1 01:00:47:09 01:00:48:20 WOMAN:
Measurement is the process
2 01:00:48:22 01:00:51:23 of quantifying properties
of objects.
3 01:00:51:25 01:00:54:20 And to do that,
we have set procedures
4 01:00:54:22 01:00:56:21 that enable us to measure.
5 01:00:56:23 01:00:58:13 Oh.
6 01:00:58:15 01:01:00:26 Measuring helps you
to understand
7 01:01:00:28 01:01:03:08 how things relate to each other.
8 01:01:03:10 01:01:07:06 Our volume of a sphere
actually has a formula
9 01:01:07:08 01:01:09:17 of four-thirds pi r-cubed.
10 01:01:09:19 01:01:13:03 This course really made me think
about how I approach measurement
11 01:01:13:05 01:01:16:06 and how I can use measurement
every day in the classroom.
12 01:01:24:16 01:01:28:00 NARRATOR:
16 elementary and middle school
teachers have gathered together
13 01:01:28:02 01:01:29:24 to participate
in a mathematics course
14 01:01:29:26 01:01:33:02 investigating key concepts
in measurement.
15 01:01:33:04 01:01:36:19 Their instructor
is Professor Suzanne Chapin.
16 01:01:36:21 01:01:40:10 CHAPIN:
The overall goals
of the measurement course
17 01:01:40:12 01:01:43:01 involve looking
at fundamental ideas
18 01:01:43:03 01:01:46:02 that are inherent
in whenever we measure.
19 01:01:46:04 01:01:50:06 There are multiple concepts
that we're going to investigate.
20 01:01:50:08 01:01:53:27 Some of them involve things
like ratio and proportion.
21 01:01:53:29 01:01:56:06 There are others
that are going to look
22 01:01:56:08 01:01:57:24 at accuracy and precision,
23 01:01:57:26 01:01:59:04 and help us understand
24 01:01:59:06 01:02:01:14 the approximate nature
of measurement.
25 01:02:01:16 01:02:05:17 We're also going to delve
into areas of measurement,
26 01:02:05:19 01:02:08:10 whether it be volume,
surface area,
27 01:02:08:12 01:02:11:27 ideas of circles with area
and circumference,
28 01:02:11:29 01:02:14:18 and the relationships
that are inherent
29 01:02:14:20 01:02:17:05 in some of those
particular situations
30 01:02:17:07 01:02:19:24 that we hope to investigate
in detail.

31 01:02:19:26 01:02:22:00 NARRATOR:
In this session,
the class will begin
32 01:02:22:02 01:02:23:16 by exploring the questions,
33 01:02:23:18 01:02:26:06 "What can be measured?" and
"What does it mean to measure?"
34 01:02:26:08 01:02:27:20 CHAPIN:
Now, to do that,
35 01:02:27:22 01:02:30:05 we're going to be looking
at something very simple,
36 01:02:30:07 01:02:32:07 namely some rocks
that are at your table.
37 01:02:32:09 01:02:35:29 And what I'd like you to do
with the person next to you
38 01:02:36:01 01:02:38:04 is take a minute and make a list
39 01:02:38:06 01:02:40:25 of what are some
of the characteristics,
40 01:02:40:27 01:02:43:22 what are some of the attributes
of the rock that you have.
41 01:02:44:24 01:02:46:19 Um, it's, um...
42 01:02:46:21 01:02:48:06 It has a shape.
43 01:02:50:24 01:02:52:03 "Has a shape."
44 01:02:52:05 01:02:54:03 WOMAN:
"Has a shape."
45 01:02:56:03 01:02:57:02 Um... it's hard.
46 01:02:57:04 01:02:58:02 "It's hard."
47 01:02:58:04 01:02:59:03 Hard.
48 01:03:01:01 01:03:05:21 Doesn't seem
to be very heavy.
49 01:03:05:23 01:03:06:24 "Not too heavy."
50 01:03:06:26 01:03:07:27 "Not too heavy."
51 01:03:07:29 01:03:10:04 CHAPIN:
In our very first activity,
52 01:03:10:06 01:03:13:17 I asked the participants
to examine their rocks
53 01:03:13:19 01:03:16:29 and to list attributes of their
rocks, or characteristics.
54 01:03:17:01 01:03:18:06 And the point of this
55 01:03:18:08 01:03:20:19 was for all of us
to start to think about
56 01:03:20:21 01:03:23:14 that there are some
characteristics or attributes
57 01:03:23:16 01:03:24:23 that are measurable,
58 01:03:24:25 01:03:27:12 and there are others
that are not measurable,
59 01:03:27:14 01:03:30:17 and how do we differentiate
between those two things.
60 01:03:30:19 01:03:34:06 Uh, gray and reddish

