1	01:00:47:08	01:00:48:21	CHAPIN: Measurement is the process
2	01:00:48:23	01:00:52:26	of quantifying properties of objects, and to do that,
3	01:00:52:28	01:00:56:22	we have set procedures that enable us to measure.
4	01:00:56:24	01:00:58:14	Oh.
5	01:00:58:16	01:01:00:28	Measuring helps you to understand
6	01:01:01:00	01:01:03:09	how things relate to each other.
7	01:01:03:11	01:01:07:07	CHAPIN: Our volume of a sphere actually has a formula
8	01:01:07:09	01:01:09:08	of four-thirds pi r-cubed.
9	01:01:09:10	01:01:12:22	This course really made me think about how I approach measurement
10	01:01:12:24	01:01:15:22	and how I can use measurement every day in the classroom.
11	01:01:21:24	01:01:25:10	In this session we are going to be talking about volume,
12	01:01:25:12	01:01:28:16	and in particular solid volume,
13	01:01:28:18	01:01:31:16	where we are filling something with cubic units.
14	01:01:31:18	01:01:33:20	CHAPIN: I had a number of different goals.
15	01:01:33:22	01:01:35:29	One was to help people understand
16	01:01:36:01	01:01:39:04	how we can use cubic units
17	01:01:39:06	01:01:41:10	to find the volume of solid objects,
18	01:01:41:12	01:01:45:06	both by using layers and by using formulas.
19	01:01:45:08	01:01:47:21	We also were interested in looking at
20	01:01:47:23	01:01:50:17	how could we take small packages
21	01:01:50:19	01:01:52:01	and fit them into larger packages,
22	01:01:52:03	01:01:53:19	and what's the relationship here
23	01:01:53:21	01:01:55:29	between the volume of each of those.
24	01:01:56:01	01:01:58:21	And finally, we were interested in volume formulas--
25	01:01:58:23	01:02:00:03	how do we make sense of them,
26	01:02:00:05	01:02:02:04	and how is the volume of one shape
27	01:02:02:06	01:02:06:02	related to the volume of another solid figure.
28	01:02:06:04	01:02:08:17	NARRATOR: To begin the first activity,
29	01:02:08:19	01:02:11:04	the class is given three different-size nets
30	01:02:11:06	01:02:12:12	of open boxes.
31	01:02:12:14	01:02:14:27	Nets are two-dimensional representations
32	01:02:14:29	01:02:16:28	of three-dimensional objects.

33	01:02:17:00	01:02:20:26	CHAPIN: Now, first off, you have three of these nets at your table,
34	01:02:20:28	01:02:23:18	and I'd like you and your partner, when we begin,
35	01:02:23:20	01:02:27:27	to estimate what is the volume of the box
36	01:02:27:29	01:02:29:16	by simply looking at the net.
37	01:02:29:18	01:02:30:29	Don't fold it.
38	01:02:31:01	01:02:34:06	But then feel free, after you've made an estimate,
39	01:02:34:08	01:02:39:13	to go ahead, cut it out, fold it up and fill it.
40	01:02:39:15	01:02:42:28	And maybe start out with the dimensions
41	01:02:43:00	01:02:46:14	of your beginning box and its volume,
42	01:02:46:16	01:02:50:16	and then play around with what dimensions could I have
43	01:02:50:18	01:02:52:09	that would double the volume?
44	01:02:52:11	01:02:58:17	What dimensions could I have that would quadruple the volume?
45	01:02:58:19	01:03:00:25	And, likewise, what could I do
46	01:03:00:27	01:03:02:14	in terms of the dimensions of a box
47	01:03:02:16	01:03:05:05	that's based on-- starting with this one--
48	01:03:05:07	01:03:08:11	that would have eight times the volume.
49	01:03:08:13	01:03:11:02	The surface area is 54.
50	01:03:11:04	01:03:13:10	54 blocks squared, whatever.
51	01:03:13:12	01:03:16:07	And if we folded it up like a box
52	01:03:16:09	01:03:17:24	and made it into volume,
53	01:03:17:26	01:03:23:05	then how much... cubic squares would that take up?
54	01:03:23:07	01:03:25:13	Would it be the same?
55	01:03:25:15	01:03:26:13	No.
56	01:03:26:15	01:03:27:17	No, because you got all...
57	01:03:27:19	01:03:29:01	you just got the outside surfaces.
58	01:03:29:03	01:03:30:17	Right.
59	01:03:30:19	01:03:34:05	CHAPIN: We were giving the participants nets of open boxes
60	01:03:34:07	01:03:36:02	and asking them to look at the nets
61	01:03:36:04	01:03:39:04	and then estimate the volume of the box
62	01:03:39:06	01:03:42:05	once they actually folded the net up.
63	01:03:42:07	01:03:46:20	This pointed out the difference

