1	01:00:47:10	01:00:48:20	WOMAN: Measurement is the process
2	01:00:48:22	01:00:51:23	of quantifying properties of objects.
3	01:00:51:25	01:00:54:20	And to do that, we have set procedures
4	01:00:54:22	01:00:56:21	that enable us to measure.
5	01:00:56:23	01:00:58:13	Oh.
6	01:00:58:15	01:01:00:27	Measuring helps you to understand
7	01:01:00:29	01:01:03:08	how things relate to each other.
8	01:01:03:10	01:01:07:06	Our volume of a sphere actually has a formula
9	01:01:07:08	01:01:09:17	of four-thirds pi r-cubed.
10	01:01:09:19	01:01:13:03	This course really made me think about how I approach measurement
11	01:01:13:05	01:01:16:06	and how I can use measurement every day in the classroom.
12	01:01:21:15	01:01:23:03	Today in our session,
13	01:01:23:05	01:01:26:19	we are going to be looking at some measurement fundamentals.
14	01:01:26:21	01:01:29:13	We also are going to consider
15	01:01:29:15	01:01:32:10	the role of ratio in measurement.
16	01:01:32:12	01:01:33:20	And finally,
17	01:01:33:22	01:01:36:26	we will be discussing in a little more detail
18	01:01:36:28	01:01:38:20	precision and accuracy.
19	01:01:38:22	01:01:42:01	To begin, let's look at some of the fundamentals
20	01:01:42:03	01:01:44:02	that underlie how we measure.
21	01:01:44:04	01:01:46:29	One of them is this idea of a unit,
22	01:01:47:01	01:01:50:12	and, in some senses, unit iteration.
23	01:01:50:14	01:01:55:09	Unit iteration is the repetition of a single unit.
24	01:01:55:11	01:01:58:26	So when we are measuring, we are actually taking a unit
25	01:01:58:28	01:02:02:19	and repeatedly placing it end to end
26	01:02:02:21	01:02:05:09	so that we have a complete, for example, length.
27	01:02:05:11	01:02:08:14	Or if we have square units, such as square centimeters,
28	01:02:08:16	01:02:10:22	we're placing them side by side
29	01:02:10:24	01:02:14:02	so they have a complete covering.
30	01:02:14:04	01:02:16:10	Now, what's interesting about measurement is
31	01:02:16:12	01:02:18:02	we can continually take our units
32	01:02:18:04	01:02:22:13	and subdivide them farther and farther and farther.
33	01:02:22:15	01:02:24:17	And this is a very important aspect

34	01:02:24:19	01:02:27:27	of being able to become more and more precise.
35	01:02:27:29	01:02:30:25	Now, let's us experiment what that means,
36	01:02:30:27	01:02:33:00	to take a unit and divide it--
37	01:02:33:02	01:02:36:15	or "partition" it is sometimes the term we use--
38	01:02:36:17	01:02:40:27	into smaller and smaller subunits.
39	01:02:40:29	01:02:43:11	NARRATOR: To explore the idea of partitioning,
40	01:02:43:13	01:02:44:27	the class is given a task:
41	01:02:44:29	01:02:49:28	Find 17/48 on this unit without the benefit of a measuring tool.
42	01:02:50:00	01:02:54:07	This leads many participants to begin the activity by folding.
43	01:02:54:09	01:02:56:03	Do you want to fold the paper to start...
44	01:02:56:05	01:02:57:23	Just to see what happens.
45	01:02:57:25	01:02:58:24	Okay.
46	01:02:58:26	01:02:59:24	Because it's equal...
47	01:02:59:26	01:03:01:05	that's equidistant, anyway.
48	01:03:01:07	01:03:02:15	It looks like it's pretty close.
49	01:03:02:17	01:03:07:17	So there's our halfway point.
50	01:03:07:19	01:03:11:02	And it would actually be slightly less than that.
51	01:03:11:04	01:03:13:00	I'm all about approximation.
52	01:03:13:02	01:03:14:21	(*both laughing*)
53	01:03:14:23	01:03:18:29	Um... I'm not sure how we'd find exactly that point, but...
54	01:03:19:01	01:03:20:00	Then we could fold...
55	01:03:20:02	01:03:21:08	You were talking about doing
56	01:03:21:10	01:03:22:16	accordion-style more.
57	01:03:22:18	01:03:24:14	We could start folding, and each time we fold
58	01:03:24:16	01:03:26:19	or do an accordion, we'd get a little bit more...
59	01:03:26:21	01:03:27:19	Closer.
60	01:03:27:21	01:03:28:22	Closer to breaking it down.
61	01:03:28:24	01:03:30:09	Okay.
62	01:03:30:11	01:03:33:28	CHAPIN: In the folding activity, what we were interested in doing
63	01:03:34:00	01:03:36:16	was thinking about

