1 01:00:47:07 01:00:48:20 CHAPIN:
Measurement is the process
2 01:00:48:22 01:00:52:25 of quantifying properties
of objects, and to do that,
3 01:00:52:27 01:00:56:21 we have set procedures
that enable us to measure.
4 01:00:56:23 01:00:58:13 Oh.
5 01:00:58:15 01:01:00:27 Measuring helps you
to understand
6 01:01:00:29 01:01:03:08 how things relate to each other.
7 01:01:03:10 01:01:09:07 CHAPIN:
Our volume of a sphere actually
has a formula of 4/3 pi r-cubed.
8 01:01:09:09 01:01:12:21 This course really made me think
about how I approach measurement
9 01:01:12:23 01:01:15:21 and how I can use measurement
every day in the classroom.
10 01:01:21:01 01:01:24:17 NARRATOR:
In today's session,
the class is going to examine
11 01:01:24:19 01:01:27:09 the circumference
and area of a circle.
12 01:01:27:11 01:01:29:27 They will investigate formulas
for these measures,
13 01:01:29:29 01:01:32:09 and make sense
of the unique relationship
14 01:01:32:11 01:01:35:02 between circles and
the irrational number pi.
15 01:01:35:04 01:01:37:02 So we're going to kind
of start our day today
16 01:01:37:04 01:01:39:04 by doing a little estimating
17 01:01:39:06 01:01:42:08 to just get ourselves
thinking in circles.
18 01:01:42:10 01:01:45:23 Now, as you notice, some
of you are bike riders,
19 01:01:45:25 01:01:49:28 and I have a bicycle wheel here,
and we're going to look
20 01:01:50:00 01:01:55:02 at what we estimate to be
the circumference of this wheel.
21 01:01:55:04 01:01:58:11 I've put a mark
on the floor here,
22 01:01:58:13 01:02:04:03 and I've marked the edge,
or the one point on the circle,
23 01:02:04:05 01:02:06:27 and then we're going
to roll it along.
24 01:02:06:29 01:02:09:05 CHAPIN:
Most adults do not estimate
25 01:02:09:07 01:02:13:16 the length of a circumference
of a circular object very often.
26 01:02:13:18 01:02:16:17 We started the session by asking
participants to estimate
27 01:02:16:19 01:02:20:18 the length of the circumference
of a bicycle wheel.
28 01:02:20:20 01:02:23:13 Where do you think,
along that line...
29 01:02:25:14 01:02:27:24 one rotation would take you?

30 01:02:27:26 01:02:28:24 All right.
31 01:02:28:26 01:02:29:24 Right here.
32 01:02:29:26 01:02:30:28 Right there,
okay.
33 01:02:31:00 01:02:33:01 Now I need
another volunteer.
34 01:02:33:03 01:02:34:10 Katy, come on up.
35 01:02:34:12 01:02:36:05 I think he's
a little short.
36 01:02:36:07 01:02:37:05 Okay.
37 01:02:38:10 01:02:40:19 So this is where
it's going to start.
38 01:02:47:12 01:02:48:10 Right there.
39 01:02:48:12 01:02:49:18 All right.
40 01:02:49:20 01:02:51:11 CHAPIN:
We laid the wheel on the ground,
41 01:02:51:13 01:02:54:12 and after a couple of
participants made estimates,
42 01:02:54:14 01:02:56:06 we then rolled it out.
43 01:02:56:08 01:02:59:06 Not surprisingly,
one of the participants
44 01:02:59:08 01:03:02:05 had way underestimated
the actual circumference,
45 01:03:02:07 01:03:04:00 and this is very typical.
46 01:03:04:02 01:03:07:25 We actually don't often use
relationships in a circle
47 01:03:07:27 01:03:10:16 to help us estimate
the circumference,
48 01:03:10:18 01:03:13:22 and, as a result,
do underestimate its length.
49 01:03:13:24 01:03:17:05 But now I'd like to think
about, how can we investigate
50 01:03:17:07 01:03:19:25 the relationship
between the circumference
51 01:03:19:27 01:03:22:17 and the diameter of a circle?
52 01:03:22:19 01:03:26:14 And one way is
that we can look at circles
53 01:03:26:16 01:03:29:22 that are inscribed
in other shapes.
54 01:03:29:24 01:03:32:19 If you check out your packet
on page two,
55 01:03:32:21 01:03:37:06 you will see a diagram
of three different circles
56 01:03:37:08 01:03:40:05 that are inscribed in a square,
57 01:03:40:07 01:03:44:15 and then inscribed
in the circle is a hexagon.
58 01:03:44:17 01:03:46:29 Now, we're going to take
a couple of measures,
59 01:03:47:01 01:03:50:27 and we're going to put it
up here on our poster
60 01:03:50:29 01:03:53:19 to look at
the relationships here
61 01:03:53:21 01:04:00:17 between circles, squares
and hexagons, okay?
62 01:04:00:19 01:04:03:26 So, let's take a look,
and let's work together