			between surface area and volume
64	01:03:46:22	01:03:48:09	and led people to start thinking about
65	01:03:48:11	01:03:51:20	where do these volume formulas for rectangular prisms
66	01:03:51:22	01:03:53:05	come from?
67	01:03:53:07	01:03:56:17	How is it that we use length times width times height,
68	01:03:56:19	01:04:00:21	and how is that related to the actual dimensions of the box?
69	01:04:00:23	01:04:05:02	So if you double...
70	01:04:05:04	01:04:07:00	the volume comes out how?
71	01:04:07:02	01:04:08:16	It came out eight times bigger?
72	01:04:08:18	01:04:09:18	Eight times.
73	01:04:09:20	01:04:14:19	And when we triple it, it came out...
74	01:04:14:21	01:04:16:19	BOTH: ...27 times bigger.
75	01:04:16:21	01:04:18:14	Oh.
76	01:04:18:16	01:04:20:16	So, well, if we quadrupled it,
77	01:04:20:18	01:04:23:22	we would expect it to be...
78	01:04:23:24	01:04:26:01	Four times four to the third,
79	01:04:26:03	01:04:27:21	so, four to the 16th.
80	01:04:27:23	01:04:28:21	Oh, look at that.
81	01:04:28:23	01:04:30:12	So if we multiply something
82	01:04:30:14	01:04:31:19	by a factor of two,
83	01:04:31:21	01:04:35:17	the volume is two cubed by three,
84	01:04:35:19	01:04:37:16	three cubed by four.
85	01:04:37:18	01:04:38:20	Four cubed.
86	01:04:38:22	01:04:41:17	So if we multiplied it by K, any factor,
87	01:04:41:19	01:04:43:03	then it would be...
88	01:04:43:05	01:04:44:27	K to the third power.
89	01:04:44:29	01:04:46:23	Right, that's perfect.
90	01:04:46:25	01:04:49:02	Perfect, right.
91	01:04:49:04	01:04:51:03	CHAPIN: I noticed that many of you were coming up
92	01:04:51:05	01:04:56:11	with a relationship between increasing the dimensions
93	01:04:56:13	01:04:58:06	and the increasing volume.
94	01:04:58:08	01:05:01:26	And so let's explore and look at that in a little more detail.
95	01:05:01:28	01:05:07:08	I have up here the dimensions of package A: 2 x 2 x 4,
96	01:05:07:10	01:05:10:18	and I have built it right here--
97	01:05:10:20	01:05:15:01	two length, two back--

			or two width-- and four high.
98	01:05:15:03	01:05:16:21	I'm just going to hold it this way,
99	01:05:16:23	01:05:18:06	so we're all kind of clear on it.
100	01:05:18:08	01:05:23:21	Our volume is 16 units cubed, or "cubic units," all right?
101	01:05:23:23	01:05:28:04	Now, if we increase the dimensions of one of those--
102	01:05:28:06	01:05:32:23	let's double this one so it's, um...
103	01:05:32:25	01:05:38:27	actually, 4 x 2 x 4.
104	01:05:38:29	01:05:43:24	What I'm doing is I'm taking another and putting it across.
105	01:05:43:26	01:05:48:12	Now I have four across, two back, and four high.
106	01:05:48:14	01:05:50:17	Right?
107	01:05:50:19	01:05:53:12	And what's happened to the volume?
108	01:05:53:14	01:05:54:15	It's doubled.
109	01:05:54:17	01:05:55:26	All right?
110	01:05:55:28	01:06:00:03	So we know that our volume is 32 cubic units.
111	01:06:00:05	01:06:02:26	Now let's take another dimension and double it.
112	01:06:02:28	01:06:04:24	How about this one?
113	01:06:04:26	01:06:09:08	So I'm going to keep this first dimension doubled,
114	01:06:09:10	01:06:12:19	so we have two, so we have 4 x 4 x 4.
115	01:06:12:21	01:06:14:15	So let's look again.
116	01:06:14:17	01:06:19:09	Four across, so now we want to go four back
117	01:06:19:11	01:06:21:28	and four high.
118	01:06:22:00	01:06:29:14	And you can see, we have a four times increase in the volume.
119	01:06:29:16	01:06:32:05	Now, finally, what is 4 x 4 x 4?
120	01:06:32:07	01:06:33:26	Is that... 64.
121	01:06:33:28	01:06:34:27	Right?
122	01:06:34:29	01:06:35:28	MAN: Cubic units.
123	01:06:36:00	01:06:37:01	Cubic units, thanks.
124	01:06:37:03	01:06:38:11	Now let's do the last one.
125	01:06:38:13	01:06:43:08	So we have 4 x 4 x-- double this-- 8.
126	01:06:43:10	01:06:45:12	CHAPIN: What's this, 128?
127	01:06:45:14	01:06:50:16	Now, how is these two related?
128	01:06:50:18	01:06:52:20	How many times is...
129	01:06:52:22	01:06:54:12	16 times what is 128?
130	01:06:54:14	01:06:55:17	STUDENTS: Eight.
131	01:06:55:19	01:06:57:06	CHAPIN: Eight.
132	01:06:57:08	01:06:58:28	Where did the eight come from?
133	01:06:59:00	01:07:01:29	It may help to

