# Appendix

# Glossary

## A

**absolute comparison** An absolute comparison is an additive comparison between quantities. In an absolute comparison, 7 out of 10 is considered to be larger than 4 out of 5 because 7 is larger than 4.

**algebraic numbers** Algebraic numbers are all the numbers that are solutions to polynomial equations where the polynomials have rational coefficients (e.g., $1/2x^3 - 3x^2 + 17x + 5/8$). They include all integers, rational numbers, and some irrational numbers (e.g., $\pm\sqrt{2}$, which are the solutions to $x^2 - 2 = 0$). Note that algebraic numbers also include some complex numbers (e.g., $\pm\sqrt{1}$, the solution to $x^2 + 1 = 0$).

**algorithm** An algorithm is a recipe or a description of a mechanical set of steps for performing some task.

**area model for multiplication** The area model for multiplication is a method of multiplying fractions (between 0 and 1) by representing the multiplied fractions as areas of a whole. The same model can be used to divide fractions that are between 0 and 1.

**asymmetrical multiplication** An asymmetrical multiplication problem is one where the order of the operands is important. Switching the order of operands in this type of problem presents a different situation, even though the product is the same. For example, buying 10 tickets at $5 each is quite different from buying 5 tickets at $10 each, although the total cost (i.e., the product) is identical.

## B

**base** The base of a number system is the number representing the value of each place in a representation. For example, "base ten" tells us that each digit in a number is some value of 10. In base ten, the number 1,234 represents four different values of 10: $(1 \cdot 10^3) + (2 \cdot 10^2) + (3 \cdot 10^1) + (4 \cdot 10^0)$. Meanwhile, 1234 in base five represents $(1 \cdot 5^3) + (2 \cdot 5^2) + (3 \cdot 5^1) + (4 \cdot 5^0)$, and so on. These representations may appear identical, but if you perform the calculations, you'll see that 1,234 in base ten is a different number from 1234 in base five.

## C

**closed set** A closed set under a given operation is such that the result of the operation is always within the set.

**common denominator model for division** The common denominator model for division is a method of dividing fractions by finding a common denominator and then dividing the numerators.

**complex numbers** Complex numbers are numbers formed by the addition of imaginary and real number elements. They are expressed in the form $a + bi$, where $a$ and $b$ are real numbers, and $i$ can be represented as $i^2 = -1$ (a number such that when you square it, you get -1). Graphically, complex numbers are typically shown as a plane, where the real part is represented horizontally and the imaginary part is represented vertically. The complex numbers are uncountably infinite, are closed under the four basic operations (other than dividing by 0), and have additive and multiplicative identity and inverses (other than 0). Additionally, the complex numbers are closed under any polynomial; that is, any polynomial with complex number coefficients will have all its roots in the complex numbers.

**composite number** A counting number is called a composite number if it has more than two factors. For example, 16 is composite because it has five factors (1, 2, 4, 8, and 16).

**countably infinite set** A countably infinite set can be put into one-to-one correspondence with the counting numbers {1, 2, 3, 4, 5, ...}. For example, the positive even numbers are countably infinite since we can find a one-to-one mapping of {2, 4, 6, 8, 10, ...} onto the counting numbers. Some examples of countably infinite sets are the whole numbers, integers, and rational numbers.

**counting numbers** Counting numbers are the same as natural numbers (i.e., 1, 2, 3, 4, …).

**cubic number** A cubic number is obtained as a result of multiplying a number by itself three times. For example, 1 (i.e., $1^3$ or $1 \cdot 1 \cdot 1$), 8 (i.e., $2^3$ or $2 \cdot 2 \cdot 2$), 27 (i.e., $3^3$ or $3 \cdot 3 \cdot 3$), 64 (i.e., $4^3$ or $4 \cdot 4 \cdot 4$), and so on, are cubic numbers. Cubic numbers of dots can be arranged to make a cube.

# Glossary, cont'd.

## D

**dense set** A dense set is such that for any two elements you choose, you can always find another element of the same type between the two. For example, integers are not dense; rational numbers are.

**divisibility test** A divisibility test is a rule that determines whether a given number is divisible by a set factor. For example, we can use a divisibility test to determine if a large number like 23,456 is or is not divisible by 2, by 3, or by 5. Some divisibility tests involve the last digits of a number, while others involve the sum of the digits.

## E

***e*** *e* is a transcendental number with the decimal approximation $e = 2.7183$. It is the base of natural logarithms. The value of *e* is found by taking the limit of $(1 + 1/n)^n$ as *n* approaches infinity. This number arises in many applications—for example, in calculus as a function whose value and slope are everywhere equal, and in compound interest as a base when interest is computed continuously.

**even numbers** Even numbers are integers divisible by 2. Any number that ends with the digit 0, 2, 4, 6, or 8 is an even number.

**exponent** An exponent is a superscript number that indicates repeated multiplication of the base number or variable. It is also referred to as the power to which the base number or variable is raised.

$$a^b$$

base exponent

## F

**factor** A factor of a number is a counting number that divides evenly into that number. For example, 3 is a factor of 15 because 3 divides evenly into 15 (five times). Four is not a factor of 15, but it is a factor of 16.