63 01:04:03:28 01:04:06:08 to put this information up here.
64 01:04:06:10 01:04:09:13 In design one, what's the diameter of the circle
65 01:04:09:15 01:04:11:20 in that design?
66 01:04:11:22 01:04:12:29 Two.
67 01:04:13:01 01:04:14:11 Two centimeters.
68 01:04:13:01 01:04:14:11 Centimeters.
69 01:04:14:13 01:04:15:23 CHAPIN: All right.
70 01:04:15:25 01:04:19:21 What's the perimeter of the hexagon in that shape?
71 01:04:19:23 01:04:21:15 Six centimeters.
72 01:04:23:11 01:04:24:17 Okay.
73 01:04:24:19 01:04:26:29 CHAPIN: We were looking at a circle
74 01:04:27:01 01:04:29:13 that was bounded by a square and a hexagon,
75 01:04:29:15 01:04:31:10 each with the same diameter.
76 01:04:31:12 01:04:34:19 Now, we could use that information to help us look
77 01:04:34:21 01:04:38:02 for relationships between the perimeter of the square,
78 01:04:38:04 01:04:42:08 the perimeter of the hexagon and the circumference of the circle.
79 01:04:42:10 01:04:45:17 And what we found was that the circumference of the circle
80 01:04:45:19 01:04:47:27 was bounded by these other two perimeters.
81 01:04:47:29 01:04:52:00 The perimeter of the square was four times its diameter,
82 01:04:52:02 01:04:55:25 and the circumference, or perimeter, of the hexagon,
83 01:04:55:27 01:04:58:00 was three times the diameter,
84 01:04:58:02 01:05:02:09 and thus, in between was the circumference of the circle.
85 01:05:02:11 01:05:04:18 Now, we're going to go on and look at another way
86 01:05:04:20 01:05:06:03 that we perhaps can get closer.
87 01:05:06:05 01:05:08:12 We now know that the circumference
88 01:05:08:14 01:05:10:08 is between three and four,
89 01:05:10:10 01:05:13:20 but we haven't narrowed it down really precisely.
90 01:05:13:22 01:05:15:27 So let's look at the next page,
91 01:05:15:29 01:05:21:19 where you have some diagrams of regular polygons.
92 01:05:21:21 01:05:27:16 You have a square, a pentagon, a regular hexagon, octagon.
93 01:05:27:18 01:05:32:15 Now, what we're going to do is we are going to look again here
94 01:05:32:17 01:05:36:28 at the relationship between the perimeter of the shape
95 01:05:37:00 01:05:40:25 and a diagonal, or a diameter, of a shape.
96 01:05:40:27 01:05:43:07 CHAPIN: Now that we'd narrowed down

97 01:05:43:09 01:05:46:08 that circumference-diameter relationship
98 01:05:46:10 01:05:49:28 was between three and four times that diameter,
99 01:05:50:00 01:05:54:06 we went and looked at regular polygons, and we started
100 01:05:54:08 01:05:57:17 with polygons with a very small number of sides
101 01:05:57:19 01:06:00:03 and went up to many-sided polygons,
102 01:06:00:05 01:06:01:26 in this case a dodecagon.
103 01:06:01:28 01:06:05:17 In each one, we measured the perimeter and formed a ratio
104 01:06:05:19 01:06:08:06 comparing it to both the diagonal length
105 01:06:08:08 01:06:10:09 and the diameter length.
106 01:06:10:11 01:06:13:24 Okay, so I'm going to do six divided by ten?
107 01:06:13:26 01:06:15:02 You get six-tenths.
108 01:06:15:04 01:06:16:26 Yeah, this is a little bit bigger.
109 01:06:16:28 01:06:18:02 It is.
110 01:06:18:04 01:06:19:09 If you look at the...
111 01:06:19:11 01:06:21:23 looking at the picture, going straight across,
112 01:06:21:25 01:06:24:15 instead of not quite going straight across...
113 01:06:24:17 01:06:25:15 Okay.
114 01:06:25:17 01:06:26:27 It doesn't look...
115 01:06:26:29 01:06:28:14 All right, I can see that.
116 01:06:28:16 01:06:31:04 I wonder if that's the case in all of them.
117 01:06:31:06 01:06:32:23 Probably, probably, yeah.
118 01:06:32:25 01:06:36:10 CHAPIN: What people found as they converted their ratios
119 01:06:36:12 01:06:40:09 was that their numbers were coming very close to pi,
120 01:06:40:11 01:06:44:12 and as the shape had more and more sides, we realized
121 01:06:44:14 01:06:48:04 that it was becoming even a closer approximation to pi,
122 01:06:48:06 01:06:50:17 and one way we can think about it is
123 01:06:50:19 01:06:55:01 that as we make the side lengths shorter and shorter and shorter,
124 01:06:55:03 01:06:57:26 the distance between each of the vertices
125 01:06:57:28 01:07:02:06 is becoming less and less, and the actual diameter or diagonal
126 01:07:02:08 01:07:06:04 are becoming closer and closer

to a diameter of a circle,

127 01:07:06:06 01:07:08:23 because it's approximating
a circle.