			rewrite this this way.
134	01:07:02:01	01:07:03:28	Well, I'm going to put our first dimensions
135	01:07:04:00	01:07:05:13	of our original box here.
136	01:07:05:15	01:07:06:27	Okay?
137	01:07:06:29	01:07:10:07	What I first did is I took the original dimension,
138	01:07:10:09	01:07:12:18	multiplied it by two.
139	01:07:12:20	01:07:14:05	Because I doubled it.
140	01:07:14:07	01:07:18:15	I then took the second dimension-- our depth--
141	01:07:18:17	01:07:22:07	and multiplied it by two.
142	01:07:22:09	01:07:27:14	And then I took our height and multiplied it by two.
143	01:07:27:16	01:07:31:12	Now, if we just use the associative property
144	01:07:31:14	01:07:33:18	and move things around in terms of the order,
145	01:07:33:20	01:07:40:23	we could end up with 2 x 2 x 4, which is the original volume,
146	01:07:40:25	01:07:46:19	times 2 x 2 x 2, which is eight.
147	01:07:46:21	01:07:49:17	We have increased it two to the third power,
148	01:07:49:19	01:07:53:00	which is also equal to eight-- eight times greater--
149	01:07:53:02	01:07:56:04	because I have to multiply all three dimensions.
150	01:07:56:06	01:07:58:07	And so when we increase all three dimensions,
151	01:07:58:09	01:08:01:17	it really affects the volume.
152	01:08:05:16	01:08:07:02	This one's done,
153	01:08:07:04	01:08:08:15	so let's take this one.
154	01:08:08:17	01:08:09:24	Okay.
155	01:08:09:26	01:08:12:03	CHAPIN: We had five different packages,
156	01:08:12:05	01:08:14:19	made up of small unit cubes,
157	01:08:14:21	01:08:18:27	and participants were asked to estimate and then determine
158	01:08:18:29	01:08:23:16	how many of each package would fit into one box.
159	01:08:23:18	01:08:26:07	Some of the packages fit perfectly into the box,
160	01:08:26:09	01:08:28:09	filling it right to the top,
161	01:08:28:11	01:08:31:00	other packages wouldn't even fit in the box,
162	01:08:31:02	01:08:32:14	the dimensions were all off.
163	01:08:32:16	01:08:35:28	So it was looking at the relationships in this case
164	01:08:36:00	01:08:38:01	between the dimensions of a larger box
165	01:08:38:03	01:08:40:03	and the dimensions of a small package
166	01:08:40:05	01:08:41:29	and how those might be related.
167	01:08:42:01	01:08:44:18	Box number five will not fit

168	01:08:44:20	01:08:46:02	into our shipping package.
169	01:08:46:04	01:08:48:11	Right, it's too long.
170	01:08:48:13	01:08:50:00	So, zero.
171	01:08:48:13	01:08:50:00	Zero.
172	01:08:50:02	01:08:51:18	Okay.
173	01:08:51:20	01:08:54:25	But if we take the pink one, box number four,
174	01:08:54:27	01:08:56:04	and we lay it,
175	01:08:56:06	01:08:59:21	we can see that two of those would fit across
176	01:08:59:23	01:09:02:07	with some leftover space.
177	01:09:02:09	01:09:04:10	Okay?
178	01:09:04:12	01:09:05:28	We've done those two.
179	01:09:06:00	01:09:07:11	So, go with three.
180	01:09:07:13	01:09:09:13	Mm-hmm.
181	01:09:09:15	01:09:13:26	One, two, three, four.
182	01:09:13:28	01:09:15:01	Plus another layer.
183	01:09:15:03	01:09:16:29	Five, six, seven, eight,
184	01:09:17:01	01:09:18:08	and then the third layer.
185	01:09:18:10	01:09:19:14	BOTH: Nine, ten, 11, 12.
186	01:09:19:16	01:09:22:04	So that's the best fit for this shipping.
187	01:09:22:06	01:09:24:00	Right, that's a good size for that.
188	01:09:24:02	01:09:25:23	Okay, "Describe your strategy."
189	01:09:25:25	01:09:28:20	How many in package four?
190	01:09:28:22	01:09:30:20	Well, we laid it out.
191	01:09:30:22	01:09:31:22	Oh.
192	01:09:31:24	01:09:32:22	Oh, you know what?
193	01:09:32:24	01:09:34:00	I think we mess...
194	01:09:34:02	01:09:35:29	We could have stood it up like that.
195	01:09:36:01	01:09:37:13	And we could have fit more.
196	01:09:37:15	01:09:38:13	More of them.
197	01:09:38:15	01:09:39:12	Okay.
198	01:09:39:14	01:09:40:25	So that actually fits four.
199	01:09:40:27	01:09:42:06	Oh, that's interesting.
200	01:09:44:01	01:09:48:19	NARRATOR: In the next activity, the class is asked to design a box
201	01:09:48:21	01:09:51:02	that could be filled completely

