| 1 | 01:00:47:10 | 01:00:48:21 | WOMAN: Measurement is the process |
| 2 | 01:00:48:23 | 01:00:51:23 | of quantifying properties of objects. |
| 3 | 01:00:51:25 | 01:00:54:19 | And to do that, we have set procedures |
| 4 | 01:00:54:21 | 01:00:56:21 | that enable us to measure. |
| 5 | 01:00:56:23 | 01:00:58:13 | Oh. |
| 6 | 01:00:58:15 | 01:01:00:26 | Measuring helps you to understand |
| 7 | 01:01:00:28 | 01:01:03:09 | how things relate to each other. |
| 8 | 01:01:03:11 | 01:01:07:07 | Our volume of a sphere actually has a formula |
| 9 | 01:01:07:09 | 01:01:09:17 | of 4/3 pi r-cubed. |
| 10 | 01:01:09:19 | 01:01:13:04 | This course really made me think about how I approach measurement |
| 11 | 01:01:13:06 | 01:01:16:06 | and how I can use measurement every day in the classroom. |
| 12 | 01:01:21:04 | 01:01:24:21 | In this session, we are going to be exploring area. |
| 13 | 01:01:24:23 | 01:01:26:15 | Now, area is a covering, |
| 14 | 01:01:26:17 | 01:01:29:04 | and we usually cover area |
| 15 | 01:01:29:06 | 01:01:32:18 | with shapes that will leave no holes and gaps. |
| 16 | 01:01:32:20 | 01:01:36:02 | You mainly think about squares as being used for covering, |
| 17 | 01:01:36:04 | 01:01:37:26 | but we could use rectangles. |
| 18 | 01:01:37:28 | 01:01:43:05 | Sometimes people think that you can only take the area, |
| 19 | 01:01:43:07 | 01:01:46:11 | or find the area, of regular shapes, |
| 20 | 01:01:46:13 | 01:01:48:27 | like of rectangles or or squares or triangles, |
| 21 | 01:01:48:29 | 01:01:50:21 | but that's not the case at all. |
| 22 | 01:01:50:23 | 01:01:55:08 | We can find the area of irregular shapes as well. |
| 23 | 01:01:55:10 | 01:01:56:24 | This afternoon |
| 24 | 01:01:56:26 | 01:01:59:05 | we are actually going to be exploring area on a geoboard, |
| 25 | 01:01:59:07 | 01:02:00:29 | for it will enable us to make sense |
| 26 | 01:02:01:01 | 01:02:03:11 | of some of the methods and concepts |
| 27 | 01:02:03:13 | 01:02:06:01 | that are involved in working with area. |
| 28 | 01:02:06:03 | 01:02:11:16 | A geoboard is a device that has pins or nails on it |
| 29 | 01:02:11:18 | 01:02:16:14 | that allow us to divide the board up into square units. |
| 30 | 01:02:16:16 | 01:02:21:08 | Notice right here is going to be our unit of measure, |
| 31 | 01:02:21:10 | 01:02:24:02 | and that's going to be one square unit. |
| 32 | 01:02:24:04 | 01:02:26:24 | We're not going to worry about actually how long that is; |
| 33 | 01:02:26:26 | 01:02:30:11 | we're just going to call it, |

|    |             |             | as I said, a square unit. |
|----|-------------|-------------|---------------------------|
| 34 | 01:02:30:13 | 01:02:33:10 | You can see I have a larger square here, |
| 35 | 01:02:33:12 | 01:02:37:27 | and that consists of four square units, |
| 36 | 01:02:37:29 | 01:02:43:02 | where we can... count them |
| 37 | 01:02:43:04 | 01:02:45:29 | and see that it is an area of four square units. |
| 38 | 01:02:46:01 | 01:02:47:08 | Okay? |
| 39 | 01:02:47:10 | 01:02:50:09 | Now, one of the things that we are going to do |
| 40 | 01:02:50:11 | 01:02:56:02 | is construct a shape and determine its area |
| 41 | 01:02:56:04 | 01:03:00:04 | using both square units and half of square units. |
| 42 | 01:03:00:06 | 01:03:02:03 | Feel free to take any of the shapes, |
| 43 | 01:03:02:05 | 01:03:03:14 | build it on your geoboard, |
| 44 | 01:03:03:16 | 01:03:08:08 | and then we're going to find the area of your shapes. |
| 45 | 01:03:08:10 | 01:03:10:02 | So, again... |
| 46 | 01:03:10:04 | 01:03:11:15 | Are you doing the big square one? |
| 47 | 01:03:11:17 | 01:03:12:16 | Yeah, the big square one. |
| 48 | 01:03:17:20 | 01:03:19:29 | CHAPIN: The very first time we started using geoboards, |
| 49 | 01:03:20:01 | 01:03:22:01 | we were becoming comfortable with the idea |
| 50 | 01:03:22:03 | 01:03:25:07 | that we measure area using square units, |
| 51 | 01:03:25:09 | 01:03:27:28 | and that we can easily designate what are square units |
| 52 | 01:03:28:00 | 01:03:30:09 | and what are half units on the geoboard-- |
| 53 | 01:03:30:11 | 01:03:33:03 | by combining half units, we can make full units. |
| 54 | 01:03:33:05 | 01:03:36:15 | So shapes that have full units and half units, |
| 55 | 01:03:36:17 | 01:03:39:08 | it's pretty easy to determine their area. |
| 56 | 01:03:39:10 | 01:03:40:11 | And then I was looking |
| 57 | 01:03:40:13 | 01:03:41:27 | at how many triangles I had-- |
| 58 | 01:03:41:29 | 01:03:43:20 | one, two, three, four. |
| 59 | 01:03:43:22 | 01:03:46:15 | And I decided that... |
| 60 | 01:03:46:17 | 01:03:50:18 | Like, you can see it here-- these two... |
| 61 | 01:03:50:20 | 01:03:52:08 | if you moved this one here, |
| 62 | 01:03:52:10 | 01:03:53:13 | would make one square. |