128 01:07:08:25 01:07:11:16 And thus,
our perimeter-diameter ratio

129 01:07:11:18 01:07:14:24 is going to come closer
and closer to pi.

130 01:07:14:26 01:07:18:13 So we notice that when we
have a perimeter of 3.5,

131 01:07:18:15 01:07:20:29 and we had measured
that diameter,

132 01:07:21:01 01:07:23:01 and we reduce that ratio,

133 01:07:23:03 01:07:26:27 we get three point...
approximately 3.29 to one.

134 01:07:26:29 01:07:28:01 Okay.

135 01:07:28:03 01:07:30:15 It's a little bit
more than pi.

136 01:07:30:17 01:07:32:06 And I had found four
before we had done it,

137 01:07:32:08 01:07:33:06 except measured
exactly.

138 01:07:33:08 01:07:34:19 Uh-huh.

139 01:07:34:21 01:07:37:12 Now, how about you when you
did the... the diagonal?

140 01:07:37:14 01:07:39:06 What kind of a ratio
did you get?

141 01:07:39:08 01:07:40:09 I came up with 2.9.

142 01:07:40:11 01:07:41:13 Oh, interesting.

143 01:07:41:15 01:07:43:12 So one ratio is
a little less than pi,

144 01:07:43:14 01:07:45:19 and another ratio is
a little more than pi.

145 01:07:45:21 01:07:47:15 What do you think
is going to happen

146 01:07:47:17 01:07:49:15 as we get, um, to a larger...

147 01:07:49:17 01:07:51:23 a shape with more
and more sides?

148 01:07:51:25 01:07:54:07 They'll probably
become a closer...

149 01:07:54:09 01:07:55:19 closer to each other.

150 01:07:55:21 01:07:56:25 Yeah, I think.

151 01:07:56:27 01:07:58:11 So let's go and
investigate that.

152 01:07:58:13 01:08:00:25 That sounds like a great
idea of what to do next.

153 01:08:02:29 01:08:06:08 Okay, let's talk
about some of the results

154 01:08:06:10 01:08:08:09 that we have just gotten.

155 01:08:08:11 01:08:11:12 What do people notice
about the ratios

156 01:08:11:14 01:08:15:07 that they were forming
of some of these figures?

157 01:08:15:09 01:08:17:18 Remember, that's a ratio
of the perimeter

158 01:08:17:20 01:08:21:23 to either the diameter
or the diagonal.

159 01:08:21:25 01:08:24:17 As the polygons that we were measuring had more sides--
160 01:08:24:19 01:08:27:13 as we went from a pentagon to a hexagon to an octagon--
161 01:08:27:15 01:08:31:14 the ratios got closer to the number of pi.
162 01:08:31:16 01:08:36:04 How is this helping us make sense of the relationship
163 01:08:36:06 01:08:41:15 in a circle, between perimeter and diameter?
164 01:08:41:17 01:08:46:03 We were looking at, also, the narrowing of the difference
165 01:08:46:05 01:08:49:10 between the perimeter and diameter ratio,
166 01:08:49:12 01:08:52:09 and the perimeter and diagonal ratio,
167 01:08:52:11 01:08:55:22 and we were saying that it would make sense
168 01:08:55:24 01:08:56:29 that that should narrow,
169 01:08:57:01 01:08:58:21 because when you get to a circle,
170 01:08:58:23 01:09:01:28 there's no difference between those two measures,
171 01:09:02:00 01:09:05:13 because there isn't a vertex versus a middle of a side.
172 01:09:05:15 01:09:08:18 And as you increase the number of sides on these polygons,
173 01:09:08:20 01:09:10:02 you're approaching a circle,
174 01:09:10:04 01:09:12:13 in which case those two would be diameters,
175 01:09:12:15 01:09:14:15 so they should be the same.
176 01:09:14:17 01:09:16:07 CHAPIN: Exactly.
177 01:09:16:09 01:09:19:00 Well, let's now actually investigate
178 01:09:19:02 01:09:22:04 the relationship in circles, all right?
179 01:09:22:06 01:09:23:17 So our next activity is
180 01:09:23:19 01:09:25:23 that we're going to do a little bit of measuring,
181 01:09:25:25 01:09:29:02 and then we are going to collect data and put it together.
182 01:09:29:04 01:09:32:14 CHAPIN: We then wanted to actually look at the relationship
183 01:09:32:16 01:09:35:29 in circular objects, so we measured many different objects
184 01:09:36:01 01:09:38:22 and formed ratios of circumference to diameter,
185 01:09:38:24 01:09:41:26 reduced those, and we found that most of these ratios
186 01:09:41:28 01:09:45:15 were very close to an approximate value of pi--

