model. In the introduction, for example, you simulated an oil spill on the ocean using a few drops of oil in a pan of water. In this activity, you simulate oil spills on land by placing drops of oil on sheets of paper.

Exploration

In this exploration, you simulate spills involving eight different volumes of oil.
Note: Save your data, observations, and calculations for the assignment in this activity.

a. Obtain a small amount of oil, a medicine dropper, a ruler, eight sheets of absorbent paper, and some paper towels.

b. Spread the eight sheets of absorbent paper on a flat, nonabsorbent surface. Arrange the sheets so that they do not touch each other. Number the sheets from 1 through 8 and place a pencil dot in the middle of each sheet. Be careful not to fold or wrinkle the paper.

c. Carefully place 8 drops of oil on the pencil dot on sheet 8. Make the size of each drop as consistent as possible.

Continue creating oil spills of different volumes by placing 7 drops on sheet 7, 6 drops on sheet 6, and so on.

d. Without disturbing the sheets of paper, observe each spill and record your observations. Describe the general relationship between the volume of oil (number of drops) and the shape and area of the spill.

e. Determine the mean radius of each spill to the nearest 0.1 cm. Start with sheet 1 and continue in numerical order to sheet 8.

f.
1. Determine the area covered by each spill and record these values in Table **1** below.

Table 1: Volume and area of oil spills

Volume (drops)	Area (cm^2)
0	0
1	
2	
⋮	
8	

2. Create a scatterplot of the data in Table **1**. Represent the area covered by the spill on the *y*-axis and the volume of oil in drops on the *x*-axis.

3. Determine the equation of a line that reasonably approximates the data. The line should have the same *y*-intercept as the scatterplot.

g. Use your equation to predict the area of an oil spill with each of the following volumes:

1. 0.04 drops
2. 0.5 drops
3. 2.5 drops
4. 25 drops (≈ 1 mL)
5. 25,000 drops (≈ 1 L)

h. For each spill in Table **1**, calculate and record the ratio of the area covered to the volume (number of drops).

i. Dispose of the oil-soaked paper as directed by your teacher.

Discussion

a. What observations did you make concerning the spread of the oil, the shape of the spill, and the area covered by the spill?

b. What problems did you encounter in measuring the area covered by the oil spills?

c.
1. How does the precision of your measurement of the mean radius affect the accuracy of the calculated area?
2. What implications does your response have for measuring actual oil spills?

d.
1. Describe the area you predicted for an oil spill of 1 L (approximately 25,000 drops) in Part **g** of the exploration.
2. Do you think that this prediction is realistic? Explain your response.

e. Is it reasonable to assume that 0 drops of oil produce an oil spill with a surface area of 0 cm^2? Explain your response.

f.
1. Describe any pattern you observed in the ratios of surface area to volume determined in Part **h** of the exploration.
2. What method might you use to find a single number *m* to represent all these ratios?
3. If this value of *m* were used to write an equation $y = mx$, what would each variable represent in terms of the oil spills?
4. How does this equation compare to the one you found in Part **f** of the exploration?
5. Could this equation be used to accurately predict the area of an oil spill of 500 bbl on the ground? If not, how could you change the model to improve the prediction?

Mathematics Note

One quantity is **directly proportional** to another when the ratio of the two quantities is constant (the same). The constant is the **constant of proportionality** and the ratio is a **direct proportion**.

A direct proportion can be described by a linear equation of the form $y = mx$, where m is the constant of proportionality. The graph of a direct proportion always contains the origin since $y = 0$ whenever $x = 0$.

For example, in the direct proportion $y = 2x$, the constant of proportionality is 2. A graph of $y = 2x$ is shown in Figure **4**.

