Unit 18: Introduction to Probability

PREREQUISITES

Students need to know how to compute proportions (or relative frequencies) and percentages. They should be comfortable with converting proportions to percentages and percentages to proportions.

ADDITIONAL TOPIC COVERAGE

Additional coverage of probability can be found in *The Basic Practice of Statistics*, Chapter 10, Introducing Probability. Unit 19, Probability Models, continues the topic of probability.

ACTIVITY DESCRIPTION

In this activity, students will be flipping coins and tossing thumbtacks repeatedly a large number of times. This activity helps students better understand random phenomena.

MATERIALS

Coins (of the same denomination) for each student or each pair of students; thumbtacks (See Figure 18.4.).

Students can work individually or in pairs. If working in pairs, one student should flip the coin (or toss the tack) while the other records the outcomes. Then they can switch.

This activity provides plenty of opportunity to discuss random phenomena. For example, when flipping a coin, students might be quite surprised at the length of the longest run. Assuming the coin is a fair coin, many students may expect the outcomes to closely alternate between heads and tails. After students have gathered the data from 100 flips, determined the longest run, and calculated the proportion in 100 flips, have them share their results with the class.

Many students will think that 100 flips is "over the long run" but it is not. There will still be a fair amount of variability in the proportion of heads. So, at least for the coin flipping activity, members of the class should combine their results to get a better estimate of the probability of heads. Students may choose to do the same for tossing the thumbtack in Part II, even though the activity does not explicitly ask them to do so.

In Part II, students toss a tack and record whether it lands point down or point up. When tossing a tack, students will get better results if they shake the tack in closed cupped hands before releasing the tack.

THE VIDEO SOLUTIONS

1. A random phenomenon is an event in which individual outcomes are uncertain but which has a regular pattern if repeated many times.

2. For any particular instance in the future, weather is not predictable with perfect accuracy. However, over time weather exhibits patterns.

3. It can mean that he is sure that 70% of the viewing area will get rain (coverage). It can also be used as the likelihood of any rain at all (level of confidence).

4. In the short run, the proportion of heads can be quite variable. In the long run, the proportion gets close to 0.5 and stays close to 0.5 as we continue flipping the coin.

5. The event whose probability is close to one is more likely to occur than the event whose probability is close to zero.

6. Essentially zero.

UNIT ACTIVITY:
OBSERVING RANDOM PHENOMENA SOLUTIONS

1. a. Sample answers: It means that getting heads is just as likely as getting tails.
If you flip the coin many, many times, approximately 50% of the flips should be heads.
The probability of getting heads is ½ .

b. Student answers will vary. Some students might expect to see patterns close to alternating heads and tails – so, their estimate might be quite small, 2 or 3. They will probably be quite surprised by the length of the longest run when they actually flip a coin repeatedly 100 or more times.

2. a. Sample data:

```
H  T  T  H  H  T  T  T  T  H  H  H  H  T  H  T  T  H  T  H
T  H  T  T  H  T  H  T  T  T  H  H  T  H  H  T  T  H  H  H
H  H  T  H  T  T  T  H  H  H  H  H  H  T  T  H  T  H  H  T
T  T  T  T  H  H  T  T  T  T  H  T  H  T  H  T  T  T  H  H
T  H  H  T  T  H  H  T  T  H  T  T  T  T  H  H  H  T  H  T
```

b. Sample answer: Based on sample data in 2(a): There is a run of 6 heads.

c. Sample answer: 10 flips – 0.40; 20 flips – 0.50; 50 flips – 0.52; 100 flips – 0.48.

d. Sample answer: It is difficult to tell. There is quite a lot of variability in the proportions. While the proportion in 100 flips is less than 0.50, the proportion after 50 flips was the same amount above 0.50.

3. a. Sample answer: Together individuals in the class flipped the coin 2000 times. The total number of heads observed was 994. The proportion of heads was 0.497.

b. Sample answer: The proportion of 0.497 is close to 0.50. If we kept flipping the coin, the results would probably get closer to 0.50.

4. a. Sample data from 100 tosses:

```
D U D U U U U D D U U U D D U U D D D D

U U D U D D D D U D D D D U D U D U D U

D D U D D D D U U U U U D U D U D U D D

U D D U U D D U D U U D U D D U D U D D

D D U U D D U U U D D U U D D U U D D U
```

Of the 100 tosses, the tack landed point up 46 times and down 54 times.

b. Probability of point up = 0.46 and probability of point down = 0.54.

c. 0.54 + 0.46 = 1.

EXERCISE SOLUTIONS

1. Sample answers: Time it takes to get to school each day; time it takes to complete a test or quiz; whether it rains or not; the temperature; length of the line when paying for lunch.