187 01:09:45:17 01:09:47:01 3.14, approximately, to one.
188 01:09:47:03 01:09:49:07 It also led us
to be able to talk about,
189 01:09:49:09 01:09:52:05 "Well, what do you do when you
collect a lot of data
190 01:09:52:07 01:09:54:07 and it's all not
exactly the same?"
191 01:09:54:09 01:09:57:18 We could find averages,
we could look at extremes,
192 01:09:57:20 01:09:59:16 and then find a median value.
193 01:09:59:18 01:10:02:08 But we can also then use that
to just show
194 01:10:02:10 01:10:05:09 that we are going to have some
inaccuracy in our measurement
195 01:10:05:11 01:10:07:04 when we do it
in a physical manner.
196 01:10:07:06 01:10:08:22 Just about 38.
197 01:10:08:24 01:10:11:12 38.7.
198 01:10:11:14 01:10:12:17 Okay.
199 01:10:12:19 01:10:15:08 And it was almost
38 before, so...
200 01:10:15:10 01:10:17:16 We're real close.
201 01:10:17:18 01:10:19:09 So if we take...
202 01:10:19:11 01:10:21:09 Um, actually 37.7,
203 01:10:21:11 01:10:24:09 because it
wasn't quite 38,
204 01:10:24:11 01:10:29:07 so if we take 37.7,
divided by our diameter,
205 01:10:29:09 01:10:32:08 which was 12...
206 01:10:32:10 01:10:34:09 we get three point...
207 01:10:34:11 01:10:35:09 one, four.
208 01:10:35:11 01:10:36:09 3.14.
209 01:10:36:11 01:10:37:29 That makes sense.
210 01:10:38:01 01:10:41:05 CHAPIN:
One of the very interesting
things about circles
211 01:10:41:07 01:10:44:19 is that the relationship between
circumference and diameter,
212 01:10:44:21 01:10:47:28 no matter what size the circle,
is always going to be pi.
213 01:10:48:00 01:10:49:16 It's an irrational number.
214 01:10:49:18 01:10:52:24 So circumference divided
by diameter equals pi.
215 01:10:52:26 01:10:55:14 And this has fascinated
mathematicians
216 01:10:55:16 01:10:59:08 for thousands of years,
because it is a constant ratio.
217 01:10:59:10 01:11:01:09 In fact, that's one reason
218 01:11:01:11 01:11:05:03 the number pi actually has a
special name-- because of its...
219 01:11:05:05 01:11:08:02 the constant nature in terms
of the relationship
220 01:11:08:04 01:11:10:08 between circumference
and diameter.
221 01:11:14:14 01:11:17:05 Well, our next activity
is going to involve

222 01:11:17:07 01:11:20:03 us trying to make sense
of this area formula.
223 01:11:20:05 01:11:21:29 Where did it come from?
224 01:11:22:01 01:11:25:19 Why is this the formula
for area of a circle?
225 01:11:25:21 01:11:28:21 And what you're going
to work with is,
226 01:11:28:23 01:11:32:02 you each have circular disks
on your table
227 01:11:32:04 01:11:35:10 that you are going to cut
into sections
228 01:11:35:12 01:11:41:16 and re-create your sectors
into a parallelogram.
229 01:11:41:18 01:11:46:00 So we're going to see if we
can use another area formula
230 01:11:46:02 01:11:50:06 to help us make sense
of the formula for a circle.
231 01:11:50:08 01:11:52:00 So we should end up
with eight pieces?
232 01:11:52:02 01:11:53:23 And we're going
to put them together
233 01:11:53:25 01:11:54:26 to make
a parallelogram?
234 01:11:54:28 01:11:55:27 Right.
235 01:12:02:20 01:12:06:21 Okay, so, we're going
to do... this way.
236 01:12:06:23 01:12:07:28 (*class talking*)
237 01:12:08:00 01:12:11:02 So that they're
lined up like that, yeah.
238 01:12:11:04 01:12:12:24 Okay.
239 01:12:12:26 01:12:14:17 CHAPIN:
In area of a circle,
240 01:12:14:19 01:12:18:08 we were really trying to make
sense of the area formulas--
241 01:12:18:10 01:12:20:11 to justify them, to explain them
242 01:12:20:13 01:12:23:27 in terms that would make sense
to the ordinary lay person.
243 01:12:23:29 01:12:28:02 We took a circle and divided it
up into very, very small wedges.
244 01:12:28:04 01:12:29:25 We then cut the wedges out
245 01:12:29:27 01:12:32:05 and rearranged them
to form something
246 01:12:32:07 01:12:35:06 that looked approximately
like a parallelogram.
247 01:12:35:08 01:12:37:08 We then could
actually figure out
248 01:12:37:10 01:12:40:24 what was the base and the height
of that parallelogram
249 01:12:40:26 01:12:42:11 and determine the area.
250 01:12:42:13 01:12:46:24 And we linked what we found
to the area-of-a-circle formula.
251 01:12:46:26 01:12:48:13 Is that on
that point?
252 01:12:48:15 01:12:49:14 Yeah.
253 01:12:51:17 01:12:56:13 It's 29.3.
254 01:12:56:15 01:12:57:21 Okay?