			"How do we take a unit
64	01:03:36:18	01:03:39:07	and subdivide it, or partition it?"
65	01:03:39:09	01:03:42:05	One of the reasons for partitioning a unit
66	01:03:42:07	01:03:45:03	into smaller and smaller subunits
67	01:03:45:05	01:03:46:26	is for us to also consider
68	01:03:46:28	01:03:50:06	"How does that change the number of units in a measure?"
69	01:03:50:08	01:03:51:14	The units are very small,
70	01:03:51:16	01:03:54:11	we are going to have a much larger number.
71	01:03:54:13	01:03:56:10	It's kind of inversely proportional,
72	01:03:56:12	01:03:58:16	that the smaller the unit,
73	01:03:58:18	01:04:00:21	the more the unit it's going to take.
74	01:04:00:23	01:04:04:19	So, then we need to find 17/48.
75	01:04:04:21	01:04:07:03	So... this is 24.
76	01:04:07:05	01:04:09:07	We have to find the middle point, right?
77	01:04:09:09	01:04:10:09	We have to find 17.
78	01:04:10:11	01:04:12:05	Yeah, so...
79	01:04:12:07	01:04:14:00	We have 12 here, right?
80	01:04:14:02	01:04:17:13	Yeah, 12, and we need to get up to 17.
81	01:04:17:15	01:04:20:17	We just keep folding it in half, can't we?
82	01:04:20:19	01:04:22:09	This section into halves.
83	01:04:22:11	01:04:23:16	Mmm.
84	01:04:23:18	01:04:25:08	This second section?
85	01:04:25:10	01:04:26:16	Yeah, this middle...
86	01:04:26:18	01:04:28:00	Yeah, the second section.
87	01:04:28:02	01:04:30:25	So fold the two creases so they're on top.
88	01:04:30:27	01:04:31:25	Do you see that?
89	01:04:33:14	01:04:37:24	*Comme ça.*
90	01:04:37:26	01:04:40:01	And then that crease...
91	01:04:40:03	01:04:41:08	18?
92	01:04:40:03	01:04:41:08	18.
93	01:04:41:10	01:04:42:12	Mm-hmm.
94	01:04:42:14	01:04:45:04	There are six in between here, right?
95	01:04:45:06	01:04:48:18	I think I would try to divide in three.
96	01:04:48:20	01:04:49:21	Three and three?
97	01:04:49:23	01:04:52:04	Three, three.
98	01:04:52:06	01:04:53:15	Because now it's easier.
99	01:04:53:17	01:04:56:29	Three and then in three again, in the middle.

100	01:04:57:01	01:04:57:29	See?
101	01:04:58:01	01:05:00:18	So I have 12, 13, 14...
102	01:05:00:20	01:05:01:22	*Quinze, dezesseis...*
103	01:05:01:24	01:05:03:05	(*both laughing*)
104	01:05:03:07	01:05:04:27	Exactly.
105	01:05:04:29	01:05:06:05	And then 17.
106	01:05:04:29	01:05:06:05	17.
107	01:05:06:07	01:05:07:18	Here.
108	01:05:07:20	01:05:12:16	So, here is our 17/48, okay?
109	01:05:12:18	01:05:13:24	*Obrigado.*
110	01:05:13:26	01:05:14:24	*De nada.*
111	01:05:14:26	01:05:16:18	(*both chuckling*)
112	01:05:16:20	01:05:19:05	I notice many of you took the one unit
113	01:05:19:07	01:05:22:12	and divided it in half as kind of the first...
114	01:05:22:14	01:05:23:17	by folding.
115	01:05:23:19	01:05:26:19	Some of you then may have taken the half
116	01:05:26:21	01:05:32:09	and cut it up into thirds, thus to get a sixth...
117	01:05:32:11	01:05:34:09	or sixths all the way along.
118	01:05:34:11	01:05:38:08	From there, we can take each one of those sixths
119	01:05:38:10	01:05:43:08	and cut it up into four pieces so that we have 24ths.
120	01:05:43:10	01:05:44:18	And finally,
121	01:05:44:20	01:05:48:07	now that we know that this smallest unit is a 24th,
122	01:05:48:09	01:05:49:25	if we cut each one of those in half,
123	01:05:49:27	01:05:55:07	or divide it, or partition it in half, we have our 48ths,
124	01:05:55:09	01:05:59:17	and then we can figure out where along here is 17/48.
125	01:06:04:03	01:06:06:12	CHAPIN: Is a measure
126	01:06:06:14	01:06:10:17	an actual accurate number, a specific,
127	01:06:10:19	01:06:12:22	or is it more an approximation?
128	01:06:12:24	01:06:15:02	Dave?
129	01:06:15:04	01:06:17:04	It's always going to be somewhat approximate,
130	01:06:17:06	01:06:19:24	because let's say that we were going to measure something
131	01:06:19:26	01:06:21:16	that we could consider to be a foot
132	01:06:21:18	01:06:24:14	and we agree that it's a foot and you stick a ruler next to it
133	01:06:24:16	01:06:26:06	and it's 12 inches long on the nose.
134	01:06:26:08	01:06:28:12	Who's to say that if we could break down that foot
135	01:06:28:14	01:06:30:12	into hundredths of a inch

