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## CURRENT CHARGES

**AT&T Invoice Charges For Period Ending OCT 27, 1996**

### Long Distance

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**Total Itemized Calls**

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Right Hand/Left Hand Experiment

The purpose of this experiment is to explore the patterns associated between time and the number of letters one can print on paper. Students will determine the difference in speed between the right hand and left hand.

1. Gathering Data
   Follow the instructions given by your teacher. Create a table of values for each data set.

2. Displaying the Data
   Make a scatterplot that shows both sets of data on the same coordinate grid.

3. Analyzing the Data
   a. Draw a line of best fit for each set of data on your graph.
   b. Find an equation for the line of best fit for each set of data.
   c. Compare your right-hand and left-hand models. How are they alike and how are they different?
   d. Are you right-handed or left-handed? Now analyze the data for your preferred hand. Is the slope greater than or less than the slope of your non-preferred hand? What does the slope represent in this situation?
   e. Do the graphs ever intersect? If so, where? What does it mean when the graphs intersect? If the graphs do not intersect, what does this tell you?
## DISPLAYING DATA

### RIGHT-HAND DATA

<table>
<thead>
<tr>
<th>Time</th>
<th>Number of letters</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### LEFT-HAND DATA

<table>
<thead>
<tr>
<th>Time</th>
<th>Number of letters</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Homework 8  Picturing Pictures

Hassan is an artist who specializes in geometric designs. He is trying to get ready for a street fair next month.

Hassan paints both watercolors and pastels. Each type of picture takes him about the same amount of time to paint. He figures he has time to make a total of at most 16 pictures.

The materials for each pastel will cost him $5 and the materials for each watercolor will cost him $15. He has $180 to spend on materials.

Hassan makes a profit of $40 on each pastel and a profit of $100 on each watercolor.

1. Use symbols to represent Hassan’s constraints.

2. Make a graph that shows Hassan’s feasible region; that is, the graph should show all the combinations of watercolors and pastels that satisfy his constraints.

3. For at least five points on your graph, find the profit that Hassan would make for that combination.

A student at West High School in Tracy, CA, explains a plot for Picturing Cookies —Part 1.
Workshop 6 Lesson Materials

Name: ____________________________ Date: ________ Period: ______
Team: _____

SKEETERS ARE OVERRUNNING THE WORLD

How does the size of the population change over the time? In this module, you use a simple model to shed light on some complicated issues.linear,exp2.avi

Introduction

At last count, more than 5.4 billion people inhabited the Earth. If each of us laid head to toe, we would make a chain of humanity long enough to wrap around the equator 250 times.

How many people can live on Earth without destroying the environment? How many people can our planet successfully feed? Several organizations studying the ever-increasing human population are concerned with just these questions.

While foretelling the future is never a sure bet, you can gather information about the past and present, find any existing patterns, and use these patterns to make predictions.

Graph can be useful tools for determining patterns. For example, Figure 1 shows the world population since 1650.

(Power Point Presentation with the graphs and questions for discussion.)
world Pop. Slide show.ppt

EXPLORATION

Statistics like those shown in Figure 1, along with an appropriate mathematical model, allow researchers to make forecasts about population trends. For example, scientists at the United Nations predict a world population of at least 8.2 billion by the year 2020.

To help make predictions in real-world situations, researchers often use experiments known as simulations. The results of the simulations are gathered and analyzed. This data is then compared with known information about the actual population. If the result seems questionable, the simulation may be revised. This modeling process can be summarized by the following five steps:

1. creating a model
2. translating the model into mathematics
3. using the mathematics
4. relating the results to the real-world situation
5. revising the model

In the following exploration, you investigate this modeling process using a population of Skeeters.

a. Obtain a large, flat container with a lid, a sack of Skeeters, and several sheets of graph paper.
Workshop 6 Lesson Materials, cont’d.

b. Before beginning the simulation, read Steps 1-7 below and predict how you think the number of Skeeters will change.

1. Place two Skeeters in your container. This is the initial population.
2. After closing the lid, shake the container.
3. Open the lid and count the number of Skeeters with the marked side up.
4. Skeeters reproduce asexually (by themselves). Reproduction is triggered when the marked side of a Skeeter is exposed to light. Add one Skeeter to the container for each mark counted.
5. Record the total number of Skeeters now in the container. This is the end of one “shake”.
   The end of each shake represents the end of one time period. The number of Skeeters present at the end of a shake is the total population at that time. (Remember that at shake 0, the number of Skeeters was 2)
7. Repeat Parts 2-5 for 10 shakes.

c. 1. Create a scatterplot to display the data you recorded. Represent the shake number on the x-axis. Select a scale for each axis that will allow you to make predictions for shake numbers through 20.
2. Describe any patterns you see in your data.

d. 1. Use the pattern described in Part c to predict the number of Skeeters after shake 20.
2. How large a box would be necessary to hold this population? Explain how you came to this conclusion.
3. Predict how many shakes it would take for the Skeeter population to reach 1000. Describe how you reached your prediction.