255 01:12:57:23 01:13:00:15 So how does the area
of the figure compare
256 01:13:00:17 01:13:02:12 with the area of the circle?
257 01:13:05:05 01:13:07:06 Well, we can use
the formula to start,
258 01:13:07:08 01:13:09:24 to figure out what
the area of the circle was
259 01:13:09:26 01:13:10:24 and then...
260 01:13:10:26 01:13:11:24 Should we try that?
261 01:13:11:26 01:13:12:24 Yeah, let's do that.
262 01:13:12:26 01:13:13:29 Okay.
263 01:13:14:01 01:13:16:24 So pi... or ra...
264 01:13:16:26 01:13:19:20 Area equals pi radius-squared.
265 01:13:19:22 01:13:21:03 Right.
266 01:13:21:05 01:13:23:23 Okay, and the radius was 9.5.
267 01:13:25:00 01:13:30:22 "9.5 times 9.5
equals 90.25"
268 01:13:30:24 01:13:34:01 is our radius-squared
times pi.
269 01:13:34:03 01:13:40:18 (*class discussing*)
270 01:13:40:20 01:13:44:16 283.53...
271 01:13:44:18 01:13:45:23 Point five three,
yeah.
272 01:13:45:25 01:13:47:22 We can take it to
the hundredths place.
273 01:13:47:24 01:13:50:12 All right, so that's the area
according to the formula--
274 01:13:50:14 01:13:51:14 the circle.
275 01:13:51:16 01:13:53:07 Okay.
276 01:13:54:16 01:14:00:21 Then, to find the area
of the parallelogram...
277 01:14:00:23 01:14:02:18 I think it's
base times height.
278 01:14:02:20 01:14:10:20 Okay, so 9.5 times 29...
times 29.3.
279 01:14:10:22 01:14:12:06 Okay.
280 01:14:12:08 01:14:17:12 Area... I got 278.35.
281 01:14:17:14 01:14:18:23 Yeah.
282 01:14:18:25 01:14:22:02 When we did the area of
the circle with the formula,
283 01:14:22:04 01:14:25:15 we came up with about
five centimeters more.
284 01:14:27:22 01:14:30:02 So how does the area
of the figure compare
285 01:14:30:04 01:14:31:18 with the area
of the circle?
286 01:14:32:27 01:14:36:00 It's point... or, it's
five centimeters less?
287 01:14:36:02 01:14:37:04 Less-- right.
288 01:14:37:06 01:14:38:05 Okay.
289 01:14:38:07 01:14:39:24 Can everybody look up here?
290 01:14:39:26 01:14:43:05 What I'd like us now to do
is talk a little bit
291 01:14:43:07 01:14:48:06 about how we can make sense of
this area of a circle formula,
292 01:14:48:08 01:14:49:22 pi r-squared.