			or thousandths of an inch
136	01:06:30:14	01:06:33:21	that it wouldn't be one foot and 1/1,000 of an inch?
137	01:06:33:23	01:06:35:03	Or take it even crazier than that:
138	01:06:35:05	01:06:36:09	You can keep breaking it down
139	01:06:36:11	01:06:37:26	into smaller and smaller partitions
140	01:06:37:28	01:06:39:23	and maybe get a greater degree of accuracy,
141	01:06:39:25	01:06:42:12	but who's to say that that isn't the perfect accuracy?
142	01:06:42:14	01:06:44:24	And you can break that hundredth of an inch
143	01:06:44:26	01:06:46:14	down into hundredths of that
144	01:06:46:16	01:06:49:02	and maybe it'll be a portion of that.
145	01:06:49:04	01:06:51:17	So it's always going to be a little bit of approximation.
146	01:06:51:19	01:06:53:29	You just kind of have to take a leap of faith
147	01:06:54:01	01:06:55:23	and accept it as truth eventually.
148	01:06:55:25	01:06:56:23	Right, yeah.
149	01:06:56:25	01:06:58:14	Katie?
150	01:06:58:16	01:06:59:20	The purpose will affect
151	01:06:59:22	01:07:01:20	how approximate you're allowing it to be.
152	01:07:01:22	01:07:04:09	So, you know, he was talking about "a foot is a foot,"
153	01:07:04:11	01:07:07:14	but if you're trying to, you know, build something
154	01:07:07:16	01:07:09:16	and create a watertight seal,
155	01:07:09:18	01:07:11:09	how long your foot is is different
156	01:07:11:11	01:07:13:25	than someone who's just cutting a foot of string
157	01:07:13:27	01:07:16:00	to be able to play a game with,
158	01:07:16:02	01:07:19:08	or someone who's creating metalwork that needs to be,
159	01:07:19:10	01:07:22:27	you know, so precise, or medical instruments, or whatever.
160	01:07:22:29	01:07:26:21	You know, each person in each of those industries or functions
161	01:07:26:23	01:07:30:16	would have a different sense of whether they're ready to say,
162	01:07:30:18	01:07:34:15	"Yes, that foot is a foot."
163	01:07:34:17	01:07:37:15	But if we are measuring something
164	01:07:37:17	01:07:41:18	with the instruments we have, can I ever say
165	01:07:41:20	01:07:45:14	"That's absolutely exactly x length long"?
166	01:07:45:16	01:07:47:22	What's it going to depend upon?
167	01:07:47:24	01:07:51:23	It's going to depend on

the measuring tool you use.
168 01:07:51:25 01:07:54:25 And the precision of the unit
that we have.
169 01:07:54:27 01:07:58:01 And as a result,
the smaller and smaller unit
170 01:07:58:03 01:08:00:26 is going to give us
more and more precision,
171 01:08:00:28 01:08:02:11 but we can always think
172 01:08:02:13 01:08:04:28 about getting smaller
and smaller, okay?
173 01:08:05:00 01:08:07:10 NARRATOR:
Since all measurements
are approximate,
174 01:08:07:12 01:08:11:12 one way to express this is
by the maximum possible error.
175 01:08:11:14 01:08:13:27 This is always half
of a measuring unit.
176 01:08:13:29 01:08:16:11 For example, if
a measurement were made
177 01:08:16:13 01:08:18:01 to the nearest centimeter,
178 01:08:18:03 01:08:21:00 the maximum possible error
would be one-half centimeter,
179 01:08:21:02 01:08:23:09 or .5 centimeters.
180 01:08:23:11 01:08:25:20 A measure of approximately
ten centimeters
181 01:08:25:22 01:08:30:03 would be stated as ten
plus or minus .5 centimeters.
182 01:08:30:05 01:08:33:18 This means that the measurements
are between 9.5 centimeters
183 01:08:33:20 01:08:35:25 and 10.5 centimeters.
184 01:08:35:27 01:08:38:00 There are different
mathematical entities,
185 01:08:38:02 01:08:43:21 and in particular we sometimes
separate counts versus measures.
186 01:08:43:23 01:08:45:06 And there's a difference.
187 01:08:45:08 01:08:47:26 We can count, for example,
the number of people
188 01:08:47:28 01:08:49:05 and be very exact.
189 01:08:49:07 01:08:50:27 We can count
the number of chairs.
190 01:08:50:29 01:08:52:26 We can count the number
of apples we eat.
191 01:08:52:28 01:08:55:12 We can also make measurements,
192 01:08:55:14 01:08:59:11 but we need to be aware that
those measurements are not exact
193 01:08:59:13 01:09:02:29 because we can
continually narrow down
194 01:09:03:01 01:09:05:13 the size of our unit.
195 01:09:05:15 01:09:09:28 And by having smaller and
smaller subunits, we can become
196 01:09:10:00 01:09:13:29 more and more accurate,
though perhaps never exact
197 01:09:14:01 01:09:16:22 in terms of the
actual measurement.
198 01:09:21:00 01:09:22:18 All measurement is a ratio,
199 01:09:22:20 01:09:27:18 because we are comparing