by each of the packages,
202 01:09:51:04 01:09:53:20 using them one at a time.
203 01:09:53:22 01:09:55:16 CHAPIN:
So they had to imagine
building a box
204 01:09:55:18 01:09:58:22 that package one
would completely fill.
205 01:09:58:24 01:10:01:11 And then if they took all
of package one out of that,
206 01:10:01:13 01:10:04:05 they could also use package two
and completely fill the box,
207 01:10:04:07 01:10:05:23 so there were
multiple dimensions
208 01:10:05:25 01:10:08:08 that had to be addressed.
209 01:10:08:10 01:10:10:26 MAN:
Um, the tallest one
is two.
210 01:10:10:28 01:10:13:07 You probably just
have to come up
two levels.
211 01:10:13:09 01:10:14:07 WOMAN:
Okay.
212 01:10:14:09 01:10:15:26 So if we put
another layer
213 01:10:15:28 01:10:17:27 of the five
candy bar on,
214 01:10:17:29 01:10:19:27 we would have
24 candy bars.
215 01:10:19:29 01:10:21:04 Mm-hmm.
216 01:10:19:29 01:10:21:04 All right.
217 01:10:21:06 01:10:22:29 Now let's try it
with each one.
218 01:10:23:01 01:10:26:20 Put the five away,
and number... one.
219 01:10:26:22 01:10:29:01 Let's start
with number one.
220 01:10:33:13 01:10:34:19 Okay.
221 01:10:34:21 01:10:36:01 Six, eight, ten.
222 01:10:36:03 01:10:39:13 Okay, so number one,
we have a base of four--
223 01:10:39:15 01:10:41:10 one, two, three, four.
224 01:10:41:12 01:10:44:21 So each one-- one...
225 01:10:44:23 01:10:48:07 we'd have one, two,
three in that row.
226 01:10:48:09 01:10:53:02 We have three
in the next row.
227 01:10:53:04 01:10:56:14 Three in
the next row.
228 01:10:56:16 01:10:58:13 Three in
the next row.
229 01:10:58:15 01:10:59:20 Three in
the next row.
230 01:10:59:22 01:11:02:21 So we'd have
15 on the bottom.
231 01:11:02:23 01:11:05:04 And if we're only
going to have two high...

232	01:11:05:06	01:11:07:07	...that's all there would be.
233	01:11:07:09	01:11:09:16	CHAPIN: I'd like to go over that problem
234	01:11:09:18	01:11:11:26	of trying to find a box
235	01:11:11:28	01:11:15:17	that will fit all of our different packages, um...
236	01:11:15:19	01:11:17:14	and the box is filled completely,
237	01:11:17:16	01:11:20:01	but it's also the smallest box that we can use.
238	01:11:20:03	01:11:23:11	There are a number of ways that that problem can be approached.
239	01:11:23:13	01:11:27:25	One way is-- that I saw some of you using--
240	01:11:27:27	01:11:31:10	was first that you took all the dimensions of the packages
241	01:11:31:12	01:11:34:04	and you found their volumes.
242	01:11:34:06	01:11:36:04	And then, I noticed,
243	01:11:36:06	01:11:42:10	that you found the least-common multiple of these volumes.
244	01:11:42:12	01:11:46:15	Namely, what number will all of these divide into,
245	01:11:46:17	01:11:47:29	what's the smallest,
246	01:11:48:01	01:11:53:16	which in this case turns out to be 120 cubic units.
247	01:11:53:18	01:11:54:20	All right?
248	01:11:54:22	01:11:59:05	Now that we have our volume of a box,
249	01:11:59:07	01:12:02:24	the next thing to think about is the dimensions.
250	01:12:02:26	01:12:05:22	Anyone give us one size box
251	01:12:05:24	01:12:09:14	that actually works using this volume?
252	01:12:09:16	01:12:10:29	Katie.
253	01:12:11:01	01:12:14:21	We found a 30 x 2 x 2.
254	01:12:14:23	01:12:15:25	30 x 2 x 2.
255	01:12:15:27	01:12:19:27	Now, how did you come up with that?
256	01:12:19:29	01:12:21:10	We actually came up with it
257	01:12:21:12	01:12:24:18	by looking at the dimensions of the various boxes.
258	01:12:24:20	01:12:26:16	So we wanted it to be a multiple of five
259	01:12:26:18	01:12:28:29	and also a multiple of two and also a multiple of three.
260	01:12:29:01	01:12:30:08	CHAPIN: Okay.
261	01:12:30:10	01:12:33:09	Anyone have another dimension that would work?
262	01:12:33:11	01:12:36:02	10 x 2 x 6.
263	01:12:36:04	01:12:37:29	CHAPIN: 10 x 2 x 6,
264	01:12:38:01	01:12:39:19	which will also give...
265	01:12:39:21	01:12:43:04	these dimensions will also give us this volume,