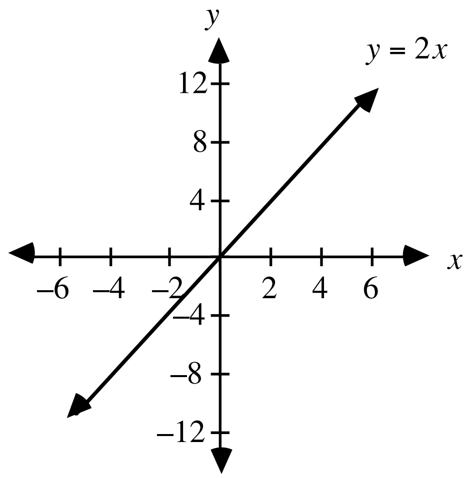


Figure 4: Graph of a direct proportion

g. Given the direct proportion $y = mx$, describe how the values of y change as:

1. the values of x increase when $m > 0$

2. the values of x increase when $m < 0$

h. Are an oil spill's area and volume directly proportional? Explain your response.

i. If you know any point with coordinates (p,q) on the graph of a direct proportion, what is the slope of the line?

Assignment

2.1 Determine if x and y are directly proportional in each of the following relationships. Defend your responses.

a.

x	1	3	4	6	25
y	3	9	12	18	75

b.

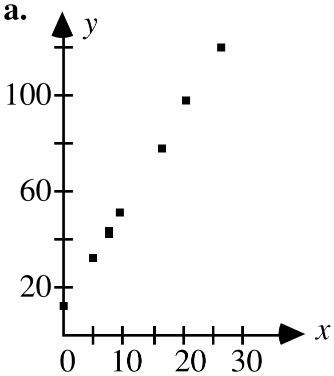
x	1	3	4	6	25
y	8	14	17	23	80

c.

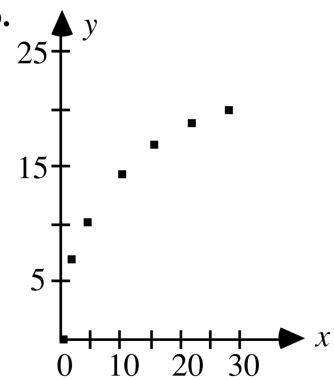
x	0	3	4	5	7
y	0	10	17	26	50

2.2 For each of the scatterplots determine if they appear to represent a direct proportion. Defend your answer.

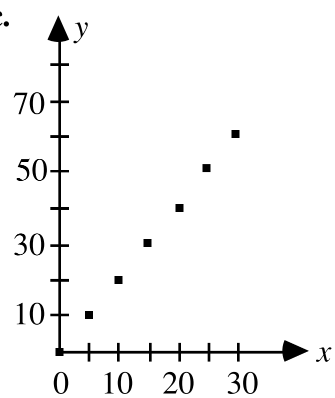
a.



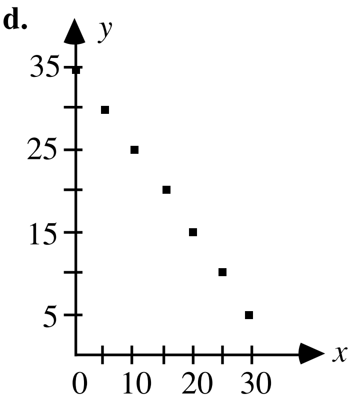
b.



c.



d.



2.3 **a.** Graph each of the following equations of direct proportions on the same coordinate system, with $0 \le x \le 10$. Compare the graphs.

1. $y = 0.5x$

2. $y = 5x$

3. $y = 1x$

4. $y = -0.25x$

5. $y = 1.75x$

6. $y = -3.5x$

b. In your own words, describe the characteristics of the graph of a direct proportion.

2.4 Use your data from the exploration to answer the following questions.

a. Is the mean radius of an oil spill directly proportional to the volume of the spill?

b. Is the square of the mean radius directly proportional to the volume?

2.5 When oil is spilled on a sheet of absorbent paper, the spill can be modeled by a cylinder. In this situation, the base of the cylinder is the shape of the spill, while the height of the cylinder is the thickness of the paper.

The volume of a cylinder is determined by multiplying the area of the base by the height. Is the volume of the spill directly proportional to the area covered by the spill? Explain your response.

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2.6 In a circle, the circumference (C) is directly proportional to the diameter (d). The constant of proportionality is π.

a. Write an equation for this direct proportion.

b. Is the relationship for each of the following a direct proportion? If so, write an equation for the proportion and identify the constant of proportionality. If not, explain why not.

1. the circumference of a circle and its radius

2. the area of a circle and its radius

2.7 The relationship between temperature measured in degrees Fahrenheit (F) and temperature measured in degrees Celsius (C) is:

$$C = \frac{5}{9}F - \frac{160}{9}$$

Explain why this relationship is not a direct proportion.