			a measure to a standard unit.
200	01:09:27:20	01:09:30:25	Now, we then also can set up proportions
201	01:09:30:27	01:09:33:08	of where two ratios are equal to each other.
202	01:09:33:10	01:09:35:24	And I know that many of you are very familiar
203	01:09:35:26	01:09:41:26	with thinking about looking at this proportion.
204	01:09:41:28	01:09:43:05	You may be thinking of it
205	01:09:43:07	01:09:45:02	in terms of equivalent fractions as well,
206	01:09:45:04	01:09:47:12	because a fraction is a form of a ratio.
207	01:09:47:14	01:09:54:14	All right, now we are going to use proportion to look at scale,
208	01:09:54:16	01:09:56:08	because scale is one area
209	01:09:56:10	01:10:00:13	where ratio is used a great deal in measurement.
210	01:10:00:15	01:10:04:29	We make scale models, scale drawings all the time.
211	01:10:05:01	01:10:06:13	Now, if you look in your packet,
212	01:10:06:15	01:10:10:20	you have some pictures of some grasshoppers.
213	01:10:10:22	01:10:14:05	And if we have a scale of one to one,
214	01:10:14:07	01:10:19:00	we then have a ratio where the scale drawing, or the model,
215	01:10:19:02	01:10:23:12	is exactly the same size as the original figure.
216	01:10:23:14	01:10:25:26	If we have a scale of one to two,
217	01:10:25:28	01:10:28:10	you can see we have a reduction.
218	01:10:28:12	01:10:29:18	Okay?
219	01:10:29:20	01:10:32:24	And a scale of two to one is an enlargement--
220	01:10:32:26	01:10:35:26	that that grasshopper is twice as long
221	01:10:35:28	01:10:39:28	and twice as high as the original one.
222	01:10:40:00	01:10:42:13	Well, now, we're going to use this
223	01:10:42:15	01:10:45:10	and, thinking about how we use proportional reasoning,
224	01:10:45:12	01:10:48:14	to think about making some scale drawings.
225	01:10:48:16	01:10:52:00	NARRATOR: Professor Chapin gives the class body measurements of herself
226	01:10:52:02	01:10:55:14	to introduce the next activity on scale drawings.
227	01:10:55:16	01:10:58:03	Her height is approximately 68 inches;
228	01:10:58:05	01:11:01:13	head, nine inches; arms, 28 inches;
229	01:11:01:15	01:11:04:17	and height to navel, 42 inches.

230 01:11:04:19 01:11:08:06 CHAPIN:
So, how am I going to calculate
how long this figure--
231 01:11:08:08 01:11:11:12 this scale model or
scale drawing-- should be?
232 01:11:11:14 01:11:13:15 WOMAN:
I would just multiply
233 01:11:13:17 01:11:15:13 68 inches by 1/8.
234 01:11:15:15 01:11:19:15 And when you do that...
I just did it on my calculator
235 01:11:19:17 01:11:22:03 and you get
8½ inches.
236 01:11:22:05 01:11:23:26 So we now
can draw
237 01:11:23:28 01:11:27:18 kind of the top of the person
and the bottom of the person
238 01:11:27:20 01:11:30:05 on our paper,
which would be 8½ inches.
239 01:11:30:07 01:11:31:14 All right?
240 01:11:31:16 01:11:35:19 How big is my head going to be
on this paper?
241 01:11:35:21 01:11:37:04 I would set up a proportion.
242 01:11:37:06 01:11:40:23 We know the scale is one
to eight, so we have one ratio,
243 01:11:40:25 01:11:43:19 and then the other side
of the proportion,
244 01:11:43:21 01:11:48:14 we know that the actual
measurement is nine inches,
245 01:11:48:16 01:11:49:21 so we're going to put that
246 01:11:49:23 01:11:52:15 into a fraction.
247 01:11:52:17 01:11:53:25 We are looking for the scale,
248 01:11:53:27 01:11:55:18 so we are looking
for the numerator,
249 01:11:55:20 01:11:58:10 and the nine inches was
the actual measurement,
250 01:11:58:12 01:12:00:03 so that is the denominator.
251 01:12:00:05 01:12:03:25 It sometimes helps to label,
in terms of a ratio,
252 01:12:03:27 01:12:05:25 what each thing
is representing--
253 01:12:05:27 01:12:09:00 in terms of this is
our scale drawing,
254 01:12:09:02 01:12:13:25 and this is the real me,
255 01:12:13:27 01:12:17:25 In terms of just making sense
of how we want to set that up.
256 01:12:17:27 01:12:19:10 All right?
257 01:12:19:12 01:12:21:28 So then we can solve this
in a lot of different ways.
258 01:12:22:00 01:12:24:08 We can use our cross products
and divide.
259 01:12:24:10 01:12:26:13 We can think about
going across here...
260 01:12:26:15 01:12:29:22 So, what is the length
of my head?
261 01:12:29:24 01:12:30:22 1 1/8 inches.
262 01:12:30:24 01:12:31:25 1 1/8 inches.