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| 63 | 01:03:53:15 | 01:03:54:25 | So there's one square, |
| 64 | 01:03:54:27 | 01:03:56:26 | and then these two make another square. |
| 65 | 01:03:56:28 | 01:04:00:08 | So we had six, seven, eight, nine, ten. |
| 66 | 01:04:00:10 | 01:04:02:07 | CHAPIN: So this shape has an area |
| 67 | 01:04:02:09 | 01:04:03:24 | of ten square units, |
| 68 | 01:04:03:26 | 01:04:05:22 | and you can see how those units |
| 69 | 01:04:05:24 | 01:04:07:11 | cover that complete shape |
| 70 | 01:04:07:13 | 01:04:09:18 | without any holes or gaps. |
| 71 | 01:04:09:20 | 01:04:11:18 | CHAPIN: Now, as you noticed, |
| 72 | 01:04:11:20 | 01:04:13:23 | when we can do half units... |
| 73 | 01:04:13:25 | 01:04:15:26 | you can find the area in full units |
| 74 | 01:04:15:28 | 01:04:18:03 | in a pretty straightforward manner. |
| 75 | 01:04:18:05 | 01:04:21:15 | Now, we can also find areas of shapes |
| 76 | 01:04:21:17 | 01:04:24:19 | that cannot be partitioned so easily |
| 77 | 01:04:24:21 | 01:04:27:20 | into halves and whole square units. |
| 78 | 01:04:27:22 | 01:04:29:02 | So we're next going to move |
| 79 | 01:04:29:04 | 01:04:31:08 | to looking at how do we go about doing that. |
| 80 | 01:04:31:10 | 01:04:33:25 | Now, that one thing I'm going to ask you to do |
| 81 | 01:04:33:27 | 01:04:36:28 | is as we move forward to finding the area of new shapes, |
| 82 | 01:04:37:00 | 01:04:40:26 | some of you have knowledge of relationships |
| 83 | 01:04:40:28 | 01:04:44:27 | in terms of how to find areas of different shapes using formulas. |
| 84 | 01:04:44:29 | 01:04:47:03 | Please do not use them. |
| 85 | 01:04:47:05 | 01:04:51:27 | Please try to find the area using more informal methods, |
| 86 | 01:04:51:29 | 01:04:53:06 | rather than formal methods. |
| 87 | 01:04:53:08 | 01:04:55:11 | We're going to move to the formal methods, |
| 88 | 01:04:55:13 | 01:04:56:26 | but right now we want to see |
| 89 | 01:04:56:28 | 01:04:58:28 | if we can make sense of this informally. |
| 90 | 01:04:59:00 | 01:05:00:17 | Now, let's take a look at... |
| 91 | 01:05:00:19 | 01:05:07:04 | If I put that right triangle on the geoboard, |
| 92 | 01:05:07:06 | 01:05:11:29 | notice that right from the beginning... |
| 93 | 01:05:12:01 | 01:05:16:13 | that doesn't cover a full square unit. |
| 94 | 01:05:16:15 | 01:05:20:26 | And then even this one here... |

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| 95 | 01:05:20:28 | 01:05:24:15 | that doesn't actually look like a half a square unit. |
| 96 | 01:05:24:17 | 01:05:26:22 | So I have to come up with a different method |
| 97 | 01:05:26:24 | 01:05:31:10 | of being able to find the area of this shape. |
| 98 | 01:05:31:12 | 01:05:34:17 | Anyone have a suggestion of what we could do? |
| 99 | 01:05:34:19 | 01:05:36:10 | Laura, you want to come on up? |
| 100 | 01:05:39:12 | 01:05:43:03 | Because the geoboard works so well |
| 101 | 01:05:43:05 | 01:05:45:22 | with squares and rectangles, |
| 102 | 01:05:45:24 | 01:05:50:25 | I would use my rubber band |
| 103 | 01:05:50:27 | 01:05:56:02 | and make one large rectangle. |
| 104 | 01:05:56:04 | 01:05:59:20 | And I can figure out the area of this easily enough-- |
| 105 | 01:05:59:22 | 01:06:03:10 | I can see it's four square units. |
| 106 | 01:06:03:12 | 01:06:08:00 | But this original triangle makes a diagonal |
| 107 | 01:06:08:02 | 01:06:11:25 | and I can almost flip this triangle over... |
| 108 | 01:06:11:27 | 01:06:13:12 | flip it over. |
| 109 | 01:06:13:14 | 01:06:18:00 | This diagonal creates two equal triangles, |
| 110 | 01:06:18:02 | 01:06:22:15 | so I have two halves of this rectangle. |
| 111 | 01:06:22:17 | 01:06:24:29 | So if I know the area of the whole rectangle, |
| 112 | 01:06:25:01 | 01:06:28:18 | I can say the area of one triangle's half the rectangle. |
| 113 | 01:06:28:20 | 01:06:33:16 | So if the rectangle is four square units, |
| 114 | 01:06:33:18 | 01:06:36:19 | the triangle must be two square units. |
| 115 | 01:06:36:21 | 01:06:39:13 | CHAPIN: We're going to call this the rectangle method, |
| 116 | 01:06:39:15 | 01:06:40:25 | or "Laura's method"... |
| 117 | 01:06:40:27 | 01:06:42:00 | ( class chuckling ) |
| 118 | 01:06:42:02 | 01:06:44:04 | in her honor for coming on up here. |
| 119 | 01:06:45:07 | 01:06:47:09 | CHAPIN: In a rectangle method, |
| 120 | 01:06:47:11 | 01:06:51:20 | we take right triangles and we can make them into rectangles, |
| 121 | 01:06:51:22 | 01:06:53:13 | of which the right triangle |
| 122 | 01:06:53:15 | 01:06:56:05 | is exactly half the area of the rectangle. |
| 123 | 01:06:56:07 | 01:06:59:23 | Rectangles are pretty easy to find the area of. |