**factor tree** A factor tree can be used to factor a number into prime factors. To create a factor tree, start with the smallest prime factor of the given number and then split the number into factors. With 30, the smallest prime factor is 2, so 30 = 2 • 15. Then factor 15 into prime numbers: 30 = 2 • 15 and 15 = 3 • 5. So 30 = 2 • 3 • 5, which is its prime factorization.

**Fibonacci sequence** The Fibonacci sequence is a series of numbers in which the first two elements are 1, and each additional element is the sum of the previous two. The sequence is 1, 1, 2, 3, 5, 8, 13, 21, ....

**figurate number** A figurate number is a number of dots which form a geometric shape. If you make a square with five dots on a side, there will be 25 dots; this makes the number 25 a square number. If you make a triangle with four dots on a side, there will be 10 dots; this makes 10 a triangular number. Figurate numbers can be formed from pentagons, hexagons, cubes, pyramids, and other geometric shapes.

## G

**golden mean** The golden mean is the limit of the ratio between two consecutive Fibonacci numbers. It is exactly $\frac{1+\sqrt{5}}{2}$ and approximately 1.618. Often, the Greek letter phi (ø) is used to represent the golden mean.

**golden rectangle** A golden rectangle is a rectangle whose sides are in the ratio of 1 to ø, where ø is the golden mean. A golden rectangle can be cut into a square and a smaller golden rectangle.

**greatest common factor** The greatest common factor of two numbers is the largest number that is a factor of both given numbers. For example, 4 is the greatest common factor of 20 and 28, since it is a factor of both 20 and 28, and no number larger than 4 is a factor of both.

## I

**identity element** If *I* is an identity element for operation *, then $a * I = I * a = a$ for all elements *a* in the set. The identity element for addition of real numbers is 0, and the identity element for multiplication of real numbers is 1.

**infinite set** An infinite set can be put into one-to-one correspondence with a proper subset of itself. For example, the counting numbers {1, 2, 3, 4, 5, ...} can be put into one-to-one correspondence with the subset {2, 3, 4, 5, 6, ...}, so the set must be infinite. Some examples of infinite sets are the integers and real numbers.

**integers** Integers include positive and negative whole numbers, and 0.

# Glossary, cont'd.

**inverse element** If $b$ is the inverse element for $a$ for operation *, then $a * b = b * a = I$, the identity element for that operation. The inverse for element $a$ for addition is $-a$, because $a + -a = -a + a = 0$ for all values of $a$. The inverse for element $a$ for multiplication is $1/a$, because $a * (1/a) = 1/a * a = 1$ for all values of $a$ except 0. Zero does not have an inverse for multiplication.

**irrational numbers** Irrational numbers are numbers that can't be expressed as a quotient of two integers, such as pi or square roots; they can only be expressed as infinite, non-repeating decimals.

## L

**laws of exponents** The laws of exponents are rules regarding simplification of expressions involving exponents. One such rule is that multiplying exponential expressions with the same base is equivalent to adding the exponents, so, for example, $x^3 / x^4 = x^7$. Another rule is that dividing exponential expressions with the same base is equivalent to subtracting the exponents, so, for example, $x^3 / x^4 = x^{-1}$.

**least common multiple** The least common multiple of two numbers is the smallest number that is a multiple of both given numbers. For example, 56 is the least common multiple of 8 and 14, since 8 and 14 are both factors of 56, and no number smaller than 56 has both 8 and 14 as factors.

**logarithm** A logarithm is an exponent. The notation $\log_2 8 = 3$ states that two is the base, 3 is the exponent, and 8 is the result. Logarithms can simplify complex exponentiation and multiplication problems numerically by using the laws of exponents to convert the more complicated operations into addition and subtraction. Most calculators are programmed with the LOG key (to perform logarithms to base ten) and the LN key (to perform logarithms to base $e$, approximately 2.718).

## O

**odd numbers** Odd numbers are integers not divisible by 2. Any number that ends with the digit 1, 3, 5, 7, or 9 is an odd number.

## P

**partitive division** A partitive division problem is one where you know the total number of groups, and you are trying to find the number of items in each group. If you have 30 popsicles and want to divide them equally among your 5 best friends, figuring out how many popsicles each person would get is a partitive division problem.

**part-part interpretation** Part-part interpretation of a fraction is the notion of comparing one quantity within a whole to another quantity within that whole. For example, if, on a field trip, there are three adults for every 10 students, the part-part interpretation of this relationship would be 3/10 (which could also be written as 3:10).

**part-whole interpretation** Part-whole interpretation of a fraction represents one or more parts of a single unit. For example, the fraction 4/5 represents the part-whole relationship in the following phrase: "Four out of five dentists prefer Blasto toothpaste."

**percent** Percent means some part out of 100. It can also be represented as a fraction or decimal. For example, 45% means 45 out of 100, 0.45, and 45/100.

**period** The period of a repeating decimal is the total number of digits in the group of digits that repeats. For example, 0.123123123... has a period of three digits (the repeating part is "123"), while 0.06151515... has a period of two digits ("15"—the "06" does not repeat and thus is not part of the period).