263	01:12:31:27	01:12:32:25	All right?
264	01:12:32:27	01:12:34:09	So now we've got--
265	01:12:34:11	01:12:36:21	and I'm just making this... this isn't accurate here--
266	01:12:36:23	01:12:41:28	that this was the 8½ inches on the paper that might be me.
267	01:12:42:00	01:12:44:25	And then we're going to go down one end,
268	01:12:44:27	01:12:48:11	you know, 1/8 inch and put the head.
269	01:12:48:13	01:12:49:18	All right?
270	01:12:49:20	01:12:51:27	Now, what I'm going to ask you to do
271	01:12:51:29	01:12:54:15	is to choose one of you at your table
272	01:12:54:17	01:12:58:20	and to sketch out a scale drawing of one of you
273	01:12:58:22	01:13:01:01	on your paper.
274	01:13:01:03	01:13:02:01	(*people murmuring*)
275	01:13:02:03	01:13:03:02	How's this?
276	01:13:03:04	01:13:04:19	Uh... okay.
277	01:13:04:21	01:13:06:03	Okay?
278	01:13:06:05	01:13:07:27	Your head up a little bit.
279	01:13:07:29	01:13:09:04	Put my head up?
280	01:13:09:06	01:13:10:10	It's about nine.
281	01:13:10:12	01:13:11:25	Nine inches?
282	01:13:11:27	01:13:14:06	(*classroom chatter continues* *in background*)
283	01:13:14:08	01:13:15:14	Twenty-six and a half.
284	01:13:15:16	01:13:16:14	Okay.
285	01:13:16:16	01:13:17:17	Take the slack out of it.
286	01:13:17:19	01:13:18:17	41.
287	01:13:18:19	01:13:19:18	41?
288	01:13:19:20	01:13:20:28	Yeah.
289	01:13:19:20	01:13:20:28	41 it is.
290	01:13:21:00	01:13:25:04	Well, actually, eight... eight inches.
291	01:13:25:06	01:13:30:15	Measure point to point is 17 inches.
292	01:13:34:13	01:13:38:25	CHAPIN: We looked at the idea that a ratio is used in measurement
293	01:13:38:27	01:13:40:23	as a scale, as a comparison
294	01:13:40:25	01:13:43:24	of either an enlargement or a reduction,
295	01:13:43:26	01:13:46:18	and we investigated how we could make
296	01:13:46:20	01:13:48:24	a scale drawing of ourselves
297	01:13:48:26	01:13:51:14	that would fit on a piece of paper
298	01:13:51:16	01:13:53:11	and how we could, by using ratios,
299	01:13:53:13	01:13:55:24	make the drawing of ourselves proportional

300	01:13:55:26	01:13:58:25	and so it looked right-- that our head was not too big,
301	01:13:58:27	01:14:00:21	and our arms were not too long,
302	01:14:00:23	01:14:03:01	and our legs were about the right length.
303	01:14:03:03	01:14:07:15	I noticed that many of you were able to make a scale drawing,
304	01:14:07:17	01:14:10:06	and, Gin, can you share with us yours?
305	01:14:10:08	01:14:14:12	Take a look here how proportional this person is
306	01:14:14:14	01:14:16:16	in terms of the head, the arm length,
307	01:14:16:18	01:14:18:24	the leg length, and that it is a reduction.
308	01:14:18:26	01:14:20:29	What was your scale factor that you used?
309	01:14:21:01	01:14:22:07	One to seven.
310	01:14:22:09	01:14:25:06	One to seven, or one-seventh reduction.
311	01:14:25:08	01:14:26:16	All right.
312	01:14:26:18	01:14:29:08	We do want to, though, look at some constants
313	01:14:29:10	01:14:31:05	that are involved with ratio.
314	01:14:31:07	01:14:34:23	One that we'll look at in other sessions is pi, but today
315	01:14:34:25	01:14:38:17	we will explore what takes place in isosceles right triangles.
316	01:14:38:19	01:14:42:24	You have at your table some sheets of right triangles.
317	01:14:42:26	01:14:46:04	You also have a chart similar to this one,
318	01:14:46:06	01:14:50:06	and what I would like us to do is, using our rulers
319	01:14:50:08	01:14:55:04	and measuring to the nearest tenth of a centimeter,
320	01:14:55:06	01:15:00:17	or a millimeter, we're going to measure the hypotenuse
321	01:15:00:19	01:15:04:05	and form a ratio of the side to side
322	01:15:04:07	01:15:07:20	and the hypotenuse to side on that triangle.
323	01:15:07:22	01:15:10:17	What you may want to do is take that ratio, if you get it,
324	01:15:10:19	01:15:12:13	and reduce it using your calculator.
325	01:15:13:21	01:15:16:29	We're measuring this one with the one-inch...
326	01:15:17:01	01:15:19:21	This one is one inch, one centimeter.
327	01:15:19:23	01:15:21:13	And then this is one...
328	01:15:21:15	01:15:23:26	so this would be 13 centimeters--
329	01:15:23:28	01:15:26:03	this would be 1.3 centimeters.
330	01:15:26:05	01:15:29:25	And the other one