| 124 | 01:06:59:25 | 01:07:01:22 | We just either count square units, |
| 125 | 01:07:01:24 | 01:07:04:09 | or we can multiply length times width. |
| 126 | 01:07:04:11 | 01:07:06:24 | When we wanted to find the area of, then, these triangles, |
| 127 | 01:07:06:26 | 01:07:09:18 | all we have to do is take the area of the rectangle |
| 128 | 01:07:09:20 | 01:07:11:27 | and divide it by two. |
| 129 | 01:07:11:29 | 01:07:14:14 | I took a look at these two also. |
| 130 | 01:07:14:16 | 01:07:15:29 | CHAPIN: Okay, those two rectangles. |
| 131 | 01:07:16:01 | 01:07:17:11 | These equal two. |
| 132 | 01:07:17:13 | 01:07:19:04 | Two square units. |
| 133 | 01:07:19:06 | 01:07:20:26 | Right, and so this line cuts it in half, |
| 134 | 01:07:20:28 | 01:07:22:26 | so this space is equal to one. |
| 135 | 01:07:22:28 | 01:07:23:26 | One... |
| 136 | 01:07:23:28 | 01:07:25:04 | Square unit. |
| 137 | 01:07:23:28 | 01:07:25:04 | Square unit. |
| 138 | 01:07:25:06 | 01:07:27:07 | And this space is also equal to one square unit. |
| 139 | 01:07:27:09 | 01:07:28:07 | Okay. |
| 140 | 01:07:28:09 | 01:07:30:07 | So altogether we have four... |
| 141 | 01:07:30:09 | 01:07:32:14 | Two, four, five, six, |
| 142 | 01:07:32:16 | 01:07:36:18 | to take away from our whole area, which is 12. |
| 143 | 01:07:36:20 | 01:07:39:02 | So that means the inside must be equal to six. |
| 144 | 01:07:39:04 | 01:07:40:10 | Six what, though? |
| 145 | 01:07:39:04 | 01:07:40:10 | Square units. |
| 146 | 01:07:40:12 | 01:07:42:06 | Six square units. |
| 147 | 01:07:42:08 | 01:07:44:13 | CHAPIN: The last activity that we did with geoboards |
| 148 | 01:07:44:15 | 01:07:47:08 | was where we were looking at a technique to be used |
| 149 | 01:07:47:10 | 01:07:48:27 | when you cannot take a shape |
| 150 | 01:07:48:29 | 01:07:51:21 | and easily divide it into subshapes. |
| 151 | 01:07:51:23 | 01:07:54:26 | What we wanted to do instead was surround the shape |
| 152 | 01:07:54:28 | 01:07:57:02 | and subtract out the difference |
| 153 | 01:07:57:04 | 01:07:59:08 | of finding area that we could calculate |
| 154 | 01:07:59:10 | 01:08:02:16 | and then subtracting that from the area of the rectangle |
| 155 | 01:08:02:18 | 01:08:04:07 | to find the area of the triangle |
| 156 | 01:08:04:09 | 01:08:06:08 | that was embedded in that rectangle. |
| 157 | 01:08:06:10 | 01:08:08:10 | So this is... |
| 158 | 01:08:06:10 | 01:08:08:10 | Two. |

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| 159 | 01:08:08:12 | 01:08:09:27 | This is two. |
| 160 | 01:08:09:29 | 01:08:12:14 | Because one, two, three, four. |
| 161 | 01:08:12:16 | 01:08:14:12 | So this is two. |
| 162 | 01:08:12:16 | 01:08:14:12 | Okay. |
| 163 | 01:08:14:14 | 01:08:15:12 | Right? |
| 164 | 01:08:15:14 | 01:08:16:22 | Minus two. |
| 165 | 01:08:15:14 | 01:08:16:22 | Minus two. |
| 166 | 01:08:16:24 | 01:08:18:25 | And the other one is... |
| 167 | 01:08:18:27 | 01:08:20:13 | *dois, quatro, seis, oito.* |
| 168 | 01:08:20:15 | 01:08:21:29 | ( *laughs* ) |
| 169 | 01:08:22:01 | 01:08:25:13 | Rosalie, I noticed that you had been working on this one. |
| 170 | 01:08:25:15 | 01:08:26:28 | Would you mind coming up |
| 171 | 01:08:27:00 | 01:08:30:02 | and showing us how you found the area of this triangle? |
| 172 | 01:08:34:25 | 01:08:38:23 | Okay, first we made a rectangle. |
| 173 | 01:08:48:18 | 01:08:49:25 | And in the rectangle |
| 174 | 01:08:49:27 | 01:08:53:16 | there were three, six, nine, 12 total units. |
| 175 | 01:08:55:21 | 01:09:01:13 | Then to find... one of the triangles, |
| 176 | 01:09:01:15 | 01:09:03:26 | part of the triangle, |
| 177 | 01:09:03:28 | 01:09:07:26 | there's two, four units, which means that half is two. |
| 178 | 01:09:07:28 | 01:09:09:18 | CHAPIN: So the area of the outside triangle |
| 179 | 01:09:09:20 | 01:09:10:18 | is what? |
| 180 | 01:09:10:20 | 01:09:12:18 | Two units. |
| 181 | 01:09:10:20 | 01:09:12:18 | Two square units? |
| 182 | 01:09:12:20 | 01:09:13:18 | Square units. |
| 183 | 01:09:13:20 | 01:09:15:15 | Square units, all right. |
| 184 | 01:09:15:17 | 01:09:19:04 | And then to find the other triangle... |
| 185 | 01:09:19:06 | 01:09:20:26 | I'm not seeing it. |
| 186 | 01:09:20:28 | 01:09:23:02 | Here's the outer triangle, right? |
| 187 | 01:09:23:04 | 01:09:24:05 | Yep. |
| 188 | 01:09:23:04 | 01:09:24:05 | Do you see it? |
| 189 | 01:09:24:07 | 01:09:26:03 | Now, can you make the rectangle |
| 190 | 01:09:26:05 | 01:09:27:16 | that will go with this triangle? |
| 191 | 01:09:27:18 | 01:09:28:16 | It's the whole thing. |
| 192 | 01:09:28:18 | 01:09:29:21 | Ah, it's the whole thing. |
| 193 | 01:09:29:23 | 01:09:31:27 | There we go-- so it's six. |
| 194 | 01:09:31:29 | 01:09:32:27 | Six... |
| 195 | 01:09:32:29 | 01:09:33:27 | Units, square units. |
| 196 | 01:09:33:29 | 01:09:35:03 | Okay. |
| 197 | 01:09:35:05 | 01:09:36:18 | So we had two |