**pi (or $\pi$)** is a transcendental, irrational number that represents the ratio of circumference to diameter for every circle. The decimal approximation of $\pi$ is 3.141593.

**prime number** A counting number is a prime number if it has exactly two factors: 1 and the number itself. For example, 17 is prime, 16 is not prime, and 1 itself is not prime, since it has only one factor.

**proportion** Proportion is an equation that states that two ratios are equal; for example, 2:1 = 6:3.

# Glossary, cont'd.

**pure imaginary numbers** Pure imaginary numbers are numbers of the form $b \cdot i$, where $b$ is a real number and $i$ is a number such that when squared it yields -1 (i.e., $i^2 = -1$). The pure imaginary numbers are typically shown graphically as a vertical line intersecting the number line at 0. They are uncountably infinite, closed under addition and subtraction only, and have additive identity and inverses.

## Q

**quotative division** A quotative division problem is one where you know the number of items in each group and are trying to find the number of groups. If you have 30 popsicles and want to give 5 popsicles to each person, figuring out the total number of people is a quotative division problem.

## R

**ratio** A ratio indicates the relative magnitude of two numbers. The ratio 3:1 means that the first quantity is equivalent to three times the second quantity. The ratio 2:3 means that twice the first quantity is equivalent to three times the second quantity. This relationship may be written 2:3 or as an indicated quotient (2/3).

**rational numbers** Rational numbers are numbers that can be expressed as a quotient of two integers; when expressed in a decimal form they will either terminate (1/2 = 0.5) or repeat (1/3 = 0.333…).

**real numbers** Real numbers comprise all rational and irrational numbers. They can be represented on a number line.

**relative comparison** A relative comparison is a multiplicative, or proportional, comparison between quantities. In a relative comparison, 4 out of 5 is considered to be larger than 7 out of 10 because 4/5 is larger than 7/10.

**relatively prime numbers** Two or more numbers are relatively prime numbers if their greatest common factor is 1. For example, 4 and 9 are not prime numbers, but they are relatively prime because their greatest common factor is 1.

**repeating decimal** A repeating decimal is a decimal that does not terminate but keeps repeating the same pattern. For example, 0.123123123... is a repeating decimal; the "123" will repeat endlessly. Any repeating decimal is equal to a rational number. For example, 0.123123... is equal to 123/999, or 41/333.

## S

**scientific notation** A number is written in scientific notation when it is in the form $F \cdot 10^E$, where the decimal F has exactly one non-zero digit to the left of the decimal point and E is an integer. Any real number can be written in scientific notation. For example, the number 23,831 can be written as $2.3831 \cdot 10^4$, the number 0.00123 as $1.23 \cdot 10^{-3}$.

**square number** A square number is obtained by multiplying a number by itself (e.g., 1, 4, 9, 25, ...).

**symmetrical multiplication** A symmetrical multiplication problem is one where the order of the operands is not important. Finding the area of a field that measures 150 feet by 50 feet, or finding the number of different sandwiches that can be made from 4 types of bread and 6 types of meat, are both symmetrical multiplication problems.

## T

**terminating decimal** A terminating decimal is a decimal that comes to a finite end, rather than repeating. For example, 0.5 and 0.381 are terminating decimals, while 0.123123123... is not. Any terminating decimal is equal to a rational number. For example, 0.381 is equal to 381/1,000.

**transcendental numbers** Transcendental numbers are numbers that cannot be the solution to a polynomial equation. The most common transcendental numbers are $\pi$ and $e$.

**triangular number** A triangular number is a number obtained as the sum of consecutive integers. For example, 1 (i.e., 0 + 1), 3 (i.e., 1 + 2), 6 (i.e., 1 + 2 + 3), 10 (i.e., 1 + 2 + 3 + 4), and so on are triangular numbers.

# Glossary, cont'd.

## U

**uncountably infinite set** An uncountably infinite set cannot be put into one-to-one correspondence with the counting numbers {1, 2, 3, 4, 5, ...}. Proving that a set is uncountable is typically done indirectly by first assuming that it is countable and then finding a contradiction. The real numbers and complex numbers are uncountably infinite.

## V

**Venn diagram** A Venn diagram is a graphic representation of sets. It can be used to show the union and intersection of two sets.

## W

**whole numbers** Whole numbers are the counting numbers and 0.

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**Session 3. Place Value: Number Systems for Computers**

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**Session 4. Meanings and Models for Operations: How Do Computers Divide?**

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**Session 5. Divisibility Tests and Factors: X-Ray Astronomy and Divisibility**

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Optical: NASA/HST/D.Wang et al.

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**Session 6. Number Theory: Internet Security**

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**Session 7. Fractions and Decimals: Babylonian Decimals**

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**Session 8. Rational Numbers and Proportional Reasoning: Relative Reasoning in Finance**

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**Session 9. Fractions, Percents, and Ratios: The Golden Rectangle in Architecture**

Ed Tsoi
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Victoria Miles, Abigail Adams Intermediate School, Weymouth, Massachusetts
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