in terms of color.
61 01:03:34:08 01:03:36:27 Reddish purple
at the bottom.
62 01:03:36:29 01:03:38:00 Uh...
63 01:03:38:02 01:03:39:24 WOMAN:
Roundy... rounded
and angled.

64 01:03:39:26 01:03:41:17 "Rounded and...
and angled," right.
65 01:03:41:19 01:03:49:02 We got, um...
scored, right?
66 01:03:49:04 01:03:50:04 The lines, cuts.
67 01:03:50:06 01:03:51:13 The lines in
the bottom, yeah.
68 01:03:51:15 01:03:53:10 About how big it is...
is it, do you think?
69 01:03:53:12 01:03:55:27 How... is that about
an... two inches maybe,
would you say?
70 01:03:55:29 01:03:57:25 Yeah, that was two inches.
71 01:03:57:27 01:03:59:02 In... in length.
72 01:03:59:04 01:04:00:11 And an inch high.
73 01:04:04:10 01:04:05:19 All right, let's take a look
74 01:04:05:21 01:04:07:29 at some of the attributes
that you came up with.
75 01:04:08:01 01:04:10:14 Who wants to volunteer a couple
that they've found?
76 01:04:12:16 01:04:15:17 Well, we decided that our rock
was irregularly shaped.
77 01:04:15:19 01:04:16:18 CHAPIN:
Mm-hmm.
78 01:04:16:20 01:04:19:11 It was dense.
79 01:04:19:13 01:04:21:26 All right.
80 01:04:21:28 01:04:24:00 And we were not sure if it
was light or if it was heavy.
81 01:04:24:02 01:04:25:21 Do we have another group
82 01:04:25:23 01:04:28:13 that can add some things
or attributes?
83 01:04:28:15 01:04:33:25 We looked at the texture,
and came up with
some attributes:
84 01:04:33:27 01:04:36:00 dull, shiny, speckled.
85 01:04:36:02 01:04:37:00 CHAPIN:
Okay.
86 01:04:37:02 01:04:38:17 Which actually
might be color.
87 01:04:38:19 01:04:41:18 Mm-hmm, so I'm going
to add that one,
"color", okay.
88 01:04:41:20 01:04:47:29 Anybody else add something that
we haven't got up here so far
89 01:04:48:01 01:04:49:14 of that rock?
90 01:04:49:16 01:04:50:25 Stone feels
kind of cool,
91 01:04:50:27 01:04:53:21 so we were talking
about the different
temperatures
92 01:04:53:23 01:04:55:00 that rocks could feel.
93 01:04:55:02 01:04:56:00 CHAPIN:
All right.
94 01:04:56:02 01:04:59:07 And we came up
with hardness.
95 01:04:59:09 01:05:03:07 You know, some rocks