266	01:12:43:06	01:12:47:26	and each of these packages will be able to fit in.
267	01:12:47:28	01:12:50:16	Will all of the various... if we divided,
268	01:12:50:18	01:12:53:04	or took 120 and looked at different ways
269	01:12:53:06	01:12:55:01	that we could make packages,
270	01:12:55:03	01:12:59:04	would any package that has a volume of 120 work?
271	01:12:59:06	01:13:01:06	Jan, why not?
272	01:13:01:08	01:13:04:06	Because when you have a dimension of two by two,
273	01:13:04:08	01:13:06:08	if you had a measurement of five,
274	01:13:06:10	01:13:09:23	a dimension of five at one end of your box,
275	01:13:09:25	01:13:14:09	those two by twos would not fit evenly into that dimension,
276	01:13:14:11	01:13:16:04	because you would need multiples of two.
277	01:13:16:06	01:13:17:05	CHAPIN: Okay.
278	01:13:20:10	01:13:23:25	Now, what I'd like to do is shift gears
279	01:13:23:27	01:13:29:08	and move away from working with our rectangular prisms
280	01:13:29:10	01:13:32:28	to thinking about volume of other shapes.
281	01:13:33:00	01:13:37:11	And while we do that, I'd like to move into using formulas,
282	01:13:37:13	01:13:38:26	which are very handy
283	01:13:38:28	01:13:42:01	when one is trying to calculate the volume
284	01:13:42:03	01:13:43:17	of different solids.
285	01:13:43:19	01:13:47:08	Now, often the volume of a prism can be thought of
286	01:13:47:10	01:13:50:10	as the area of the base times the height.
287	01:13:50:12	01:13:54:12	And let me just show you this example.
288	01:13:54:14	01:13:56:25	Here we have a nice rectangular prism,
289	01:13:56:27	01:13:59:02	one of the ones we were working on,
290	01:13:59:04	01:14:02:12	and if we think of the area of the base--
291	01:14:02:14	01:14:05:04	the base is two by two, all right?
292	01:14:05:06	01:14:08:06	And so if we find the area of the base,
293	01:14:08:08	01:14:10:10	which is four square units,
294	01:14:10:12	01:14:12:20	we then can think about basically layering this
295	01:14:12:22	01:14:14:21	all the way up

no matter what the height.
296 01:14:14:23 01:14:17:24 In this case,
the height is three,
297 01:14:17:26 01:14:20:16 so I would think about having
three of these layers.
298 01:14:20:18 01:14:24:17 Now, this idea of the area
of the base times the height
299 01:14:24:19 01:14:27:21 can be used with lots
of different shapes.
300 01:14:27:23 01:14:30:08 We could have it
with a triangular prism.
301 01:14:36:27 01:14:39:03 If we think of
the triangular prism,
302 01:14:39:05 01:14:43:01 one way we can think of this
is having a triangle as the base
303 01:14:43:03 01:14:47:07 and those triangles being
layered up, up, up, up, up,
304 01:14:47:09 01:14:48:23 until we get to the top.
305 01:14:48:25 01:14:51:28 And so again we could use
our formula here of this prism
306 01:14:52:00 01:14:53:24 of finding the area
of the base--
307 01:14:53:26 01:14:56:25 half base times height
for a triangle--
308 01:14:56:27 01:15:00:08 and then just multiplying it
by its height.
309 01:15:00:10 01:15:04:06 We could even use this sometimes
with our... with cylinders.
310 01:15:04:08 01:15:07:12 So in many cases,
formulas can be reduced
311 01:15:07:14 01:15:11:10 to area of the base
times the height.
312 01:15:11:12 01:15:14:29 Now, that'll work as long
as the height goes up straight
313 01:15:15:01 01:15:16:18 and is consistent.
314 01:15:16:20 01:15:20:06 It will even work
if it's off to a slant,
315 01:15:20:08 01:15:24:23 as long as the whole shape
is going up consistently.
316 01:15:24:25 01:15:26:16 We run into some difficulties,
317 01:15:26:18 01:15:31:08 we have a base, but it doesn't
go up consistently on all sides.
318 01:15:31:10 01:15:34:29 So it's not like you were
layering the same thing
319 01:15:35:01 01:15:36:05 all the way up.
320 01:15:36:07 01:15:38:01 So in our next activity,
321 01:15:38:03 01:15:41:00 we're going to start
to investigate volumes of cones,
322 01:15:41:02 01:15:47:16 volumes of cylinders
and volume of spheres.
323 01:15:47:18 01:15:51:01 Oh, we could even
measure the height
of the... the sphere.
324 01:15:51:03 01:15:52:14 It's like
two inches high.
325 01:15:52:16 01:15:53:29 It's under

two inches high.
326 01:15:54:01 01:15:55:01 Okay, so...
327 01:15:55:03 01:15:56:13 So if we cut it...
328 01:15:56:15 01:15:57:25 At three?
329 01:15:57:27 01:16:00:11 Yeah, about three,
and we can trim it
down afterwards.
330 01:16:02:03 01:16:05:00 And we don't need
all this tape.
331 01:16:05:02 01:16:07:02 I'm just going
to trim a little
bit off this, too,
332 01:16:07:04 01:16:08:09 because we had
so much extra.
333 01:16:08:11 01:16:09:24 How much extra
did we have, though?
334 01:16:09:26 01:16:10:17 A lot.
335 01:16:10:19 01:16:11:10 A lot?
336 01:16:11:12 01:16:12:16 Oh, yeah.
337 01:16:12:18 01:16:14:11 Okay.
338 01:16:14:13 01:16:17:26 CHAPIN:
We've had participants
make a sphere out of clay
339 01:16:17:28 01:16:22:02 and then build a cylinder
around that clay sphere.
340 01:16:22:04 01:16:23:19 It's important to note
341 01:16:23:21 01:16:26:29 that the height of the sphere
and the cylinder were the same,
342 01:16:27:01 01:16:30:20 and the diameter of the sphere
and the cylinder were the same.
343 01:16:30:22 01:16:35:13 They then squashed down
the sphere into their cylinder,
344 01:16:35:15 01:16:37:05 and looked at the relationship
345 01:16:37:07 01:16:39:23 and found that the volumes
were not identical.
346 01:16:39:25 01:16:43:14 In fact, the sphere's volume was
less than the cylinder's volume.
347 01:16:43:16 01:16:48:13 It was two-thirds approximately
of the cylinder's volume.
348 01:16:48:15 01:16:51:25 We also did a similar activity
349 01:16:51:27 01:16:55:10 to look at the relationship
between a cylinder and a cone.
350 01:16:55:12 01:16:57:15 We were filling
the cone with rice--
351 01:16:57:17 01:17:01:02 pouring it into the cylinder
and seeing that it would take
352 01:17:01:04 01:17:03:06 three times the volume
of the cone
353 01:17:03:08 01:17:05:15 to equal the volume
of the cylinder.
354 01:17:05:17 01:17:08:23 It led us to realize
that the relationship here is
355 01:17:08:25 01:17:11:02 a third of the volume
of a cylinder
356 01:17:11:04 01:17:13:06 is equal to the volume
of a cone.