2. a. Data set (b) appears to be random. There does not appear to be a short-term predictable pattern. Data set (a) is all T's and hence totally predictable. In data set (c), the pattern alternates between H and T, and hence is totally predictable in the short term. For data set (d) the pattern is THHT which repeats and hence is totally predictable.

3. a.

Number Flips	Number Heads	Proportion	Number Flips	Number Heads	Proportion
50	27	0.540	550	277	0.504
100	54	0.540	600	305	0.508
150	81	0.540	650	326	0.502
200	109	0.545	700	352	0.503
250	136	0.544	750	373	0.497
300	164	0.547	800	399	0.499
350	184	0.526	850	424	0.499
400	208	0.520	900	449	0.499
450	227	0.504	950	477	0.502
500	255	0.510	1000	501	0.501

b.
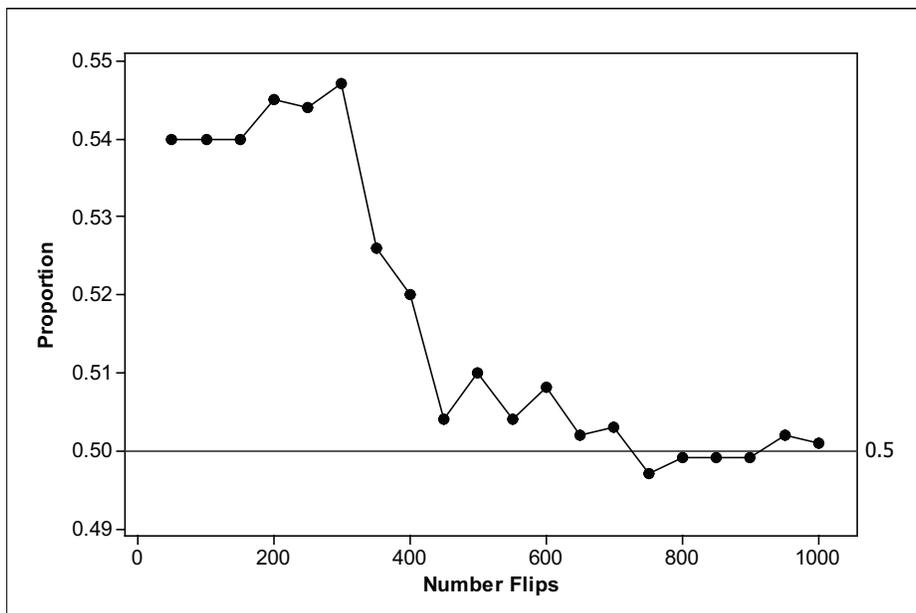
c. Yes. Although the proportion started on the high side, after many flips, the proportions appear to be oscillating about 0.5.

4. a. Alex's chances are low – 3 out of 20 would be below 1/3.

b. Sample answer: When it is humid and cloudy and the temperature drops, very often rain results. Cooler air can't hold as much moisture as warm air. So, the chance for rain is high.

c. The chances are 4 out of 52, which is low.

d. Your chances are moderate. Half the deck are red cards. So, even with one card out of the deck, there would be either 25 or 26 red cards left. So, there are still about half as many red cards as there are cards left in the deck.

REVIEW QUESTIONS SOLUTIONS

1. a. 0

b. 0.5

c. 1

d. 0.0002

2. a. Sample answer: Just because a probability is high that some event will occur, it does not mean that the event is going to occur. In this case, there is a 30% chance that the event will not occur – which is still a sizable chance.

b. Amanda is not correct. The 70% refers to the percentage of occurrence over the long run. In the short term, random phenomena such as rain are unpredictable. You won't know for certain the outcome, until you observe what happens. A 70% chance of rain does not mean that it will rain exactly 7 times in 10 days.

3. a. The spinner is most likely to land on sector 5 because it is the largest sector on the spinner.

b. Sector 4 appears to be 1/4 of the area of the entire spinner. Hence, I would expect 1/4 of the 1000 spins or 250 spins to stop on sector 4. Since a probability of 1/4 refers to the proportion over the long run – over more than 1000 spins – it is likely that the actual outcome will be slightly more or less than 250.

c. Since sector 3 appears to be twice as large as sector 2, it is twice as likely that the spinner lands on 3 than on 2.

d. Approximately six sectors the size of the 3-sector would fit in the lower half of the spinner and six in the upper half. Five of the 3-sectors would cover the combined area of the 4-sector and 2-sector. Hence, the probability of landing on an even numbered sector is around 5/12.

4. a. 14,278

b.

Response	Frequency	Probability
No	1845	0.1292
Yes/Some	2637	0.1847
Yes/Most	2648	0.1855
Yes/Nrly All	7148	0.5006

c. The sum of the probabilities is 1.

d. There were 12,433 responses that were not 'No' out of a total of 14,278 responses. The estimated probability is the proportion 12,433/14,278 ≈ 0.8708.