| | | square units |
|---|---|---|
| 198 | 01:09:36:20 | 01:09:38:25 and six square units, which is eight. |
| 199 | 01:09:38:27 | 01:09:40:13 Subtract 12, subtract eight, |
| 200 | 01:09:40:15 | 01:09:41:26 equals four square units |
| 201 | 01:09:41:28 | 01:09:43:19 for the triangle we were trying to find. |
| 202 | 01:09:43:21 | 01:09:45:03 Excellent, great job. |
| 203 | 01:09:45:05 | 01:09:46:06 All right, thank you. |
| 204 | 01:09:47:14 | 01:09:49:04 Thanks. |
| 205 | 01:09:49:06 | 01:09:51:11 Now, using this leads us |
| 206 | 01:09:51:13 | 01:09:55:07 not only in being able to find areas here, |
| 207 | 01:09:55:09 | 01:09:57:18 but to look at some relationships. |
| 208 | 01:09:57:20 | 01:10:04:00 And if we built a rectangle like that on our board, |
| 209 | 01:10:04:02 | 01:10:07:07 and then I was to make triangles, |
| 210 | 01:10:07:09 | 01:10:09:18 here's my question for you. |
| 211 | 01:10:09:20 | 01:10:13:23 I'm going to start with a triangle like that, |
| 212 | 01:10:13:25 | 01:10:18:29 has a base of what I would call two units right there. |
| 213 | 01:10:19:01 | 01:10:20:19 All right? |
| 214 | 01:10:20:21 | 01:10:22:07 Now, watch what I do. |
| 215 | 01:10:22:09 | 01:10:25:01 I'm going to take this triangle and I'm going to... |
| 216 | 01:10:25:03 | 01:10:27:11 Let's see if I can do this so you can all see. |
| 217 | 01:10:27:13 | 01:10:29:02 I'm going to take it from the top |
| 218 | 01:10:29:04 | 01:10:30:15 and I'm going to move it to here. |
| 219 | 01:10:30:17 | 01:10:31:21 Okay? |
| 220 | 01:10:31:23 | 01:10:33:12 It still has the same base, right? |
| 221 | 01:10:33:14 | 01:10:36:22 Then I'm going to move it to here. |
| 222 | 01:10:36:24 | 01:10:40:14 And then I'm going to move it to here. |
| 223 | 01:10:40:16 | 01:10:41:27 Then I'm going to move it to here. |
| 224 | 01:10:41:29 | 01:10:44:07 Oh, boy, that's a really skinny one, huh? |
| 225 | 01:10:44:09 | 01:10:45:15 Now... |
| 226 | 01:10:45:17 | 01:10:51:00 What is the area of each of those triangles? |
| 227 | 01:10:51:02 | 01:10:52:29 Come on up. |
| 228 | 01:10:55:15 | 01:11:01:17 You keep the bases the same, and the height also. |
| 229 | 01:11:01:19 | 01:11:06:19 CHAPIN: |

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| 230 | 01:11:06:21 | 01:11:10:08 | One participant came up and helped explain the relationship between base and height and area of a triangle. |
| 231 | 01:11:10:10 | 01:11:13:06 | We had two parallel lines on our geoboard, |
| 232 | 01:11:13:08 | 01:11:17:02 | and we started with a triangle that had a set base. |
| 233 | 01:11:17:04 | 01:11:20:25 | This participant then kept moving the point... |
| 234 | 01:11:20:27 | 01:11:23:16 | or the vertex point of the triangle |
| 235 | 01:11:23:18 | 01:11:26:15 | different places along the parallel line. |
| 236 | 01:11:26:17 | 01:11:28:20 | Each time, the height remained the same, |
| 237 | 01:11:28:22 | 01:11:30:04 | and the base remained the same, |
| 238 | 01:11:30:06 | 01:11:31:28 | but the triangle looked very different. |
| 239 | 01:11:32:00 | 01:11:34:08 | As we reflected on the area, though, |
| 240 | 01:11:34:10 | 01:11:37:05 | of each of these different-looking triangles, |
| 241 | 01:11:37:07 | 01:11:40:08 | we realized that the area was exactly the same in all of them, |
| 242 | 01:11:40:10 | 01:11:43:26 | because they all had the same base and the same height. |
| 243 | 01:11:46:29 | 01:11:50:27 | Now, formulas are also a very important part of measurement |
| 244 | 01:11:50:29 | 01:11:54:11 | and can be used to find the area of different shapes. |
| 245 | 01:11:54:13 | 01:11:56:00 | What we're going to first think about |
| 246 | 01:11:56:02 | 01:11:59:01 | is the area of a parallelogram. |
| 247 | 01:11:59:03 | 01:12:03:06 | We all are pretty comfortable with the area of a rectangle |
| 248 | 01:12:03:08 | 01:12:06:12 | is length times width, or base times height, |
| 249 | 01:12:06:14 | 01:12:09:02 | however we want to label the dimensions. |
| 250 | 01:12:09:04 | 01:12:13:09 | But let's look about finding the area of a parallelogram. |
| 251 | 01:12:13:11 | 01:12:15:06 | How are we going to do that? |
| 252 | 01:12:15:08 | 01:12:20:27 | Now, we can use our geoboards or the power polygons for this one. |
| 253 | 01:12:20:29 | 01:12:22:08 | Can anyone review for me |
| 254 | 01:12:22:10 | 01:12:27:13 | what the formula is for the area of a parallelogram? |
| 255 | 01:12:27:15 | 01:12:28:24 | It's base times height. |
| 256 | 01:12:28:26 | 01:12:30:26 | CHAPIN: Base times height. |
| 257 | 01:12:30:28 | 01:12:33:18 | So here's our base, and here's our height. |
| 258 | 01:12:33:20 | 01:12:39:11 | So our base is three units, and our height is one unit, |