357	01:17:13:08	01:17:15:09	So let's make a... a couple of conclusions.
358	01:17:15:11	01:17:16:17	What's it say to do?
359	01:17:16:19	01:17:18:08	Okay.
360	01:17:18:10	01:17:22:11	It says, "If a cone, cylinder and sphere have the same radius,
361	01:17:22:13	01:17:25:11	what is the relationship between the volumes?"
362	01:17:25:13	01:17:29:08	Well, we related both to the... cone, right?
363	01:17:29:10	01:17:30:09	Was this...?
364	01:17:30:11	01:17:32:00	Because we poured the cone in the...
365	01:17:32:02	01:17:34:26	Oh, we related both to the cylinder.
366	01:17:34:28	01:17:36:16	...to the cylinder.
367	01:17:36:18	01:17:40:07	So it took three cones to make one cylinder.
368	01:17:40:09	01:17:42:03	Right, so...
369	01:17:42:05	01:17:44:21	The volume of a cone...
370	01:17:44:23	01:17:46:10	Cone.
371	01:17:46:12	01:17:47:16	Is equal...
372	01:17:47:18	01:17:51:10	...times three equals the cylinder.
373	01:17:51:12	01:17:53:03	Equals the volume of the cylinder.
374	01:17:53:05	01:17:54:27	Equals the volume of a cylinder,
375	01:17:54:29	01:17:57:04	or one-third of a cylinder equals a cone.
376	01:17:58:21	01:18:03:00	Okay, let's go over some of the insights that you've gained
377	01:18:03:02	01:18:05:09	based on doing some measuring.
378	01:18:05:11	01:18:09:08	To review, when we want to find the volume of a cylinder,
379	01:18:09:10	01:18:12:07	remember we can find the area of the base,
380	01:18:12:09	01:18:19:03	which is a circle, so it's pi r-squared times the height.
381	01:18:19:05	01:18:20:18	All right?
382	01:18:20:20	01:18:25:04	Now, what did you find in terms of the relationship
383	01:18:25:06	01:18:29:20	of the sphere's volume in terms of the cylinder's?
384	01:18:29:22	01:18:31:21	Were they identical?
385	01:18:31:23	01:18:33:22	We came closer to two-thirds.
386	01:18:33:24	01:18:35:03	CHAPIN: Okay.
387	01:18:35:05	01:18:38:02	Both with the sphere and with the rice...
388	01:18:38:04	01:18:42:07	using the rice as a measure, using the two plastic shapes.
389	01:18:42:09	01:18:45:20	Where we got differences-- there is a set relationship,
390	01:18:45:22	01:18:48:06	if we could do this

very, very accurately,
391 01:18:48:08 01:18:53:02 and that relationship is
that the volume of the sphere
392 01:18:53:04 01:19:01:23 is actually equal to two-thirds
of the volume of the cylinder.
393 01:19:01:25 01:19:03:02 All right?
394 01:19:03:04 01:19:06:27 Now, it's interesting,
because in this case
395 01:19:06:29 01:19:10:13 we had a cylinder
that had a set height.
396 01:19:10:15 01:19:16:07 In fact, the height was exactly
the... to the top of the sphere.
397 01:19:16:09 01:19:19:15 How would we... how could
we record that height?
398 01:19:19:17 01:19:20:27 2r, or the diameter.
399 01:19:20:29 01:19:23:14 CHAPIN:
It was the diameter
of the sphere,
400 01:19:23:16 01:19:25:19 because it fills
the cylinder completely.
401 01:19:25:21 01:19:27:18 So we're going to call it "2r".
402 01:19:27:20 01:19:30:02 Now, if we expand this out,
403 01:19:30:04 01:19:32:23 what we end up with
is 2/3 x 2
404 01:19:32:25 01:19:37:18 is 4/3 pi,
405 01:19:37:20 01:19:42:26 and then r-squared
times r is r-cubed.
406 01:19:42:28 01:19:44:28 And so our volume of a sphere
407 01:19:45:00 01:19:48:11 actually has a formula
of 4/3 pi r-cubed,
408 01:19:48:13 01:19:50:22 which many of you
are probably familiar with.
409 01:19:50:24 01:19:52:17 Now, there was one
other relationship
410 01:19:52:19 01:19:58:07 and that was the volume
of the cone.
411 01:19:58:09 01:20:01:01 What kind of relationship
did we find here?
412 01:20:01:03 01:20:03:19 When we used the cone
to fill the cylinder,
413 01:20:03:21 01:20:06:06 it took three cones
to fill the cylinder,
414 01:20:06:08 01:20:08:21 so the cone is one-third
of the cylinder.
415 01:20:08:23 01:20:11:15 Okay,
so we could say
416 01:20:11:17 01:20:16:06 1/3 pi r-squared h,
so it's that relationship.
417 01:20:16:08 01:20:18:03 Now, one last thing.
418 01:20:18:05 01:20:21:15 If the cone is a third
of the cylinder
419 01:20:21:17 01:20:25:23 and the sphere is two-thirds
of the cylinder,
420 01:20:25:25 01:20:30:11 what do you think
the relationship of the volume
421 01:20:30:13 01:20:34:17 in this case between