is, um...
331 01:15:29:27 01:15:33:29 forty... two.
332 01:15:34:01 01:15:34:28 Forty-two?
333 01:15:37:14 01:15:41:17 DAVE:
When we did the activity with
the isosceles right triangle,
334 01:15:41:19 01:15:45:08 we had right triangles that had
legs of units one unit long
335 01:15:45:10 01:15:47:04 through units six units long,
336 01:15:47:06 01:15:49:21 and then we're also asked,
with a ruler,
337 01:15:49:23 01:15:52:29 to the nearest tenth of
a centimeter, or millimeter,
338 01:15:53:01 01:15:54:19 to measure the hypotenuse,
339 01:15:54:21 01:15:57:08 and set them up into ratios,
and we found
340 01:15:57:10 01:16:00:02 that each one of those
came out to a number
341 01:16:00:04 01:16:01:16 that was very, very close
342 01:16:01:18 01:16:03:16 to the approximation
of the square root of two.
343 01:16:03:18 01:16:05:03 But then, when you think,
344 01:16:05:05 01:16:07:24 "Well, how did I measure that
being the square root of two?"
345 01:16:07:26 01:16:09:20 That's not something
that I can measure.
346 01:16:09:22 01:16:11:28 The square root of two is
not a measurable number,
347 01:16:12:00 01:16:13:06 because it's irrational,
348 01:16:13:08 01:16:15:04 but yet here it is
right in front of me.
349 01:16:15:06 01:16:18:18 Can I have everybody take a look
up here at this chart,
350 01:16:18:20 01:16:21:29 where I've put some of
the measurements that I made?
351 01:16:22:01 01:16:25:01 You'll probably find
that yours are very similar,
352 01:16:25:03 01:16:26:19 if not exactly the same.
353 01:16:26:21 01:16:29:20 And then what I did was
I started to relate
354 01:16:29:22 01:16:34:06 the ratio of the hypotenuse in
these isosceles right triangles
355 01:16:34:08 01:16:36:00 to the side length,
356 01:16:36:02 01:16:39:00 so in the first one,
I got 1.4 to one;
357 01:16:39:02 01:16:41:09 second one I got 2.9 to two,
358 01:16:41:11 01:16:44:27 which I can then reduce
down to 1.45 to one.
359 01:16:44:29 01:16:47:11 And then we get
4.3 to three...
360 01:16:47:13 01:16:50:04 or does anybody
have a calculator?
361 01:16:50:06 01:16:52:16 What's that going
to reduce down to?
362 01:16:52:18 01:16:55:08 1 43/100 to one.
363 01:16:55:10 01:16:56:23 CHAPIN:

			Okay.
364	01:16:56:25	01:16:58:26	Now, if we keep going,
365	01:16:58:28	01:17:01:22	what do you notice about each of these numbers?
366	01:17:03:19	01:17:04:26	They're all approximations
367	01:17:04:28	01:17:06:05	for the square root of two.
368	01:17:06:07	01:17:07:05	CHAPIN: Okay.
369	01:17:07:07	01:17:08:24	So, what kind of a number
370	01:17:08:26	01:17:10:17	is the square root of two?
371	01:17:10:19	01:17:12:22	I think it is an irrational number.
372	01:17:12:24	01:17:13:22	Right.
373	01:17:13:24	01:17:15:17	Now, can anyone fill us in
374	01:17:15:19	01:17:17:27	on what is an irrational number?
375	01:17:17:29	01:17:19:16	How would we define it?
376	01:17:19:18	01:17:21:01	Yeah.
377	01:17:21:03	01:17:25:16	It's a number that you can't write in a fraction form,
378	01:17:25:18	01:17:26:24	like a under b.
379	01:17:26:26	01:17:27:24	Right.
380	01:17:31:16	01:17:33:25	We often represent numbers
381	01:17:33:27	01:17:37:13	as a over b where b does not equal zero,
382	01:17:37:15	01:17:39:20	and an irrational number is one
383	01:17:39:22	01:17:42:03	that cannot be put into that form.
384	01:17:42:05	01:17:45:02	Likewise, when it is written as a decimal,
385	01:17:45:04	01:17:49:18	it is not a terminating decimal, nor is it a repeating decimal.
386	01:17:49:20	01:17:53:04	It goes on infinitely without repeating.
387	01:17:53:06	01:17:57:03	That brings us to some interesting questions, then,
388	01:17:57:05	01:18:01:03	about finding the actual length of the hypotenuse.
389	01:18:01:05	01:18:05:00	If it is making us think that it's the square root of two,
390	01:18:05:02	01:18:06:29	maybe we would like to think
391	01:18:07:01	01:18:09:10	about using the Pythagorean theorem
392	01:18:09:12	01:18:11:21	as rather than directly measuring,
393	01:18:11:23	01:18:13:26	which is what we've just done,
394	01:18:13:28	01:18:16:11	to see about deriving the measurement
395	01:18:16:13	01:18:19:13	of some of these isosceles right triangles.
396	01:18:19:15	01:18:21:13	Just as a quick review,
397	01:18:21:15	01:18:27:02	the Pythagorean theorem is that in a right triangle,
398	01:18:27:04	01:18:33:11	we can say that a-squared plus b-squared equals c-squared...