Discussion 2

a. Discuss any similarities or differences you observe between your scatterplot and those of your classmates.

b. How did the number of Skeeters in your population change during the exploration?

c. Consider your scatterplot as describing the change in the population of Skeeters over time. Use this idea to explain the shape of the graph.

d. What other types of living creatures might show the same pattern of population growth as the Skeeters?

e. What limitations might this simulation have in modeling a real-world population?
Rallods in Rednow Land

Which is more money?

a. One billion rallods

b. The amount obtained by putting 1 rallod on one square of a chessboard, 2 rallods on the next square, 4 on the next, and so on, until all 64 squares are filled.

Solution: To find the total number of rallods on the chessboard, students must add $1 + 2 + 4 + 8 \ldots + 2^{63}$. The solution is $2^{64} - 1$, which is approximately equal to $1.84 \times 10^{19}$. However, finding the total on all 64 squares is not necessary to answer the question, since the running total surpasses 1 billion by the 30th square.
**ACTIVITY 1**

Within any population, there are differences in appearance and behavior due to genetics and environment. In this activity, you investigate some Skeeter populations with different growth characteristics.

**Exploration**

In this exploration, each color of Skeeter has its own growth characteristics and initial population. Table 1 shows a list of these characteristics for each color.

**Table 1: Skeeter growth characteristics**

<table>
<thead>
<tr>
<th>Color</th>
<th>Growth Characteristics</th>
<th>Initial Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>After every shake, for every green Skeeter with or without the mark showing, add 2 green Skeeters.</td>
<td>1 green</td>
</tr>
<tr>
<td>Yellow</td>
<td>After every shake, for every yellow Skeeter with or without the mark showing, add 1 yellow Skeeter.</td>
<td>1 yellow</td>
</tr>
<tr>
<td>Orange</td>
<td>After every shake always add 2 orange Skeeters</td>
<td>40 orange</td>
</tr>
<tr>
<td>Red</td>
<td>After every shake always add 20 red Skeeters</td>
<td>5 red</td>
</tr>
<tr>
<td>Purple</td>
<td>After every shake, for every purple Skeeter with a mark showing, add 1 purple Skeeter.</td>
<td>5 purple</td>
</tr>
</tbody>
</table>

a. Consider the information given in table 1.
   1. Predict the population of yellow Skeeters after shake 6.
   2. Predict the population of orange Skeeters after shake 6.
   3. After shake 6, which one will be larger?

b. Obtain a large, flat container with a lid, a sack of Skeeters of a chosen color, and a sheet of graph paper. Place the initial population of the chosen color (according to Table 1) in the box.

c. Place the lid on the container and shake it.

d. At the end of each shake, use the growth characteristics from Table 1 to add the appropriate number of Skeeters of the chosen color.

e. Record the total number of Skeeters at the end of six consecutive shakes, on the provided table. (Record the initial population as the number at shake 0.)

f. By filling the Pattern and Process columns of the corresponding table, make a prediction of the population of Skeeters of the chosen color, at the end of shake 11, and for any number of shakes (n).

g. After 6 shakes, graph the data for the chosen Skeeter population on a rectangular coordinate system. (Label the x-axis as Shake Number, and the y-axis as Color of Skeeter Population.)

h. Repeat Parts b-g for the other populations of Skeeters.
Table 2: Growth of the Green Skeeter Population.

<table>
<thead>
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<th>Shake Number</th>
<th>Skeeter’s Population</th>
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<th>Process</th>
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<tr>
<td>n</td>
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Table 3: Growth of the Yellow Skeeter Population.

<table>
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Table 4: Growth of the Orange Skeeter Population.

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Table 5: Growth of the Red Skeeter Population.

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Table 6: Growth of the Purple Skeeter Population.

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</tr>
</tbody>
</table>

Discussion

a. Describe the relationship between the numbers of yellow Skeeters at the end of two consecutive shakes.
b. 1. Describe the relationship between the number of yellow Skeeters at the end of a shake and the shake number.
   2. Restate this relationship as a mathematical equation.
c. Describe the relationship between the numbers of orange Skeeters at the end of two consecutive shakes.
d. 1. Describe the relationship between the number of orange Skeeters at the end of a shake and the shake number.
   2. Restate this relationship as a mathematical equation.
e. What differences and similarities do you observe between the yellow and orange Skeeter Populations?
f. At the beginning of the lesson, we presented You with 2 different models of growth in Nature. Write a mathematical equation for each example.
Thirteen/WNET New York

A major American cultural and educational institution for nearly four decades, Thirteen/WNET supplies more than one-third of all primetime programs aired on PBS, including acclaimed cultural, science, and public affairs series and specials. The award-winning Children’s and Educational Programming group is a leading and innovative provider of programming for a variety of projects, from teacher professional development to instructional television and interactive multimedia. Broadcast series that further the station’s educational mission include the daily animated PBS Kids math program Cyberchase, the history series for families Freedom: A History of US, ZOOM Local/National, What’s Up?, and In the Mix specials. Many projects promote implementation of national and state education standards. These include Science ... Simply Amazing, Learning Science Through Inquiry, and The Expanding Canon: Teaching Multicultural Literature in High School for Annenberg/CPB, and PBS TeacherLine, Mathline, and Scienceline.

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