the cone and the sphere is?
422 01:20:34:19 01:20:36:27 We actually checked
with the rice
423 01:20:36:29 01:20:39:20 and found that
the volume of the sphere
424 01:20:39:22 01:20:41:29 was about twice
that of the cone.
425 01:20:42:01 01:20:42:28 Great.
426 01:20:43:00 01:20:43:28 Could you show it?
427 01:20:44:00 01:20:45:05 Just hold it up
right there.
428 01:20:45:07 01:20:46:07 Do you have
half the...?
429 01:20:46:09 01:20:49:05 We actually still
have it in here.
430 01:20:49:07 01:20:52:03 The... we emptied
one of the cones
431 01:20:52:05 01:20:54:19 into half the sphere
exactly.
432 01:20:54:21 01:20:56:23 CHAPIN:
We live in
a three-dimensional world,
433 01:20:56:25 01:21:00:17 and thus, knowledge of volume
is very, very important.
434 01:21:00:19 01:21:04:11 Whether you are trying
to find the volume of a package
435 01:21:04:13 01:21:06:02 or the volume of a room,
436 01:21:06:04 01:21:09:06 we need to understand,
what units will we use?
437 01:21:09:08 01:21:12:23 What relationships or dimensions
must we consider?
438 01:21:12:25 01:21:14:22 And can we use a formula?
439 01:21:14:24 01:21:16:21 If we can use a formula,
440 01:21:16:23 01:21:21:04 hopefully we understand where
it came from, why it works
441 01:21:21:06 01:21:24:10 and we can apply it
to find that volume.
442 01:21:32:18 01:21:38:07 NARRATOR:
Driving in Boston-- it's stop
and go from dawn till dusk.
443 01:21:38:09 01:21:41:03 The culprit:
an antiquated elevated highway
444 01:21:41:05 01:21:42:28 known as the Central Artery
445 01:21:43:00 01:21:45:17 that cuts through
the heart of downtown.
446 01:21:45:19 01:21:49:25 Originally built in 1959
to carry 75,000 vehicles,
447 01:21:49:27 01:21:54:05 today over 200,000 cars
and trucks jam the roadway,
448 01:21:54:07 01:21:56:19 bringing traffic to a crawl.
449 01:21:56:21 01:21:58:17 Something had to be done.
450 01:21:58:19 01:22:02:24 That something became the
Central Artery/Tunnel Project,
451 01:22:02:26 01:22:05:00 better known as the Big Dig.
452 01:22:05:02 01:22:07:08 MAN:
The Big Dig is actually

two different projects.
453 01:22:07:10 01:22:09:24 One's the reconstruction
of the downtown Central Artery,
454 01:22:09:26 01:22:10:29 putting it underground.
455 01:22:11:01 01:22:12:24 And the other's
the extension of I-90,
456 01:22:12:26 01:22:14:15 which terminates
at I-93 right now,
457 01:22:14:17 01:22:16:27 and extending it
all the way to the airport.
458 01:22:16:29 01:22:20:15 NARRATOR:
The Big Dig is the most
expensive public works project
459 01:22:20:17 01:22:22:15 in the history
of the United States.
460 01:22:22:17 01:22:25:14 From the beginning,
a variety of complex design
461 01:22:25:16 01:22:28:12 and construction problems
confronted engineers.
462 01:22:28:14 01:22:31:03 One of these involved
crossing a waterway
463 01:22:31:05 01:22:33:04 known as the Fort Point Channel
464 01:22:33:06 01:22:35:26 to complete a tunnel system
to the airport.
465 01:22:35:28 01:22:38:24 BERTOULIN:
Fort Point Channel is probably
what most people consider
466 01:22:38:26 01:22:41:15 the biggest technical challenge
we had on the artery itself.
467 01:22:41:17 01:22:42:22 The Red Line subway system
468 01:22:42:24 01:22:44:19 runs right down the center
of the channel,
469 01:22:44:21 01:22:46:22 so we basically,
in order to cross the channel,
470 01:22:46:24 01:22:49:03 we had to have tunnels
go under the railroad tracks.
471 01:22:49:05 01:22:51:05 We had to cross
the Fort Point Channel itself.
472 01:22:51:07 01:22:53:16 It was a huge technical issue
for us to deal with,
473 01:22:53:18 01:22:56:18 and all of these things had to
be worked on on a daily basis.
474 01:22:56:20 01:22:59:16 NARRATOR:
Normally, the tunnels
would be cast off-site
475 01:22:59:18 01:23:01:20 and brought to
the construction area.
476 01:23:01:22 01:23:06:01 Unfortunately, access
to the waterway was blocked,
477 01:23:06:03 01:23:09:22 so a casting basin was built
along the banks of the channel.
478 01:23:09:24 01:23:11:23 Before constructing the basin,
479 01:23:11:25 01:23:15:11 engineers had to calculate
how many cubic yards of dirt
480 01:23:15:13 01:23:17:13 needed to be excavated
from the site.