2.8 Hailstones are formed when raindrops are caught in updrafts and carried into high clouds containing very cold air. The radius of a hailstone is directly proportional to the amount of time it remains in the high cloud.

a. After remaining in a high cloud for 10 sec, a hailstone has a radius of about 1.3 cm. What was the radius of the hailstone after 1 sec?

b. Write an equation for this direct proportion.

c. How long would a hailstone have to remain in high clouds to reach a radius greater than 2.5 cm?

2.9 In 1980, per capita personal income in the United States was $9948. Ten years later, per capita income rose to $18,699.

a. What was the total increase in per capita personal income from 1980 to 1990?

b. Assume that the yearly increases in per capita income and the number of years after 1980 are directly proportional.

1. What was the yearly increase in per capita personal income between 1980 and 1990?

2. Write an equation for the direct proportion between yearly increases in per capita income and number of years after 1980.

c. 1. Predict the per capita personal income in 1994.

2. Write an equation that could be used to model the per capita income for any year after 1980.

3. Does this equation define a direct proportion? Explain your response.

d. The actual per capita personal income in 1994 was $21,846. What limitations might the model in Part **c** have for predicting per capita personal income?

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Activity 3

As soon as a quantity of oil is spilled, it starts to spread. If not contained, the resulting slick can cover a very large area. As the oil continues to spread, the depth of the slick decreases. In the following exploration, you investigate the relationship between the depth of a spill and the area it covers.

Exploration

When liquid is poured into a cylindrical container, the surface of the liquid takes the same shape as the base of the container. In this exploration, you use a fixed amount of water to represent the volume of an oil spill. The spread of the spill is simulated by pouring the water into several containers with different base areas. **Note:** Save your work, including the spreadsheet, for use in the assignment.

a. Obtain a cylindrical container from your teacher. Determine and record its base area (B) in square centimeters.

b. Pour 200 mL of water into the container. Measure and record the height (h) of the water in centimeters.

c. Calculate $B \bullet h$, where B is the base area and h is the height of the water in the container. Label the product with the appropriate units and record the result.

d.
1. Collect the class data for the different containers.
2. Enter the data in a spreadsheet.
3. Sort the data so that the base areas appear in ascending order (from least to greatest).

e. Create a scatterplot of the height of the water versus the base area.

Discussion

a.
1. What should be true of each value for $B \bullet h$ calculated in Part **c** of the exploration?
2. Do the class values support this conclusion? Explain your response.

b. Describe the graph obtained in Part **e** of the exploration.

c. Consider a right circular cylinder and a right triangular prism with the same base areas. Each contains an equal volume of water. Does the height of the water depend on the shapes of the bases?

d.
1. In the exploration, what happened to the height of the water as it was poured into containers with larger base areas?
2. Is the height directly proportional to the area of the base? Explain your response.

e. Do you think that examining the heights of liquid in a series of containers with increasing base areas provides a good model of the spread of an oil spill? Explain your response.

Assignment

3.1 In the exploration, you poured 200 mL of water into containers with different base areas.

a. Let *B* represent the base area of a container and *h* represent the height of water in the container. Write an equation that describes the relationship of *B* and *h* to 200 mL.

b. Solve this equation for *h*.

c. Graph the equation on the same set of axes as the scatterplot from Part **e** of the exploration.

d. Which appears to be the better model of the experiment in the exploration—the scatterplot or the graph of the equation? Defend your choice.

e. Why can there be no negative values for *h?*

3.2 **a.** Solve the equation you wrote in Problem **3.1a** for *B*.

b. **1.** Predict the area covered by an oil spill of 200 mL if it spreads to a thickness of $2.5 \bullet 10^{-3}$ cm.

2. If the spill is circular, what is its diameter?

3.3 Consider a spill of 100 bbl of highly refined oil on a calm body of water.

a. Write an equation that models this spill in terms of *B*, *h,* and 100 bbl.

b. How many cubic centimeters of oil are there in 100 bbl? (There are approximately 160 L in 1 bbl; 1 cm^3 contains 1 mL.)

c. Rewrite your equation in Part **a** by replacing 100 bbl with its equivalent in cubic centimeters.

d. Determine the area (in square kilometers) covered by a 100-bbl spill that spreads to a thickness of $2.5 \bullet 10^{-3}$ cm.

e. If the spill is circular, what is its diameter?