399	01:18:35:08	01:18:38:14	where either of the sides are a and b
400	01:18:38:16	01:18:41:27	and the hypotenuse is always known as c.
401	01:18:41:29	01:18:45:04	We happen to be working with triangles
402	01:18:45:06	01:18:49:07	where the two sides are exactly the same, right?
403	01:18:49:09	01:18:53:01	Now, let's explore that and see what happens
404	01:18:53:03	01:18:56:10	if we put in some values for the sides,
405	01:18:56:12	01:19:00:19	not measuring but using the Pythagorean theorem.
406	01:19:00:21	01:19:06:22	If we have our side length of one, we can then put
407	01:19:06:24	01:19:13:06	one-squared plus one-squared is going to give us two.
408	01:19:14:27	01:19:18:22	And so we know that c-squared equals two.
409	01:19:18:24	01:19:20:20	Put that over here.
410	01:19:23:07	01:19:27:10	And so if we want to find the actual length of c,
411	01:19:27:12	01:19:32:07	we know that it's going to be... the square root of two.
412	01:19:32:09	01:19:33:23	CHAPIN: Here we were able
413	01:19:33:25	01:19:37:06	to use a-squared plus b-squared equals c-squared
414	01:19:37:08	01:19:39:26	to find the length of the hypotenuse
415	01:19:39:28	01:19:43:19	and write it as a value times the square root of two--
416	01:19:43:21	01:19:45:04	not terribly practical
417	01:19:45:06	01:19:48:13	when we need to actually go out and make a measurement
418	01:19:48:15	01:19:51:10	or actually find a length, but on the other hand,
419	01:19:51:12	01:19:54:18	very accurate when we are saying, "How long is this?"
420	01:19:54:20	01:19:58:03	realizing that we have got a constant-- we are multiplying
421	01:19:58:05	01:20:01:16	by the square root of two times the side length each time.
422	01:20:01:18	01:20:04:22	Well, we've had a lot of things that we've covered today.
423	01:20:04:24	01:20:06:02	We have looked
424	01:20:06:04	01:20:09:13	at some of the fundamental ideas of measurement
425	01:20:09:15	01:20:13:06	and especially the unit and how we can cut that unit up
426	01:20:13:08	01:20:15:24	into as many subunits as we want.
427	01:20:15:26	01:20:18:01	We've looked at the role of ratio

428	01:20:18:03	01:20:20:24	and how important ratio is to measurement.
429	01:20:20:26	01:20:23:14	And finally, we're starting to explore
430	01:20:23:16	01:20:26:28	some of the role of irrational numbers in measurement
431	01:20:27:00	01:20:30:07	and how that is going to impact our interpretation
432	01:20:30:09	01:20:31:23	of the actual measure.
433	01:20:36:21	01:20:41:18	(*playing Prelude from Bach's* *Suite No. 1 for Cello Solo*)
434	01:20:49:24	01:20:52:15	NARRATOR: Cellist Owen Young is playing a Bach prelude
435	01:20:52:17	01:20:54:08	on an 18th-century cello
436	01:20:54:10	01:20:57:12	made by master craftsman Gennaro Gagliano.
437	01:20:57:14	01:20:59:07	It comes from an age
438	01:20:59:09	01:21:02:22	renowned for its hand-crafted string instruments,
439	01:21:02:24	01:21:07:09	and is today highly valued for its exceptional tone and beauty.
440	01:21:07:11	01:21:10:11	In the shadow of the Old North Church in Boston,
441	01:21:10:13	01:21:11:23	there is a school
442	01:21:11:25	01:21:15:01	that is carrying on this tradition of craftsmanship.
443	01:21:15:03	01:21:18:18	MAN: The North Bennet Street School was started in 1888
444	01:21:18:20	01:21:21:21	to train immigrants to become productive workers
445	01:21:21:23	01:21:25:17	in the United States, and it continued on in that capacity
446	01:21:25:19	01:21:27:02	serving community needs
447	01:21:27:04	01:21:30:02	for probably a good part of the next 75 years,
448	01:21:30:04	01:21:32:21	and then it gradually evolved into a school
449	01:21:32:23	01:21:36:00	that provided training in traditional crafts,
450	01:21:36:02	01:21:39:02	violin making being one of those.
451	01:21:39:04	01:21:42:00	NARRATOR: As a teacher at the North Bennet Street School,
452	01:21:42:02	01:21:43:29	David Polstein provides his students
453	01:21:44:01	01:21:47:07	with the hands-on training they need to master the intricacies
454	01:21:47:09	01:21:50:04	of making an instrument such as a violin.
455	01:21:50:06	01:21:54:06	POLSTEIN: Over time, the violin has become very standardized
456	01:21:54:08	01:21:56:24	to certain measurements,

and the degree of accuracy
457 01:21:56:26 01:21:59:18 with which those measurements
is made is extremely precise.
458 01:21:59:20 01:22:02:23 An example of accurate measuring
would be
459 01:22:02:25 01:22:07:17 checking the thicknesses of the
top or back of an instrument.
460 01:22:07:19 01:22:09:28 We're using a dial caliper,
461 01:22:10:00 01:22:12:12 which is specifically used
to measure thicknesses.
462 01:22:12:14 01:22:13:26 There'd be a determination
463 01:22:13:28 01:22:16:23 of what the appropriate
thickness of a top would be,
464 01:22:16:25 01:22:19:12 and then check everywhere
with a gauge like this
465 01:22:19:14 01:22:21:29 to make sure that you had
accurately reached
466 01:22:22:01 01:22:23:24 the measurements
you set out to get.
467 01:22:23:26 01:22:25:17 On all violin-family
instruments,
468 01:22:25:19 01:22:28:22 a brace is put into the top--
glued in and fit--
469 01:22:28:24 01:22:32:06 and the location of that brace
is determined in part
470 01:22:32:08 01:22:34:16 by some proportional
relationships.
471 01:22:34:18 01:22:38:13 One would determine where the
center of the instrument was,
472 01:22:38:15 01:22:40:21 and then measuring
from the center
473 01:22:40:23 01:22:43:26 at the widest part
of the upper and lower bout,
474 01:22:43:28 01:22:47:28 and that measurement would then
be divided into seven parts,
475 01:22:48:00 01:22:49:25 and a divider could be used
476 01:22:49:27 01:22:53:24 to lay out that distance,
and then on the upper bout,
477 01:22:53:26 01:22:57:09 the divider would be at
a narrower dimension, and then
478 01:22:57:11 01:23:01:12 the inclination of this brace
relative to the center line
479 01:23:01:14 01:23:04:27 would be determined by
this one-seventh measurement.
480 01:23:06:13 01:23:09:01 A lack of precision can have
a steamrolling effect.
481 01:23:09:03 01:23:11:16 If one thing is wrong,
it leads to another thing
482 01:23:11:18 01:23:14:04 being slightly wrong,
and it grows exponentially.
483 01:23:14:06 01:23:16:20 For example,
the width of this thing
484 01:23:16:22 01:23:19:21 might not be that critical,
but for example,
485 01:23:19:23 01:23:23:09 if the sides are not actually
square to each other,

