Unit 23: Control Charts

PREREQUISITES

Unit 22, Sampling Distributions, is a prerequisite for this unit. Students need to have an understanding of the sampling distribution of the sample mean. Students should be familiar with normal distributions and the 68-95-99.7% Rule (Unit 7: Normal Curves, and Unit 8: Normal Calculations). They should know how to calculate sample means (Unit 4: Measures of Center).

ADDITIONAL TOPIC COVERAGE

Additional coverage of this topic can be found in *The Basic Practice of Statistics*, Chapter 27, Statistical Process Control.

ACTIVITY DESCRIPTION

This activity should be used at the end of the unit and could serve as an assessment of $\bar{x}$ charts. For this activity students will use the Control Chart tool from the Interactive Tools menu. Students can either work individually or in pairs.

MATERIALS

Access to the Control Chart tool. Graph paper (optional).

For the Control Chart tool, students select a mean and standard deviation for the process (from when the process is in control), and then decide on a sample size. After students have determined and entered correct values for the upper and lower control limits, the Control Chart tool will draw the reference lines on the control chart. (Remind students to enter the values for the upper and lower control limits to four decimals.) At that point, students can use the Control Chart tool to generate sample data, compute the sample mean, and then plot the mean

against the sample number. After each sample mean has been plotted, students must decide either that the process is in control and thus should be allowed to continue or that the process is out of control and should be stopped.

In order to verify their work, students need to either make sketches of their control charts or copy screenshots into a Word document. If you want students to sketch their control charts by hand, they will need graph paper. In hand-drawn sketches, students should aim for capturing the approximate pattern of the control chart and not strive for accuracy. The better approach would be to have students do a screen capture and then paste it into a Word document.

Here's how to accomplish this on a PC:

1. Hold the Alt key while pressing the Print-Screen key.
2. In an open Word document, hold the Ctrl key while pressing V to paste the screen capture into their document.

On a Mac:

1. Press "Command" + "Shift" + "3" to capture the entire screen.
2. The file will be saved on your desktop.

THE VIDEO SOLUTIONS

1. He was among the first to champion the idea of statistical process management. He helped Japanese industry rebuild after World War II.

2. A process is a chain of steps that turns inputs into outputs. For example, start with raw foods, prepare them for cooking, combine the prepared ingredients, cook them, serve them on a plate – the served food is the output.

3. When a process is running smoothly, with its variables staying within an expected range.

4. They had a 2:00 a.m. target to get all specimens logged in and ready for processing. They were rarely meeting this target.

5. The control limits were set three standard deviations above and below the center line.

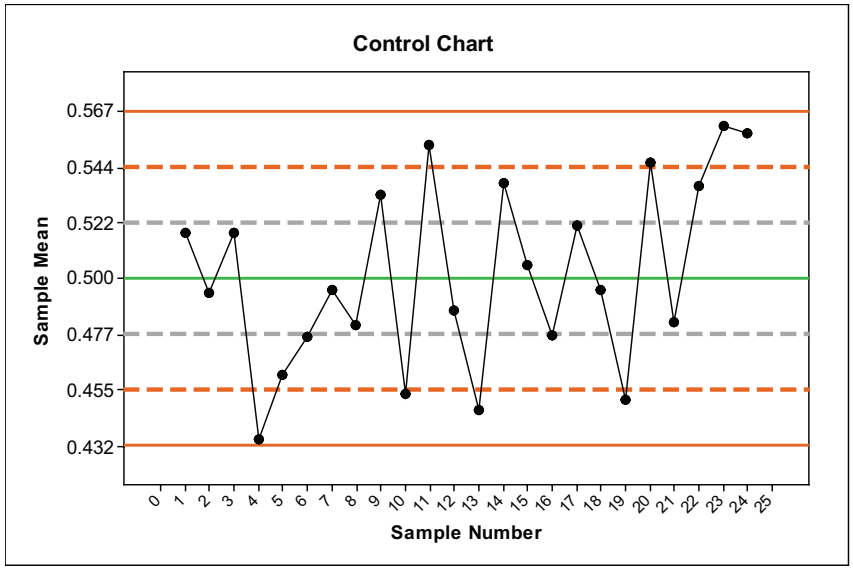
6. Quest remodeled the entire department dividing it into self-contained pods and changing staffing. These changes brought the mean finish time closer to the 2:00 a.m. target.

UNIT ACTIVITY SOLUTIONS

1. Note: Students' control charts will vary even if they make the same choices for mean, standard deviation, and sample size.