3.4 **a.** Solve the following equation for y: $20 = x \bullet y$.

b. Choose at least five different values for x. Find the corresponding y-values and organize these results in a table.

c. As the values of x increase, what happens to the corresponding values of y? Is this consistent with what you observed in the exploration? Explain your response.

Mathematics Note

One quantity is **inversely proportional** to another when the product of the two quantities is constant. An inverse proportion can be described by an equation of the form $xy = k$, where k is the **constant of proportionality**. The equation of an inverse proportion can also be written in the form $y = k/x$.

For example, the inverse proportion $xy = 2$ can be written as $y = 2/x$. The graph of $y = 2/x$ is shown in Figure **5**.

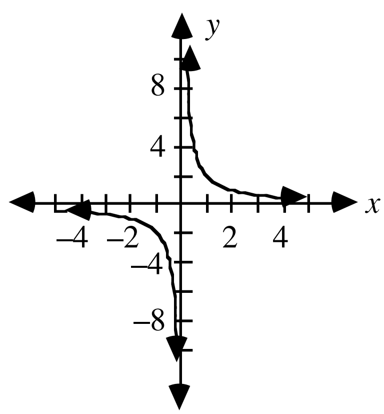

Figure 5: Graph of an inverse proportion

3.5 **a.** Solve each of the following inverse proportions for y in terms of x and identify the constants of proportionality.

1. $x \bullet y = 20$
2. $x \bullet y = 10$
3. $y \bullet x = 5$
4. $0.5 = y \bullet x$

b. Using a graphing utility, graph the equations from Part **a** on the same set of axes, where $-10 \le x \le 10$. What appears to be true about each of the graphs?

c. 1. As the x-values approach 0, what happens to the corresponding y-values?

2. As the x-values increase from 100 to 1000 to 10,000, what happens to the corresponding y-values?

3. As the x-values decrease from –100 to –1000 to –10,000, what happens to the corresponding y-values?

d. How does the constant of proportionality affect the graph?

e. What does the constant of proportionality represent in the equation you wrote in Problem **3.1**?

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3.6 **a.** Compare the shapes of the graphs of a direct proportion and an inverse proportion.

b. Compare the equations of a direct proportion and an inverse proportion.

3.7 Consider the set of ordered pairs shown in the following table.

x	y
1	100
2	50
4	25
5	20
10	10
20	5
25	4

a. Create a scatterplot of this data.

b. As the values of x become large, what happens to the values of y?

c. As the values of x become small, what happens to the values of y?

d. Is the relationship between x and y a direct proportion, an inverse proportion, or neither? Justify your conclusion.

e. Write an equation that describes this relationship and identify the constant of proportionality, if one exists.

f. Describe a real-world situation that might generate these ordered pairs.

3.8 At 0 ˚C, 32.0 g of oxygen gas occupies a volume of 22.4 L with a pressure of 1.0 atmosphere (atm). As the volume is decreased at constant temperature, the pressure changes as shown in the following table.

Volume (L)	Pressure (atm)
22.4	1.00
17.1	1.31
11.2	2.00
5.60	4.00
2.24	10.0

a. Create a scatterplot of the data.

b. At constant temperature, are pressure and volume directly proportional or inversely proportional? Explain your response.

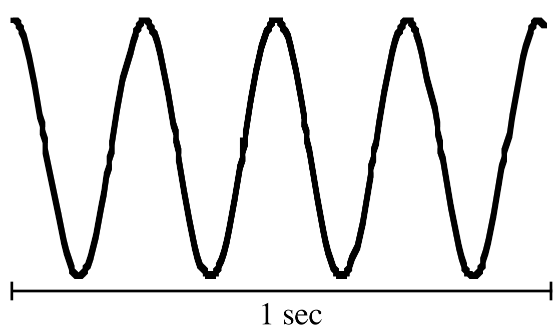
c. **1.** Write an equation that describes the relationship and identify the constant of proportionality.