486	01:23:23:11	01:23:27:17	any layout that you do might... lead to a compounding error--
487	01:23:27:19	01:23:29:17	the relationship between the two sides
488	01:23:29:19	01:23:31:03	would tend to be different.
489	01:23:31:05	01:23:33:20	A number of relationships are important
490	01:23:33:22	01:23:36:16	in laying out the basic parts of the violin.
491	01:23:36:18	01:23:39:06	There's a relationship between the length of the neck
492	01:23:39:08	01:23:40:26	and what's called the body stop,
493	01:23:40:28	01:23:43:18	which is a measurement taken from the side of the neck
494	01:23:43:20	01:23:46:20	to the center of the bridge, and that measurement should be
495	01:23:46:22	01:23:49:27	three parts to two parts the length of the neck.
496	01:23:49:29	01:23:51:29	Another ratio relationship would be
497	01:23:52:01	01:23:55:00	the length of the fingerboard, which is determined
498	01:23:55:02	01:23:57:08	by the overall length of the strings,
499	01:23:57:10	01:23:59:02	so you take the string length,
500	01:23:59:04	01:24:02:17	and the fingerboard would be five-sixths of that length.
501	01:24:02:19	01:24:05:26	Another ratio measurement would be this measurement
502	01:24:05:28	01:24:09:13	from the tailpiece to the bridge, and that measurement
503	01:24:09:15	01:24:12:16	is usually one-sixth of the total string length.
504	01:24:14:00	01:24:16:27	Because a violin-family instrument does not have frets,
505	01:24:16:29	01:24:19:22	the player has to know where to put their fingers,
506	01:24:19:24	01:24:21:07	and there are certain cues
507	01:24:21:09	01:24:23:07	in terms of the shaping of the neck,
508	01:24:23:09	01:24:25:17	and the location of particular parts.
509	01:24:25:19	01:24:27:22	(*playing prelude*)
510	01:24:27:24	01:24:32:06	As the player shifts, when they reach certain cues
511	01:24:32:08	01:24:34:10	like this part of the neck or the body,
512	01:24:34:12	01:24:36:27	they know they're going to be at a certain note,
513	01:24:36:29	01:24:38:18	and if the proportion is wrong,
514	01:24:38:20	01:24:40:13	then when they get to that point,
515	01:24:40:15	01:24:43:05	they're going to be at the wrong note.
516	01:24:43:07	01:24:45:20	(*playing descending notes*)
517	01:24:45:22	01:24:48:12	Setups of most old instruments

			don't correspond
518	01:24:48:14	01:24:50:10	to what the modern settings are.
519	01:24:50:12	01:24:51:20	Baroque settings--
520	01:24:51:22	01:24:54:06	the original settings, say, on an instrument
521	01:24:54:08	01:24:56:28	made by Stradivari, would be slightly different.
522	01:24:57:00	01:24:58:05	It's quite possible
523	01:24:58:07	01:25:00:17	the neck would have been a little shorter.
524	01:25:00:19	01:25:02:16	So these modern ratios have evolved
525	01:25:02:18	01:25:05:20	to meet the specific playing demands of modern players
526	01:25:05:22	01:25:08:03	and the evolution of the quality of strings
527	01:25:08:05	01:25:11:13	and the fact that you would be playing in a much larger hall
528	01:25:11:15	01:25:13:23	to larger audiences.
529	01:25:13:25	01:25:16:25	(*prelude continues*)
530	01:25:16:27	01:25:20:05	Making a stringed instrument has an interesting reward,
531	01:25:20:07	01:25:22:12	because not only when you're done
532	01:25:22:14	01:25:25:28	do you have possibly a visually satisfying object,
533	01:25:26:00	01:25:28:00	but presumably, you have something
534	01:25:28:02	01:25:30:13	that can be used to make beautiful music
535	01:25:30:15	01:25:34:14	and will give an individual lasting pleasure in its use
536	01:25:34:16	01:25:36:13	over many, many years.
537	01:25:36:15	01:25:39:22	(*piece concludes*)
538	01:25:41:13	01:25:46:12	Captioned by Media Access Group at WGBH access.wgbh.org