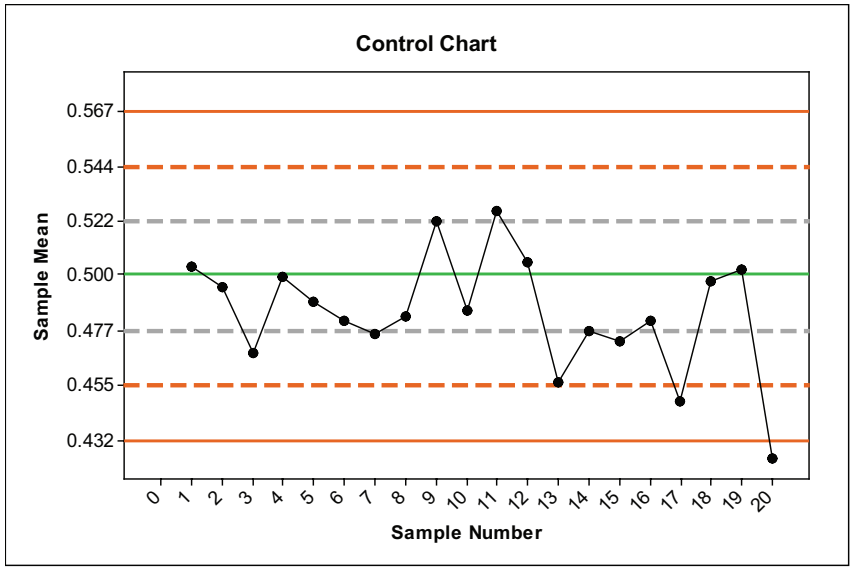
Sample answer: We selected $\mu = 0.5$, $\sigma = 0.05$, and $n = 5$.

Control limits: $0.5 \pm 3(.05/\sqrt{5})$; LCL = 0.4329 and UCL = 0.5671.



We stopped the process at sample 24. Samples 22, 23, and 24 had two of three consecutive points above the $2\sigma/\sqrt{n}$ limit of 0.544; these points were on the same side of the center line. So, using Rule 2, we made the decision to stop the process.

2. Sample answer: See solutions to question 1 for the settings and control limits.

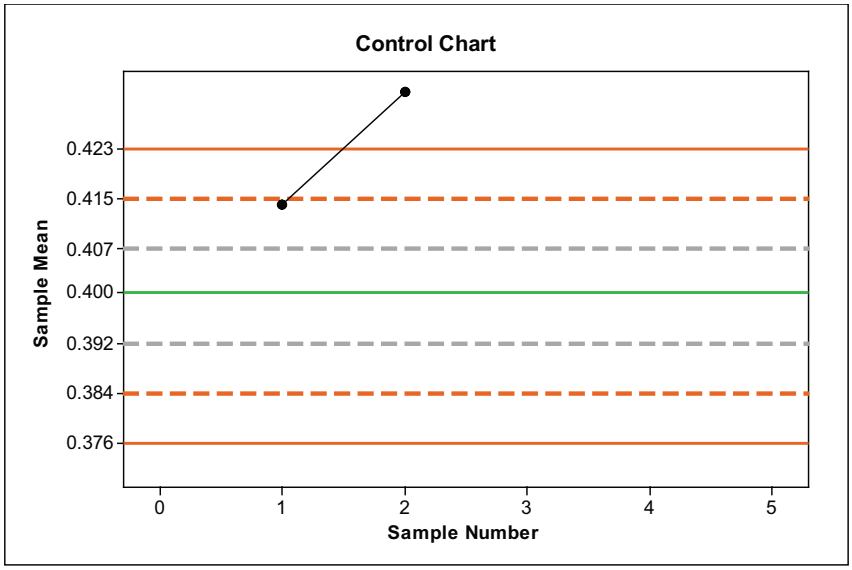


We stopped the process at sample 20. Sample 20 had a sample mean that fell below the lower control limit. So, using Rule 1, we made the decision to stop the process.

3. a. We selected $\mu = 0.4$, $\sigma = 0.025$, and n = 10.

Control limits: $0.4 \pm 3(0.025/\sqrt{10})$; LCL = 0.376 and UCL = 0.424.

b.



c. Feedback: We had made the right decision and deserved a bonus.

d. The second sample had a sample mean that was larger than the upper control limit. Based on Rule 1, we made the decision to stop the process.