2. Graph this equation on the same set of axes as the scatterplot from Part **a**.

d. Predict the pressure on 32.0 g of oxygen gas when the volume is 20.5 L.

3.9 Two characteristics of waves are wavelength and frequency. **Wavelength** is the distance between two consecutive peaks or troughs in a wave. **Frequency** is the number of wavelengths that pass a given point in a certain amount of time.

The standard unit of frequency is the **hertz** (**Hz**). One hertz is equal to 1 cycle per second. For example, the diagram below shows a wave with a frequency of 4 cycles per second, or 4 Hz.



The table below shows the wavelengths and frequencies of some different forms of electromagnetic radiation.

Electromagnetic Radiation	**Wavelength (m)**	**Frequency (Hz)**
gamma rays	1.0×10^{-12}	3.0×10^{20}
X rays	1.0×10^{-10}	3.0×10^{18}
red light	7.0×10^{-7}	4.29×10^{14}
microwaves	1.0×10^{-2}	3.0×10^{10}
radio waves	1.0×10^{2}	3.0×10^{6}

a. Is the relationship between wavelength and frequency a direct proportion or an inverse proportion? Explain your response.

b. Write an equation that describes this relationship.

c. Identify the constant of proportionality.

d. Violet light has a wavelength of about 4.0×10^{-7} m. Use your equation from Part **b** to determine the frequency of violet light.

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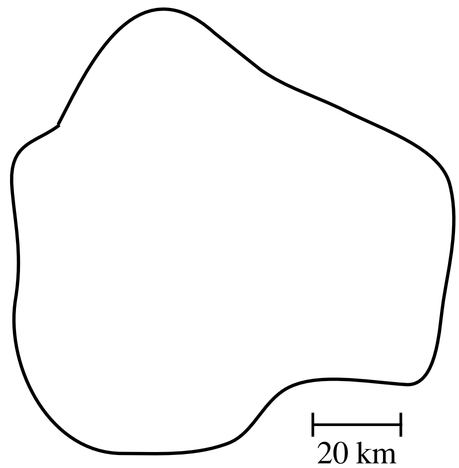
Summary Assessment

1. The diagram below shows the shape of an oil slick that spread to a thickness of $2.5 \bullet 10^{-3}$ cm.



Estimate the volume of oil in the spill in gallons.

2. **a.** Assuming that the spilled oil spreads to a thickness of $2.5 \bullet 10^{-3}$, complete the following table.

Area of Slick (km^2)	Volume of Spill (gal)
5000	
7500	
10,000	
25,000	

b. Determine the type of relationship formed by the data collected in the table. Create a graph that displays this relationship.

c. Find an equation that models the graph from Part **b.**

d. Use this equation to predict the number of gallons of oil that would create a slick of 71,000 km^2.

3. Assuming constant pressure, the time required to fill an oil tank is inversely proportional to the square of the diameter of the hose used to fill it. The table below shows the diameters of four hoses and the corresponding times to fill the tank.

Diameter (cm)	**Square of Diameter**	**Time (min)**
2		36
3		16
4		9
6		4

a. Complete the column for the squares of the diameters.

b. Create a scatterplot of the time versus the square of the diameter.

c. Find an equation that represents this inverse proportion.

d. Predict how long it would take to fill the tank using a hose with a diameter of 10 cm.

e. Predict the diameter of a pipe that could fill the tank in 30 min.

Module Summary

- A **simple closed curve** in a plane is a curve with no endpoints that does not intersect itself.
- A **cylinder** is a three-dimensional solid with bases that are congruent simple closed curves (non-polygons) in parallel planes.
- The volume (V) of a cylinder can be found by multiplying the area of its base (B) by its height (h): $V = B \bullet h$.
- One quantity is **directly proportional** to another when the ratio of the two quantities is constant (the same). The constant is the **constant of proportionality** and the ratio is a **direct proportion**.
- A direct proportion can be described by an equation of the form $y = mx$, where m is the constant of proportionality. The graph of a direct proportion always contains the origin since $y = 0$ whenever $x = 0$.
- One quantity is **inversely proportional** to another when the product of the two quantities is constant.
- An inverse proportion can be described by an equation of the form $xy = k$, where k is the **constant of proportionality**. The equation of an inverse proportion can also be written in the form $y = k/x$.

Selected References

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