293 01:14:49:24 01:14:51:08 Susan, do you mind coming up
294 01:14:51:10 01:14:54:00 and showing everyone what you and I were talking about?
295 01:14:54:02 01:14:56:21 NARRATOR: On this chart is a drawing of circle wedges
296 01:14:56:23 01:15:00:22 arranged to form what looks approximately like a rectangle.
297 01:15:00:24 01:15:01:29 With this in mind,
298 01:15:02:01 01:15:04:06 the participant gives her explanation.
299 01:15:04:08 01:15:05:24 Okay, if we know
300 01:15:05:26 01:15:12:03 that the formula for the area of a circle equals pi r-squared
301 01:15:12:05 01:15:14:25 and we examine our figure here...
302 01:15:14:27 01:15:16:08 If we notice
303 01:15:16:10 01:15:19:23 that all of these little wedges together form the base
304 01:15:19:25 01:15:22:02 and that there's two set of wedges,
305 01:15:22:04 01:15:24:09 we can think of the base
306 01:15:24:11 01:15:29:11 as being equal to half the circumference of the circle.
307 01:15:29:13 01:15:34:01 So the base equals one-half the circumference.
308 01:15:34:03 01:15:35:05 Okay?
309 01:15:35:07 01:15:37:29 Or we can also think of that
310 01:15:38:01 01:15:42:03 as being the circumference divided by two.
311 01:15:42:05 01:15:46:06 Circumference is equal to two pi r, or pi D.
312 01:15:46:08 01:15:49:21 So I'm going to say,
313 01:15:49:23 01:15:55:01 if I think of the circumference equaling pi D,
314 01:15:55:03 01:15:59:19 or two pi r...
315 01:15:59:21 01:16:03:15 now I have another way of thinking of the base.
316 01:16:03:17 01:16:05:10 So I'm going to say
317 01:16:05:12 01:16:11:11 that the base equals...
318 01:16:11:13 01:16:15:19 two pi r divided by two,
319 01:16:15:21 01:16:18:12 because I had thought of my circumference
320 01:16:18:14 01:16:20:02 as being divided by two.
321 01:16:20:04 01:16:25:07 And if I think about canceling out these two 2s,
322 01:16:25:09 01:16:29:19 I can say that the base is equal to pi r.
323 01:16:29:21 01:16:33:12 So I have my dimension for this.
324 01:16:33:14 01:16:35:04 Now I need to come back
325 01:16:35:06 01:16:39:21 and I need to think about this measurement to find the area.
326 01:16:39:23 01:16:43:08 And if I look at these wedges, I know that it's actually...
327 01:16:43:10 01:16:46:26 this measurement is actually just the radius of my circle.

328 01:16:46:28 01:16:51:18 And if we think of area as equaling length times width,
329 01:16:51:20 01:16:56:06 I can multiply my radius times my pi r.
330 01:16:56:08 01:17:02:19 Consequently, I go back to my formula of pi r-squared.
331 01:17:02:21 01:17:05:15 NARRATOR: When wedges of a circle are rearranged
332 01:17:05:17 01:17:08:10 to approximate a parallelogram like this one,
333 01:17:08:12 01:17:10:24 it helps make sense of the formula
334 01:17:10:26 01:17:12:21 for the area of a circle.
335 01:17:12:23 01:17:14:28 Now we're also going to continue to think
336 01:17:15:00 01:17:18:01 about the formula for area of a circle in one other way.
337 01:17:18:03 01:17:22:18 And that is, sometimes when you say "r-squared"
338 01:17:22:20 01:17:24:02 we think of the radius,
339 01:17:24:04 01:17:31:22 but we could also think of a square
340 01:17:31:24 01:17:35:11 in which each side is length r,
341 01:17:35:13 01:17:42:26 and thus the area of that little square there is r-squared.
342 01:17:42:28 01:17:45:15 Now, you have some sheets
343 01:17:45:17 01:17:47:28 that have some different circles on them
344 01:17:48:00 01:17:52:25 and on each one there is a blackened square
345 01:17:52:27 01:17:57:27 which represents r-squared for that circle--
346 01:17:57:29 01:18:02:06 namely, it is a square with sides of length r,
347 01:18:02:08 01:18:05:15 and its area is r-squared.
348 01:18:05:17 01:18:07:26 And what we want to do is
349 01:18:07:28 01:18:13:26 see how many of these r-squared squares will fit in one circle.
350 01:18:13:28 01:18:14:26 Like this...
351 01:18:14:28 01:18:16:09 We can just figure out this piece
352 01:18:16:11 01:18:17:25 and figure out how much of it it is
353 01:18:17:27 01:18:19:09 and we can multiply it by four.
354 01:18:19:11 01:18:21:18 Right. I'm thinking that this complements this somehow;
355 01:18:21:20 01:18:25:07 that once we know this,
356 01:18:25:09 01:18:26:12 we would know the reverse.
357 01:18:26:14 01:18:27:20 Mm-hmm.
358 01:18:27:22 01:18:28:29 Does that make...
359 01:18:29:01 01:18:31:13 And then we could figure out what we've taken away?