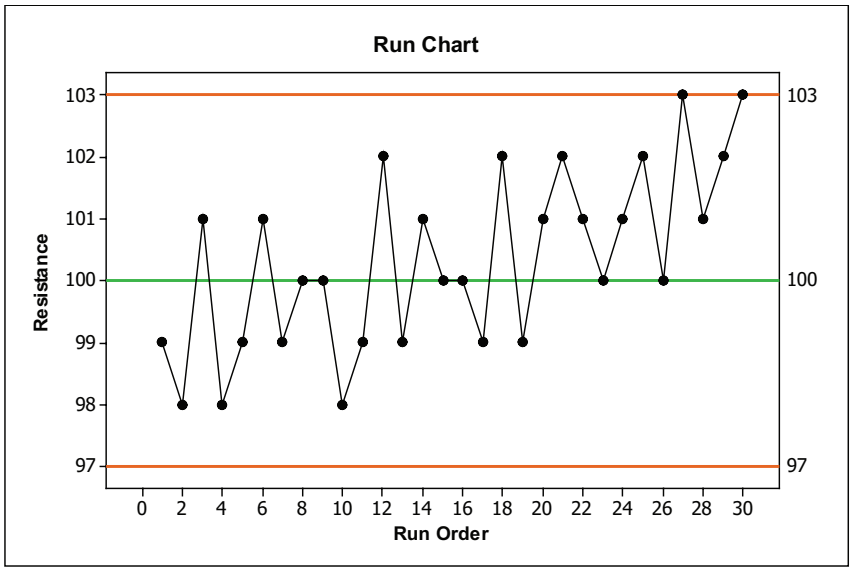
4. See sample answers to questions 1 - 3.

EXERCISE SOLUTIONS

1. a. See run chart for part (c) and refer to the portion of the plot associated with run orders 1 – 15.

b. Sample answer: There is no evidence that the process is out of control. The dots appear randomly scattered above and below the reference line of 100 ohms. No dots are outside of the tolerance interval.

c.



d. Sample answer: Although no dots are outside the tolerance limits, the values for resistance appear to be increasing over run order. The dots associated with run orders 20 to 30 all lie on or above the target value of 100 ohms. In the last four data values, two of the four values were at the upper tolerance level. Management needs to investigate the cause of the changes in the distribution of resistance.

2. a. Using the 68-95-99.7 rule, 95% of the caps will have diameters within two standard deviations of the mean – within 0.497 and 0.503 inch. Thus, around 5% of the bottles will have diameters outside of the chemical manufacturer's specification limits.

b. $\bar{x}$ will have a normal distribution with mean 0.500 inch and standard deviation 0.0015/3 = 0.0005 inch.

c. The endpoints of the acceptance interval can be written as 0.500 ± 0.001, which is equivalent to 0.005 ± 2(0.0005). Hence, 95% of the samples will have means within this interval. The production process will be stopped 5% of the time (or a proportion of 0.05).

3. a. The process is not in control. Rule 3 applies. Look at the dots corresponding to Samples 3, 4, 5, 6, and 7. The means of the last four of these samples fall below the $\sigma/\sqrt{n}$ limit and are on the same side of the center line.

b. The process is in control. None of the decision rules applies.

c. The process is not in control. A run of 9 consecutive points, Samples 7 – 15, appears below the center line. Rule 4 applies after observing the sample mean from Sample 15. At that point the process should be stopped.

4. a. The lower and upper control limits are $\mu \pm 3\sigma/n$. In this case, the limits are $6 \pm 3(0.9)/\sqrt{3}$ or approximately LCL = 4.44 and UCL = 7.56 (rounded to two decimals).

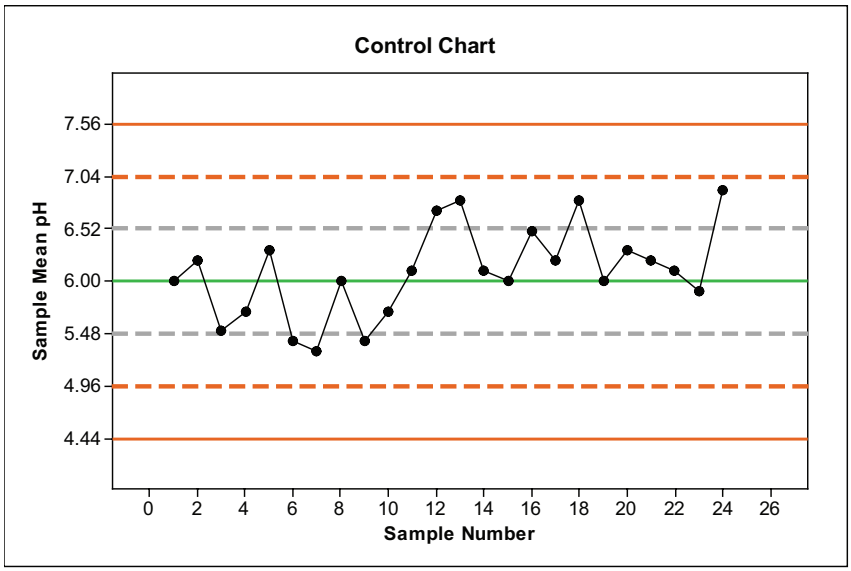
b.