| 259 | 01:12:39:13 | 01:12:40:26 | so three by one. |
| 260 | 01:12:40:28 | 01:12:43:14 | And let's double-check with our more informal method, |
| 261 | 01:12:43:16 | 01:12:44:28 | see if that works. |
| 262 | 01:12:45:00 | 01:12:49:13 | There's one unit, two units, |
| 263 | 01:12:49:15 | 01:12:51:02 | and those half units can go together; |
| 264 | 01:12:51:04 | 01:12:52:09 | we have three square units. |
| 265 | 01:12:52:11 | 01:12:54:20 | How could we show that this always works? |
| 266 | 01:12:54:22 | 01:12:57:22 | How could we make sense of that? |
| 267 | 01:12:57:24 | 01:13:01:04 | I put a rectangle on top of my parallelogram |
| 268 | 01:13:01:06 | 01:13:07:19 | to show that this triangle over here... |
| 269 | 01:13:07:21 | 01:13:10:07 | that's sticking out of my rectangle, |
| 270 | 01:13:10:09 | 01:13:14:07 | came from the one that is not covered on my rectangle. |
| 271 | 01:13:14:09 | 01:13:16:27 | CHAPIN: We investigated two area formulas. |
| 272 | 01:13:16:29 | 01:13:21:05 | One was area of a parallelogram, which is base times height, |
| 273 | 01:13:21:07 | 01:13:23:22 | and the other formula that we investigated |
| 274 | 01:13:23:24 | 01:13:26:29 | was area of a triangle, which is half base times height. |
| 275 | 01:13:27:01 | 01:13:30:12 | So we linked the area of a triangle |
| 276 | 01:13:30:14 | 01:13:32:02 | to the area of a parallelogram |
| 277 | 01:13:32:04 | 01:13:34:10 | in trying to make sense of these formulas. |
| 278 | 01:13:34:12 | 01:13:36:10 | This would be the base times the height, |
| 279 | 01:13:36:12 | 01:13:37:29 | and that would give it to us. |
| 280 | 01:13:38:01 | 01:13:39:16 | So, it would be half of it, right, |
| 281 | 01:13:39:18 | 01:13:41:16 | if it was doing a triangle? |
| 282 | 01:13:41:18 | 01:13:43:25 | ( *other students chatting in background* ) |
| 283 | 01:13:43:27 | 01:13:46:07 | Right, the area of a triangle, right. |
| 284 | 01:13:46:09 | 01:13:47:14 | One-half. |
| 285 | 01:13:46:09 | 01:13:47:14 | One-half of it. |
| 286 | 01:13:47:16 | 01:13:49:21 | Same thing with this-- it would be half of it. |
| 287 | 01:13:49:23 | 01:13:51:12 | WOMAN: Make it a rectangle. |
| 288 | 01:13:51:14 | 01:13:54:10 | MAN: Right, and those should be the same shape |
| 289 | 01:13:54:12 | 01:13:55:19 | that you're making. |
| 290 | 01:13:55:21 | 01:13:58:08 | Right, and then |

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| 291 | 01:13:58:10 | 01:14:00:08 | if I move this piece over here, I end up having that whole rectangle... |
| 292 | 01:14:00:10 | 01:14:01:15 | Right. |
| 293 | 01:14:01:17 | 01:14:03:09 | And then I can do just base times height. |
| 294 | 01:14:03:11 | 01:14:04:12 | Right. |
| 295 | 01:14:04:14 | 01:14:06:20 | And then, since it took two triangles |
| 296 | 01:14:06:22 | 01:14:08:10 | to make that parallelogram, |
| 297 | 01:14:08:12 | 01:14:10:17 | if I wanted to find the area of one, |
| 298 | 01:14:10:19 | 01:14:12:01 | I'd just take the half. |
| 299 | 01:14:12:03 | 01:14:13:01 | Gotcha. |
| 300 | 01:14:16:15 | 01:14:19:29 | Well, we're going to shift gears ever so slightly |
| 301 | 01:14:20:01 | 01:14:23:27 | and now look at one other topic having to do with area, |
| 302 | 01:14:23:29 | 01:14:26:09 | and that is, we're going to look |
| 303 | 01:14:26:11 | 01:14:30:24 | at what happens to the area of shapes when we scale things up. |
| 304 | 01:14:30:26 | 01:14:34:08 | We're going to do some enlargement of similar shapes, |
| 305 | 01:14:34:10 | 01:14:37:11 | so we're kind of tying back in with what we did |
| 306 | 01:14:37:13 | 01:14:41:02 | in terms of learning about similar shapes earlier today. |
| 307 | 01:14:42:21 | 01:14:45:12 | Similar figures have the same shape |
| 308 | 01:14:45:14 | 01:14:47:15 | but not necessarily the same size. |
| 309 | 01:14:47:17 | 01:14:50:24 | We're going to use the power polygons, |
| 310 | 01:14:50:26 | 01:14:56:04 | and for example, let me show you one, which is the rectangle. |
| 311 | 01:14:56:06 | 01:15:00:05 | I can make a similar, but larger, rectangle, |
| 312 | 01:15:00:07 | 01:15:02:29 | by maintaining the same shape |
| 313 | 01:15:03:01 | 01:15:05:02 | and also, namely, the same angles |
| 314 | 01:15:05:04 | 01:15:08:02 | and then have corresponding sides in proportion. |
| 315 | 01:15:10:17 | 01:15:14:20 | Are those two rectangles similar? |
| 316 | 01:15:17:24 | 01:15:19:06 | CLASS: No. |
| 317 | 01:15:19:08 | 01:15:20:24 | No. |
| 318 | 01:15:20:26 | 01:15:23:23 | Twice the length, but not twice the height. |
| 319 | 01:15:23:25 | 01:15:27:09 | Remember, all dimensions have to be in proportion. |
| 320 | 01:15:27:11 | 01:15:28:29 | So to build a rectangle |
| 321 | 01:15:29:01 | 01:15:32:06 | that's twice the length and twice the height, |