481 01:23:17:15 01:23:18:27 BERTOULIN:
We had this plan
482 01:23:18:29 01:23:21:19 to monitor progress
from the Fort Point Channel--
483 01:23:21:21 01:23:23:15 our immersed tube construction,
484 01:23:23:17 01:23:25:27 our tunnels on either side
tying it in.
485 01:23:25:29 01:23:27:09 But what it gets right down to
486 01:23:27:11 01:23:28:24 is the shape
of our casting basin.
487 01:23:28:26 01:23:30:23 As you can see,
it's a very irregular shape.
488 01:23:30:25 01:23:33:10 But what we as engineers do
is we break irregular shapes
489 01:23:33:12 01:23:35:28 into a series of regular shapes
for volume calculations--
490 01:23:36:00 01:23:37:19 triangular shapes,
rectangular shapes.
491 01:23:37:21 01:23:41:00 And what we do-- because we have
a depth, a width and a length--
492 01:23:41:02 01:23:43:23 you can very quickly calculate
the volumes overall
493 01:23:43:25 01:23:45:08 from a very irregular shape.
494 01:23:45:10 01:23:48:04 We can even cheat a little bit
by taking a shape out here
495 01:23:48:06 01:23:51:12 and going with the best fit to
create our last triangular shape
496 01:23:51:14 01:23:53:26 to create the total volume
for the casting basin.
497 01:23:53:28 01:23:56:29 NARRATOR:
That total was
450,000 cubic yards,
498 01:23:57:01 01:24:01:03 or 30,000 truckloads of material
excavated from the site.
499 01:24:01:05 01:24:05:23 After 18 months of construction,
the casting basin was completed
500 01:24:05:25 01:24:09:00 and building of
the immersed tube tunnels began.
501 01:24:09:02 01:24:11:12 BERTOULIN:
Typically, our immersed tubes
502 01:24:11:14 01:24:14:25 have about a four-foot-thick
floor, four-foot-thick walls
503 01:24:14:27 01:24:16:23 and about
a four-foot-thick roof.
504 01:24:16:25 01:24:19:08 NARRATOR:
Once the tunnels were assembled,
505 01:24:19:10 01:24:22:00 engineers had to determine
how much water was necessary
506 01:24:22:02 01:24:23:17 to flood the casting basin
507 01:24:23:19 01:24:26:18 so the tubes could be floated
into place across the channel.
508 01:24:26:20 01:24:29:05 BERTOULIN:
Basically, we had a volume
of the casting basin
509 01:24:29:07 01:24:31:05 that we had filled up
with the immersed tubes,

510	01:24:31:07	01:24:33:18	so you could subtract the volume of the immersed tubes
511	01:24:33:20	01:24:35:10	out of the volume of the casting basin--
512	01:24:35:12	01:24:37:19	that told us how much water then had to go into it.
513	01:24:37:21	01:24:40:24	And we used a series of pumps-- 10,000 gallons per minute.
514	01:24:40:26	01:24:42:06	The casting basin itself
515	01:24:42:08	01:24:44:20	would hold about 70 million gallons of water,
516	01:24:44:22	01:24:48:00	but once you subtracted out the volume of the immersed tubes,
517	01:24:48:02	01:24:52:24	only about half of that had to be pumped in as far as water.
518	01:24:52:26	01:24:55:19	NARRATOR: Afterwards, the casting basin was drained
519	01:24:55:21	01:24:58:06	and became part of the tunnel system itself.
520	01:24:58:08	01:25:00:22	(*truck beeping*)
521	01:25:05:22	01:25:08:27	Over a hundred truckloads of concrete were used each day,
522	01:25:08:29	01:25:12:09	while construction workers laid the foundations of the road
523	01:25:12:11	01:25:15:07	that will eventually carry vehicles
524	01:25:15:09	01:25:17:09	to and from the airport.
525	01:25:17:11	01:25:21:09	BERTOULIN: Once the tunnels are completed, then we have to backfill them.
526	01:25:21:11	01:25:22:18	But, of course, where we took
527	01:25:22:20	01:25:24:04	450,000 cubic yards of material out,
528	01:25:24:06	01:25:25:15	now we've built tunnels there.
529	01:25:25:17	01:25:27:16	So it's only going to take a fraction of that
530	01:25:27:18	01:25:29:25	to fill it back up and return it to the natural grade
531	01:25:29:27	01:25:31:27	that it was before the start of our project.
532	01:25:35:23	01:25:39:11	Captioned by Media Access Group at WGBH access.wgbh.org