Sample	pH level			Sample mean
1	5.8	6.2	6.0	6.0
2	6.4	6.9	5.3	6.2
3	5.8	5.2	5.5	5.5
4	5.7	6.4	5.0	5.7
5	6.5	5.7	6.7	6.3
6	5.2	5.2	5.8	5.4
7	5.1	5.2	5.6	5.3
8	5.8	6.0	6.2	6.0
9	4.9	5.7	5.6	5.4
10	6.4	6.3	4.4	5.7
11	6.9	5.2	6.2	6.1
12	7.2	6.2	6.7	6.7
13	6.9	7.4	6.1	6.8
14	5.3	6.8	6.2	6.1
15	6.5	6.6	4.9	6.0
16	6.4	6.1	7.0	6.5
17	6.5	6.7	5.4	6.2
18	6.9	6.8	6.7	6.8

(Continued...)

19	6.2	7.1	4.7	6.0
20	5.5	6.7	6.7	6.3
21	6.6	5.2	6.8	6.2
22	6.4	6.0	5.9	6.1
23	6.4	4.6	6.7	5.9
24	7.0	6.3	7.4	6.9

c.

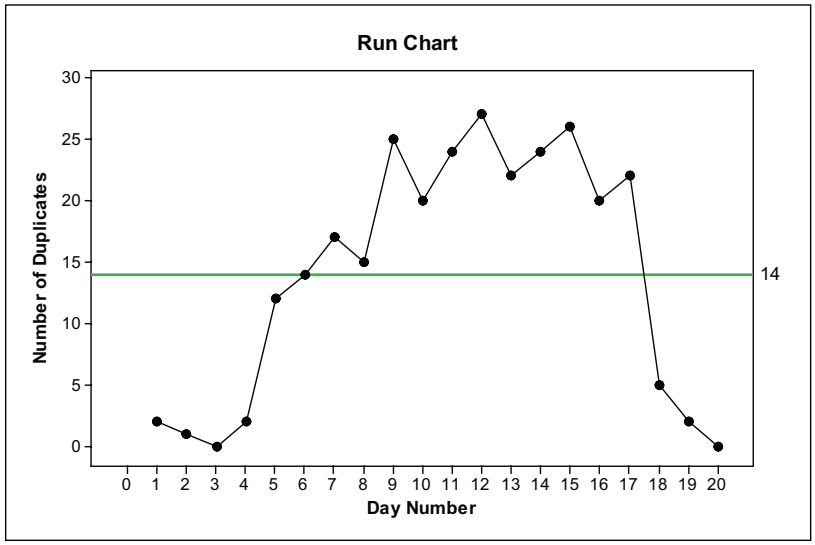


d. The process is in control. None of the decision rules applies.

REVIEW QUESTIONS SOLUTIONS

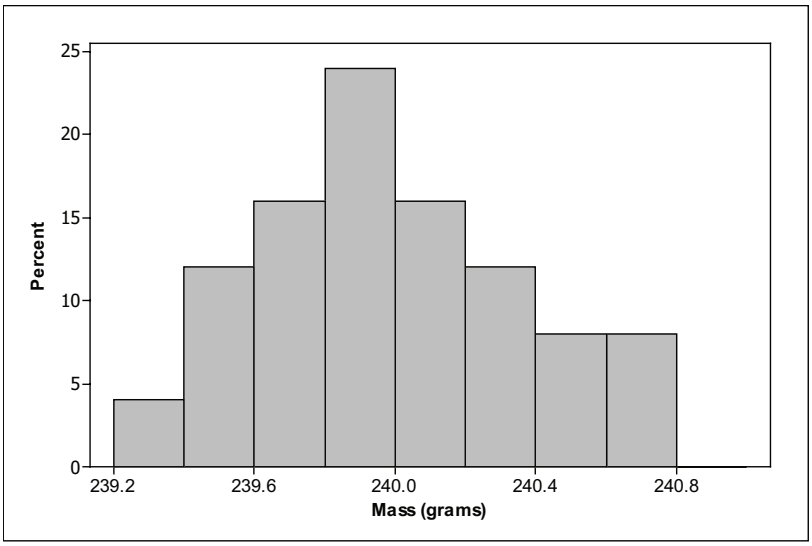
1. a. 14 duplicate e-mails.

b.