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| 322 | 01:15:32:08 | 01:15:34:04 | I'm going to go like this. |
| 323 | 01:15:36:07 | 01:15:39:18 | Is the area twice as much as the area of this one? |
| 324 | 01:15:39:20 | 01:15:40:25 | CLASS: No. |
| 325 | 01:15:40:27 | 01:15:43:22 | No, it actually looks to be quite a bit more. |
| 326 | 01:15:43:24 | 01:15:46:13 | So what we're going to experiment with |
| 327 | 01:15:46:15 | 01:15:50:02 | is looking at what happens to the area of shapes |
| 328 | 01:15:50:04 | 01:15:53:25 | when we enlarge them by different scale factors. |
| 329 | 01:15:53:27 | 01:15:56:11 | The base has to increase by three, |
| 330 | 01:15:56:13 | 01:15:59:17 | so if this is one, we need three this time. |
| 331 | 01:15:59:19 | 01:16:01:00 | Okay... let's see |
| 332 | 01:16:01:02 | 01:16:03:14 | if we have enough in here-- yep. |
| 333 | 01:16:03:16 | 01:16:08:20 | CHAPIN: Our final objective was to investigate scale factor, |
| 334 | 01:16:08:22 | 01:16:12:21 | and how is scale factor affected when we are looking at area. |
| 335 | 01:16:12:23 | 01:16:16:00 | So for example, if we increase the size of a shape |
| 336 | 01:16:16:02 | 01:16:17:23 | with a scale factor of two, |
| 337 | 01:16:17:25 | 01:16:20:12 | what does that do to the area of the shape? |
| 338 | 01:16:20:14 | 01:16:22:06 | Is it twice as great, or not? |
| 339 | 01:16:24:02 | 01:16:25:04 | Okay. |
| 340 | 01:16:26:13 | 01:16:27:16 | How about that? |
| 341 | 01:16:27:18 | 01:16:30:02 | So we get one-two, one-two, one-two... |
| 342 | 01:16:30:04 | 01:16:32:16 | We've made each side twice as long. |
| 343 | 01:16:32:18 | 01:16:34:28 | To figure out the area increase... |
| 344 | 01:16:35:00 | 01:16:36:05 | Look at this. |
| 345 | 01:16:36:07 | 01:16:38:24 | It takes... three of these |
| 346 | 01:16:38:26 | 01:16:40:26 | to make a hexagon. |
| 347 | 01:16:40:28 | 01:16:44:06 | So each one of these is a third. |
| 348 | 01:16:44:08 | 01:16:45:06 | Okay. |
| 349 | 01:16:45:08 | 01:16:47:08 | So we have one, two, three. |
| 350 | 01:16:47:10 | 01:16:49:02 | And then these three combined make four. |
| 351 | 01:16:49:04 | 01:16:50:09 | Is four. |
| 352 | 01:16:50:11 | 01:16:52:14 | So when we double the length of a side, |
| 353 | 01:16:52:16 | 01:16:53:28 | we get four times the area. |
| 354 | 01:16:54:00 | 01:16:56:04 | Okay... which fits in |

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| 355 | 01:16:56:06 | 01:16:59:00 | with our predictions so far. |
| 356 | 01:16:59:02 | 01:17:00:19 | Want to make it bigger than that? |
| 357 | 01:17:00:21 | 01:17:01:20 | Sure. |
| 358 | 01:17:01:22 | 01:17:03:14 | Let's take these out. |
| 359 | 01:17:10:10 | 01:17:12:21 | Slide... yeah, slide... |
| 360 | 01:17:12:23 | 01:17:13:23 | No. |
| 361 | 01:17:13:25 | 01:17:15:06 | No, slide that piece back in |
| 362 | 01:17:15:08 | 01:17:16:27 | where we had him, in the middle. |
| 363 | 01:17:16:29 | 01:17:18:22 | Make, like, a soccer ball out of it. |
| 364 | 01:17:18:24 | 01:17:21:03 | Yep, and throw that one back in. |
| 365 | 01:17:21:05 | 01:17:25:29 | I think we're going to need another couple of hexagons. |
| 366 | 01:17:26:01 | 01:17:28:16 | That one will go right there. |
| 367 | 01:17:28:18 | 01:17:31:03 | This one will go right here. |
| 368 | 01:17:31:05 | 01:17:32:27 | Now we should just fill in the pieces |
| 369 | 01:17:32:29 | 01:17:33:28 | with blue ones. |
| 370 | 01:17:35:00 | 01:17:36:09 | Okay. |
| 371 | 01:17:36:11 | 01:17:38:17 | So we have sides of three units each... |
| 372 | 01:17:38:19 | 01:17:39:16 | Okay. |
| 373 | 01:17:39:18 | 01:17:41:18 | so a scale factor of three, |
| 374 | 01:17:41:20 | 01:17:43:28 | and our area now has increased |
| 375 | 01:17:44:00 | 01:17:47:08 | by one, two, three, four, five, six, seven. |
| 376 | 01:17:47:10 | 01:17:50:18 | We have seven hexagons, and eight... |
| 377 | 01:17:50:20 | 01:17:52:09 | And three is nine. |
| 378 | 01:17:52:11 | 01:17:54:02 | And then each three makes one more, |
| 379 | 01:17:54:04 | 01:17:55:08 | so we have eight, nine. |
| 380 | 01:17:55:10 | 01:17:57:08 | So it was nine times when we tripled it. |
| 381 | 01:17:58:19 | 01:18:01:11 | Do we have enough pieces to make the big one? |
| 382 | 01:18:01:13 | 01:18:04:22 | So we can safely predict that the area, if we were to do it, |
| 383 | 01:18:04:24 | 01:18:06:07 | would be 16 times as great. |
| 384 | 01:18:06:09 | 01:18:07:14 | It would be 16 times bigger. |
| 385 | 01:18:07:16 | 01:18:08:19 | That kind of makes sense, |
| 386 | 01:18:08:21 | 01:18:10:19 | because if we're |

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| | | taking a length, |
| 387 | 01:18:10:21 | 01:18:12:26 and then we're looking for the area, |
| 388 | 01:18:12:28 | 01:18:15:27 length are regular units, area are square units, |
| 389 | 01:18:15:29 | 01:18:19:07 so when we square a number, we're squaring the length |
| 390 | 01:18:19:09 | 01:18:22:16 to get the area, so going from four times the size, |
| 391 | 01:18:22:18 | 01:18:23:26 16 times the area-- |
| 392 | 01:18:23:28 | 01:18:25:10 if we went nine times bigger, |
| 393 | 01:18:25:12 | 01:18:27:19 it would be 81 times the area, and so on from there. |
| 394 | 01:18:27:21 | 01:18:28:19 And so on from there. |
| 395 | 01:18:28:21 | 01:18:30:08 Good. |
| 396 | 01:18:30:10 | 01:18:32:10 CHAPIN: All right, can I pull everybody back together again? |
| 397 | 01:18:32:12 | 01:18:35:09 Let's just look at some summary statements |
| 398 | 01:18:35:11 | 01:18:36:28 that maybe we can make |
| 399 | 01:18:37:00 | 01:18:40:24 about the relationship of when we have a scale factor, |
| 400 | 01:18:40:26 | 01:18:43:21 and we increase and make a similar figure |
| 401 | 01:18:43:23 | 01:18:48:12 with that scale factor, what it does to the area. |
| 402 | 01:18:48:14 | 01:18:54:05 When we have a scale factor of two, the enlargement has |
| 403 | 01:18:54:07 | 01:18:57:25 four times the area of the original rectangle, |
| 404 | 01:18:57:27 | 01:19:00:20 and when the scale factor is three, |
| 405 | 01:19:00:22 | 01:19:03:06 the enlargement has nine times. |
| 406 | 01:19:03:08 | 01:19:07:09 It seems to be that it's the scale factor squared. |
| 407 | 01:19:07:11 | 01:19:08:17 Why? |
| 408 | 01:19:08:19 | 01:19:09:25 Why is that happening? |
| 409 | 01:19:09:27 | 01:19:12:00 It happened with all the figures. |
| 410 | 01:19:12:02 | 01:19:15:27 Basically, if you double the size, |
| 411 | 01:19:15:29 | 01:19:22:24 you are taking the area of the unit area, two by two, |
| 412 | 01:19:22:26 | 01:19:25:10 which makes it four. |
| 413 | 01:19:25:12 | 01:19:29:03 And if you triple the size, |
| 414 | 01:19:29:05 | 01:19:31:01 the area becomes three by three, |
| 415 | 01:19:31:03 | 01:19:33:22 which is nine, and that's what we're getting. |
| 416 | 01:19:33:24 | 01:19:36:22 And if you do it four by... four times, |
| 417 | 01:19:36:24 | 01:19:39:24 that will be four by four, which is 16. |