360 01:18:31:15 01:18:34:03 Maybe.
361 01:18:34:05 01:18:35:20 Because it's
symmetric, too.
362 01:18:35:22 01:18:38:18 Like, if we
could figure out
how much this piece is,
363 01:18:38:20 01:18:40:23 then that piece
and that piece are
kind of obvious
364 01:18:40:25 01:18:42:04 because they're
whole squares.
365 01:18:42:06 01:18:43:15 Squares, right.
366 01:18:43:17 01:18:45:26 And then this one
and this one are
exactly the same.
367 01:18:45:28 01:18:47:06 And if you look...
well, yeah.
368 01:18:49:05 01:18:53:07 What I'm wondering is
if two of these
complete this.
369 01:18:55:11 01:18:56:14 Well, we can...
370 01:18:56:16 01:18:58:08 Maybe that's what we
can cut out and try.
371 01:18:58:10 01:18:59:16 Try.
372 01:18:59:18 01:19:01:08 CHAPIN:
In the next activity,
373 01:19:01:10 01:19:04:13 what if we take squares
that are r-squared size,
374 01:19:04:15 01:19:05:14 and cover a circle?
375 01:19:05:16 01:19:07:04 How many of them will it take?
376 01:19:07:06 01:19:10:25 Participants took squares,
they cut them out,
377 01:19:10:27 01:19:14:01 they tried to fit them
onto the circles
378 01:19:14:03 01:19:17:28 and they found that three
and a little bit more
379 01:19:18:00 01:19:21:19 of those r-squared squares fit
on the circle.
380 01:19:21:21 01:19:24:13 Now, if we relate that
to the formula,
381 01:19:24:15 01:19:28:16 we know the formula for area is
area equals pi r-squared,
382 01:19:28:18 01:19:32:07 so the pi is approximately
a little more than three--
383 01:19:32:09 01:19:35:13 3.1 or so-- and we found
that that was exactly
384 01:19:35:15 01:19:39:04 how many of those r-squares
squared we needed
385 01:19:39:06 01:19:40:19 to cover the surface.
386 01:19:40:21 01:19:42:04 CHAPIN:
Did anybody have any luck
387 01:19:42:06 01:19:46:01 doing that, or, uh,
skilled precision
388 01:19:46:03 01:19:51:19 in terms of about how many
of these r-square squares
389 01:19:51:21 01:19:58:12 were they able to fit on, um...

cover the area of their circle?

390 01:19:58:14 01:20:00:12 I used the negative space

391 01:20:00:14 01:20:03:11 to find out how much less
than four squares I used.

392 01:20:03:13 01:20:04:14 Ah, so you...

393 01:20:04:16 01:20:05:25 Okay.

394 01:20:05:27 01:20:07:29 So I cut off
the little bits...

395 01:20:08:01 01:20:09:27 Little curves around, yeah.

396 01:20:09:29 01:20:12:22 And found out how many square
units were in those little bits

397 01:20:12:24 01:20:13:25 and then subtracted it.

398 01:20:13:27 01:20:16:02 Ah, and so then
you also got...

399 01:20:16:04 01:20:19:04 I got that it was
three and one-ninth.

400 01:20:19:06 01:20:21:02 Okay, so three
and a little bit more.

401 01:20:21:04 01:20:24:26 Does this three and one-ninth,
three and a little bit more,

402 01:20:24:28 01:20:28:18 remind us of any number that
we've been talking about today?

403 01:20:28:20 01:20:29:26 (*laughter*)

404 01:20:29:28 01:20:33:19 Pi. And so again,
what's happening here is

405 01:20:33:21 01:20:37:06 it's about pi number
of these r-squares

406 01:20:37:08 01:20:40:09 that we are able to place
in here--

407 01:20:40:11 01:20:42:26 pi being a little more
than three--

408 01:20:42:28 01:20:47:17 and thus that's one reason
why in our approximate attempts,

409 01:20:47:19 01:20:49:26 that's about what we're coming
up with.

410 01:20:49:28 01:20:53:03 In our session today,
we've investigated circles;

411 01:20:53:05 01:20:56:09 in particular we've tried
to make sense of the formulas

412 01:20:56:11 01:20:58:15 for circumference
and area of circles.

413 01:20:58:17 01:21:02:05 We use these geometric shapes
all the time

414 01:21:02:07 01:21:04:21 and it's important for us
to have actually...

415 01:21:04:23 01:21:05:29 to be able to justify:

416 01:21:06:01 01:21:07:27 Where do these formulas
come from?

417 01:21:07:29 01:21:09:18 How do we make sense of them?

418 01:21:09:20 01:21:11:16 Which is what we've focused on
today.

419 01:21:24:29 01:21:28:17 WOMAN:
The circle is a very important
shape in Islam.