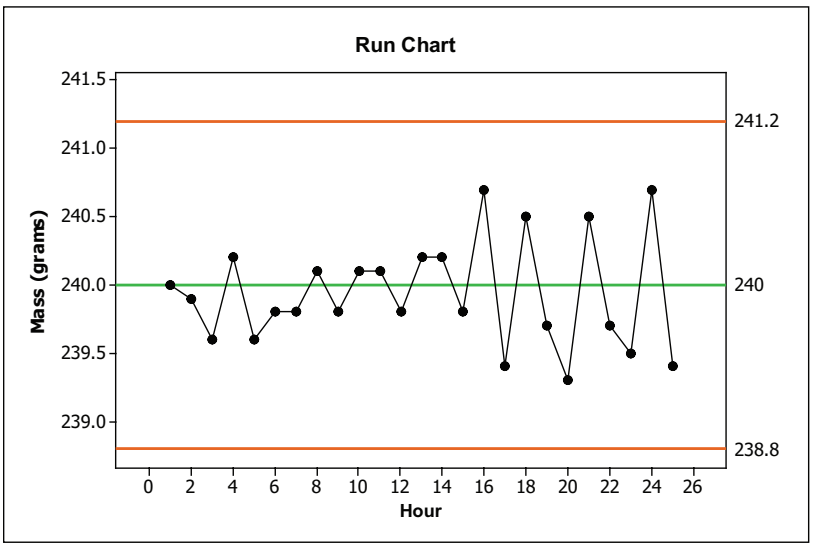


c. Sample answer: Yes – the data points corresponding to Day 7 – Day 17 all lie above the center line. So, this most likely is an example of special cause variation. Perhaps there may be a problem with the spam filter or someone has the manager duplicated on a group list.

2. a. The histogram below gives no evidence that the process is not in control. The histogram appears centered around 240.0 grams. All the data is contained in the interval 240.0 grams ± 1.2 grams (or from 238.8 grams to 241.2 grams).



b. The upper and lower control limits are at 238.8 and 241.2.



c. Sample answer: The variability of the masses about the center line appears to have increased after hour 15. So, that may indicate there is some problem.

3. a. LCL = 0.91 μm, L2 = 0.94 μm, L1 = 0.97 μm, U1 = 1.03 μm, U2 = 1.06 μm, UCL = 1.09 μm

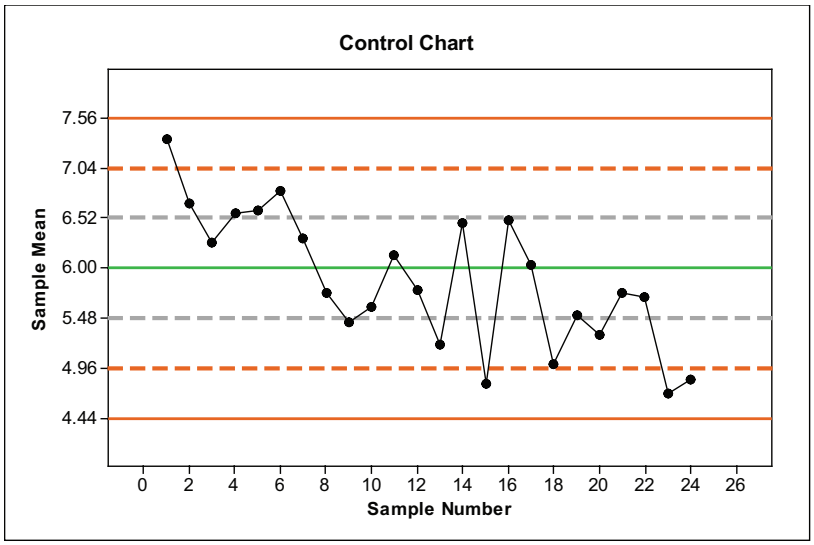
b. The process is not in control. Rule 1 applies. The process should have been shut down after the results from the 9th sample.

4. a. The sample means are as follows, given in the order of the sample numbers:

7.33 6.67 6.27 6.57 6.60 6.80 6.30 5.73 5.43 5.60 6.13 5.77

5.20 6.47 4.80 6.50 6.03 5.00 5.50 5.30 5.73 5.70 4.70 4.83

b.



c. The process is not in control. Rule 3 applies right from the start. Four out of five means from samples 1 – 5 fall above 6.52, and lie on the same side of the center line. The pH level in the liquid is too high at the start of the sampling and appears to be getting lower over time.