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| 418 | 01:19:39:26 | 01:19:41:14 | So it's always going to be 16 times |
| 419 | 01:19:41:16 | 01:19:43:00 | whatever the original area was. |
| 420 | 01:19:43:02 | 01:19:44:22 | WOMAN: Because it's four times as big |
| 421 | 01:19:44:24 | 01:19:45:25 | in this direction |
| 422 | 01:19:45:27 | 01:19:47:20 | and four times as big in that direction. |
| 423 | 01:19:47:22 | 01:19:50:12 | CHAPIN: That's right, so it's... we're getting two dimensions |
| 424 | 01:19:50:14 | 01:19:54:14 | are being affected here, and when we have two dimensions, |
| 425 | 01:19:54:16 | 01:19:58:00 | that scale factor is affected by both dimensions, |
| 426 | 01:19:58:02 | 01:20:01:17 | and that's why we have that very nice relationship |
| 427 | 01:20:01:19 | 01:20:04:08 | of it being what we sometimes refer to |
| 428 | 01:20:04:10 | 01:20:06:04 | as the scale factor is k, |
| 429 | 01:20:06:06 | 01:20:09:19 | and so the enlargement is k-squared times as big. |
| 430 | 01:20:09:21 | 01:20:12:00 | CHAPIN: We use area every day. |
| 431 | 01:20:12:02 | 01:20:15:15 | It is a very practical application of mathematics. |
| 432 | 01:20:15:17 | 01:20:18:14 | However, there are aspects of using area |
| 433 | 01:20:18:16 | 01:20:21:18 | that in many cases we make mistakes about, |
| 434 | 01:20:21:20 | 01:20:25:07 | and in part it's because we do not have a sound grounding |
| 435 | 01:20:25:09 | 01:20:27:26 | in the mathematics that goes behind them. |
| 436 | 01:20:27:28 | 01:20:31:00 | Hopefully today, we'll have given participants |
| 437 | 01:20:31:02 | 01:20:32:09 | some more background |
| 438 | 01:20:32:11 | 01:20:35:11 | in some of those aspects of understanding area, |
| 439 | 01:20:35:13 | 01:20:38:07 | so as they use it in their day-to-day lives, |
| 440 | 01:20:38:09 | 01:20:41:18 | they will make careful and thoughtful calculations. |
| 441 | 01:20:49:16 | 01:20:52:00 | ( *seagulls cawing* ) |
| 442 | 01:20:52:02 | 01:20:54:18 | NARRATOR: Among the rustic clapboard houses |
| 443 | 01:20:54:20 | 01:20:57:07 | that inhabit the coast of Martha's Vineyard |
| 444 | 01:20:57:09 | 01:21:00:07 | resides the Gannon and Benjamin Marine Railway. |
| 445 | 01:21:00:09 | 01:21:03:05 | ( *sawing* ) |
| 446 | 01:21:03:07 | 01:21:07:09 | This unique company specializes in building wooden sailboats |
| 447 | 01:21:07:11 | 01:21:12:01 | the old-fashioned way, custom- |

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| | | | making each vessel by hand. |
| 448 | 01:21:12:03 | 01:21:16:10 | Owned and operated by Nathaniel Benjamin and Ross Gannon, |
| 449 | 01:21:16:12 | 01:21:19:06 | the company grew out of a love for sailing. |
| 450 | 01:21:19:08 | 01:21:21:23 | BENJAMIN: Ross and I became interested |
| 451 | 01:21:21:25 | 01:21:24:06 | in design and construction of wooden boats |
| 452 | 01:21:24:08 | 01:21:27:04 | really through sailing and repairing wooden boats. |
| 453 | 01:21:27:06 | 01:21:28:17 | That's where we started. |
| 454 | 01:21:28:19 | 01:21:32:20 | So we both became just really fascinated |
| 455 | 01:21:32:22 | 01:21:36:10 | with the whole engineering and style |
| 456 | 01:21:36:12 | 01:21:40:26 | of this wooden boat, this mysterious object. |
| 457 | 01:21:40:28 | 01:21:42:12 | ( *hammering* ) |
| 458 | 01:21:45:13 | 01:21:50:04 | NARRATOR: In the workshop, craftsmen use traditional tools and techniques |
| 459 | 01:21:50:06 | 01:21:52:11 | to construct their latest project, |
| 460 | 01:21:52:13 | 01:21:55:08 | a 31-foot canoe-stern yawl. |
| 461 | 01:21:55:10 | 01:21:58:28 | Building a sailboat like this can take several months, |
| 462 | 01:21:59:00 | 01:22:01:17 | from original design to final rigging. |
| 463 | 01:22:01:19 | 01:22:03:25 | BENJAMIN: What's unique about this business |
| 464 | 01:22:03:27 | 01:22:06:15 | is that we do pretty much everything right here. |
| 465 | 01:22:06:17 | 01:22:08:29 | We design the boats, build the boats. |
| 466 | 01:22:10:20 | 01:22:12:17 | We do a lot of the metal fabrication. |
| 467 | 01:22:12:19 | 01:22:15:18 | We do a lot of custom bronze hardware. |
| 468 | 01:22:15:20 | 01:22:17:29 | We don't build racing boats. |
| 469 | 01:22:18:01 | 01:22:19:28 | We build cruising boats, |
| 470 | 01:22:20:00 | 01:22:23:20 | family boats, day sailers, made to order. |
| 471 | 01:22:23:22 | 01:22:27:01 | NARRATOR: Whether it's a day sailer or a large schooner, |
| 472 | 01:22:27:03 | 01:22:29:24 | there are a number of measurements to consider |
| 473 | 01:22:29:26 | 01:22:32:04 | when designing and constructing a boat. |
| 474 | 01:22:32:06 | 01:22:35:06 | One of the more important is the area of the sail. |
| 475 | 01:22:35:08 | 01:22:38:11 | BENJAMIN: The sail area of a boat is really your engine. |