420 01:21:28:19 01:21:30:28 It represents, visually,
a metaphor

421 01:21:31:00 01:21:33:19 for the religious philosophy of Islam,
422 01:21:33:21 01:21:37:04 as well as, physically, it informs the art
423 01:21:37:06 01:21:39:17 and architecture of Islam.
424 01:21:39:19 01:21:44:16 The circle starts with a center point that determines its shape.
425 01:21:44:18 01:21:48:19 This center point can also be seen as God, or Allah.
426 01:21:48:21 01:21:52:22 God radiates his message through the Koran,
427 01:21:52:24 01:21:54:27 as you can see here in the radius.
428 01:21:54:29 01:21:59:00 The Koran is then given to Prophet Muhammad
429 01:21:59:02 01:22:05:16 to disseminate, or to spread, the knowledge of God's word.
430 01:22:05:18 01:22:09:04 By doing so, Prophet Muhammad creates a community,
431 01:22:09:06 01:22:12:15 or a circumference, of the circle, or a circle.
432 01:22:12:17 01:22:16:17 And that circumference represents the Ummah in Islam.
433 01:22:16:19 01:22:20:14 The center is further empowered by the five daily prayers
434 01:22:20:16 01:22:26:22 that Muslims must perform facing Mecca, or the Kaba in Mecca.
435 01:22:26:24 01:22:31:14 The circle also represents, visually, the dome of heaven.
436 01:22:31:16 01:22:36:19 Here in this plan of a mosque, we can see the dome of heaven
437 01:22:36:21 01:22:40:07 as represented by a full and complete circle.
438 01:22:40:09 01:22:44:05 All points on the circumference of a circle are equidistant
439 01:22:44:07 01:22:45:13 from the center.
440 01:22:45:15 01:22:48:17 To determine how large the dome will be,
441 01:22:48:19 01:22:53:03 the architect must first figure out the area of the circle.
442 01:22:53:05 01:22:56:28 And the area is derived by "area equals pi r-squared,"
443 01:22:57:00 01:22:59:03 and that determines for him
444 01:22:59:05 01:23:03:11 the space that he needs to allow for the dome to fit.
445 01:23:03:13 01:23:08:17 In this plan we see a series of circles and semicircles.
446 01:23:08:19 01:23:11:19 The semicircles are used structurally
447 01:23:11:21 01:23:13:23 to support the main dome.
448 01:23:13:25 01:23:17:26 The circle again becomes the point of departure,
449 01:23:17:28 01:23:22:05 or the beginning, of a decorative pattern in Islam.
450 01:23:22:07 01:23:25:04 This is a 12-pointed star.
451 01:23:25:06 01:23:28:15 The circle is divided equally

452 01:23:28:17 01:23:32:21 using the diagonals,
or the diameters, of the circle,
453 01:23:32:23 01:23:36:19 which then measure the points,
or place the points
454 01:23:36:21 01:23:39:08 where the squares are
superimposed
455 01:23:39:10 01:23:41:25 to reveal stars, or star shapes.
456 01:23:41:27 01:23:46:06 The artisan then blacks out
some points of the star
457 01:23:46:08 01:23:48:29 and leaves others open,
or white,
458 01:23:49:01 01:23:52:04 to create a wonderfully rich
pattern.
459 01:23:52:06 01:23:54:19 The students in my class
also practice
460 01:23:54:21 01:23:58:08 this age-old Islamic tradition
of pattern-making
461 01:23:58:10 01:24:00:24 by first starting
with the circle
462 01:24:00:26 01:24:03:11 in which they superimpose
squares
463 01:24:03:13 01:24:07:06 by dividing the diameters
of the circle.
464 01:24:07:08 01:24:09:18 This then generated
a star pattern
465 01:24:09:20 01:24:12:23 which then evolved into a more
organic shape
466 01:24:12:25 01:24:14:18 of leaves and petals.
467 01:24:14:20 01:24:17:03 Once the motif was perfected,
468 01:24:17:05 01:24:21:08 it was then used to fabricate
this Islamic panel.
469 01:24:21:10 01:24:26:04 The corners of the panels remain
constant to the original design,
470 01:24:26:06 01:24:30:12 but the connectors, or the vines
that connect the corners,
471 01:24:30:14 01:24:33:08 are derivatives
of the original motif.
472 01:24:38:29 01:24:41:24 The classical arch is
the Roman arch,
473 01:24:41:26 01:24:46:05 and that's constructed
through the use of semicircles.
474 01:24:46:07 01:24:49:08 But the Islamic arch is pointed
at the apex
475 01:24:49:10 01:24:52:11 and doesn't have a complete
and rounded top
476 01:24:52:13 01:24:53:26 like the Roman arch.
477 01:24:53:28 01:24:57:04 To arrive at the Islamic arch,
478 01:24:57:06 01:25:01:11 we intersected
two perfect semicircles;
479 01:25:01:13 01:25:03:27 their point of intersection
became
480 01:25:03:29 01:25:06:24 the center point
of the Islamic arch.
481 01:25:09:19 01:25:14:07 This allowed for a perfect
Islamic pointed arch
482 01:25:14:09 01:25:17:11 to be perceived in the design.

483 01:25:18:28 01:25:21:28 All the points of these arches essentially go back

484 01:25:22:00 01:25:24:01 to the source, the center.

485 01:25:24:03 01:25:27:19 The success of this project allowed this panel to be chosen

486 01:25:27:21 01:25:29:01 for the new mosque

487 01:25:29:03 01:25:32:00 that will be completed in Boston in the near future.

488 01:25:33:22 01:25:37:10 Captioned by
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