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| 476 | 01:22:38:13 | 01:22:39:21 | That's your horsepower. |
| 477 | 01:22:39:23 | 01:22:41:11 | That's what drives the boat. |
| 478 | 01:22:41:13 | 01:22:44:16 | So to determine what type of sail plan goes on a boat, |
| 479 | 01:22:44:18 | 01:22:46:07 | it's analogous to determining |
| 480 | 01:22:46:09 | 01:22:49:02 | what size of an engine to put into an automobile. |
| 481 | 01:22:49:04 | 01:22:51:23 | If it's a great, big, heavy truck, you need a lot more power |
| 482 | 01:22:51:25 | 01:22:53:12 | than you do in a little sports car. |
| 483 | 01:22:53:14 | 01:22:56:18 | So your sail area really is determined |
| 484 | 01:22:56:20 | 01:22:58:26 | by the weight of the boat. |
| 485 | 01:22:58:28 | 01:23:02:05 | NARRATOR: Determining the weight of the boat involves |
| 486 | 01:23:02:07 | 01:23:04:10 | finding the volume of displacement. |
| 487 | 01:23:04:12 | 01:23:08:00 | This is the amount of water a vessel displaces when afloat. |
| 488 | 01:23:08:02 | 01:23:12:07 | BENJAMIN: Essentially, you're calculating the cubic feet |
| 489 | 01:23:12:09 | 01:23:16:22 | below the load waterline, the underwater body of the boat, |
| 490 | 01:23:16:24 | 01:23:19:13 | and that volume multiplied times 64 |
| 491 | 01:23:19:15 | 01:23:22:09 | will give you the weight of the boat, |
| 492 | 01:23:22:11 | 01:23:25:28 | because a cubic foot of water weighs 64 pounds, |
| 493 | 01:23:26:00 | 01:23:30:04 | and Archimedes figured out that a vessel will displace |
| 494 | 01:23:30:06 | 01:23:33:22 | in cubic feet of water, exactly what it weighs. |
| 495 | 01:23:33:24 | 01:23:36:05 | NARRATOR: Once the boat's weight is known, |
| 496 | 01:23:36:07 | 01:23:39:00 | the size and shape of the sails can be measured. |
| 497 | 01:23:39:02 | 01:23:41:18 | If it's a jib or a mainsail or mizzen |
| 498 | 01:23:41:20 | 01:23:43:13 | and triangular in section, |
| 499 | 01:23:43:15 | 01:23:46:15 | you simply multiply the base times the height, |
| 500 | 01:23:46:17 | 01:23:49:00 | divide it by two, and that's your area. |
| 501 | 01:23:49:02 | 01:23:52:27 | So you have so many square feet, and the sailmaker orders |
| 502 | 01:23:52:29 | 01:23:56:28 | the appropriate amount of cloth to build it. |
| 503 | 01:23:57:00 | 01:23:59:18 | NARRATOR: In the loft above the workshop, |
| 504 | 01:23:59:20 | 01:24:02:25 | sailmaker Gretchen Snyder cuts, sews and shapes |
| 505 | 01:24:02:27 | 01:24:05:06 | most of the sails |

|     |            |            | for the company. |
| 506 | 01:24:05:08 | 01:24:08:29 | SNYDER: |
|     |            |            | I get a sail plan, |
|     |            |            | which gives me the dimensions. |
| 507 | 01:24:09:01 | 01:24:12:19 | I take those measurements, |
|     |            |            | lie them down on the floor, |
| 508 | 01:24:12:21 | 01:24:16:11 | and then I lay the panels, |
|     |            |            | and I then can put shape in. |
| 509 | 01:24:16:13 | 01:24:20:16 | NARRATOR: |
|     |            |            | Adding the right amount of shape |
|     |            |            | is the art of sailmaking |
| 510 | 01:24:20:18 | 01:24:24:13 | and is essential to creating an |
|     |            |            | efficient and manageable sail. |
| 511 | 01:24:24:15 | 01:24:28:05 | Well, in order for the sail |
|     |            |            | to move the boat, |
| 512 | 01:24:28:07 | 01:24:31:12 | it can't just be |
|     |            |            | a flat piece of cloth. |
| 513 | 01:24:31:14 | 01:24:36:05 | It needs some belly in it... |
|     |            |            | kind of an airfoil. |
| 514 | 01:24:38:14 | 01:24:42:10 | NARRATOR: |
|     |            |            | Wooden vessels of all kinds |
|     |            |            | have been built in this boatyard |
| 515 | 01:24:42:12 | 01:24:43:27 | for over 20 years. |
| 516 | 01:24:45:14 | 01:24:49:02 | During that time, Gannon and |
|     |            |            | Benjamin have quietly led |
| 517 | 01:24:49:04 | 01:24:53:16 | a renaissance in traditional |
|     |            |            | artistry and design. |
| 518 | 01:24:53:18 | 01:24:57:04 | BENJAMIN: |
|     |            |            | The actual building |
|     |            |            | and designing of a boat |
| 519 | 01:24:57:06 | 01:25:00:22 | is a process that is so engaging |
|     |            |            | to all your senses |
| 520 | 01:25:00:24 | 01:25:02:22 | that it's limitless, really. |
| 521 | 01:25:02:24 | 01:25:07:07 | You know, you're just constantly |
|     |            |            | learning and seeing new things, |
| 522 | 01:25:07:09 | 01:25:10:28 | and new shapes, and I find it |
|     |            |            | very... very enjoyable. |
| 523 | 01:25:19:00 | 01:25:22:18 | Captioned by |
|     |            |            | Media Access Group at WGBH |
|     |            |            | access.wgbh.org |