| | | | |
|---|---|---|---|
| | | | are really hard, |
| 96 | 01:05:03:09 | 01:05:04:19 | and this one seems to be, |
| 97 | 01:05:04:21 | 01:05:07:03 | but there might be some that more, um... brittle, |
| 98 | 01:05:07:05 | 01:05:08:18 | so we talked about hardness. |
| 99 | 01:05:08:20 | 01:05:09:25 | CHAPIN: Okay. |
| 100 | 01:05:09:27 | 01:05:11:24 | And, um... types of veins |
| 101 | 01:05:11:26 | 01:05:13:23 | that you see in the rock. |
| 102 | 01:05:13:25 | 01:05:16:11 | There's a lot of different angles |
| 103 | 01:05:16:13 | 01:05:19:12 | because it's more like a quartz type. |
| 104 | 01:05:19:14 | 01:05:21:28 | We didn't know if that was a type of stone |
| 105 | 01:05:22:00 | 01:05:24:10 | or what other term that could be used. |
| 106 | 01:05:27:04 | 01:05:28:17 | Its length and its height. |
| 107 | 01:05:28:19 | 01:05:29:16 | CHAPIN: Okay. |
| 108 | 01:05:29:18 | 01:05:30:17 | And then its width. |
| 109 | 01:05:30:19 | 01:05:31:19 | So its dimensions. |
| 110 | 01:05:31:21 | 01:05:32:20 | Dimensions. |
| 111 | 01:05:32:22 | 01:05:33:20 | Yep. |
| 112 | 01:05:35:01 | 01:05:36:28 | How about its surface area? |
| 113 | 01:05:37:00 | 01:05:40:13 | Some of you got very bumpy rocks, don't you, yep? |
| 114 | 01:05:40:15 | 01:05:42:17 | So that... that's another. |
| 115 | 01:05:42:19 | 01:05:47:08 | Well, if you look here, and we could probably keep going, |
| 116 | 01:05:47:10 | 01:05:51:29 | we have many different attributes that we can examine |
| 117 | 01:05:52:01 | 01:05:54:18 | and use to compare these rocks. |
| 118 | 01:05:54:20 | 01:05:57:21 | Now, some of these are measurable attributes, |
| 119 | 01:05:57:23 | 01:05:59:02 | and some are not, |
| 120 | 01:05:59:04 | 01:06:02:10 | and that's what we really want to investigate |
| 121 | 01:06:02:12 | 01:06:05:15 | is what makes something measurable. |
| 122 | 01:06:05:17 | 01:06:08:22 | And let's take a look, at the moment, at this one, |
| 123 | 01:06:08:24 | 01:06:13:00 | which I'm going to summarize as we could figure out the weight-- |
| 124 | 01:06:13:02 | 01:06:15:07 | how light or heavy. |
| 125 | 01:06:15:09 | 01:06:20:24 | And, in fact, we might even want to call it the mass-- |
| 126 | 01:06:20:26 | 01:06:27:01 | how much stuff is there. |
| 127 | 01:06:27:03 | 01:06:28:28 | Is this a measurable attribute? |
| 128 | 01:06:29:00 | 01:06:29:28 | CLASS: Yes. |

129 01:06:30:00 01:06:31:12 How come?
130 01:06:31:14 01:06:35:16 Because we have standards
by which to measure those.
131 01:06:35:18 01:06:38:01 We have ounces to measure mass.
132 01:06:38:03 01:06:42:17 CHAPIN:
Yeah, so we could pick
a standard unit, all right,
133 01:06:42:19 01:06:44:06 that would make sense for us
134 01:06:44:08 01:06:46:14 to gather some information
about that,
135 01:06:46:16 01:06:49:09 and then we could actually
somehow collect data
136 01:06:49:11 01:06:54:08 on how many of those standard
units that rock, in this case,
137 01:06:54:10 01:06:56:08 has a mass of or weighs.
138 01:06:56:10 01:06:57:24 All right?
139 01:06:57:26 01:07:02:16 Now, which of these are going
to be a lot more difficult
140 01:07:02:18 01:07:05:08 to actually take a measure from?
141 01:07:05:10 01:07:07:21 Shape and texture
were probably difficult
to measure
142 01:07:07:23 01:07:09:20 because it's a personal
preference thing.
143 01:07:09:22 01:07:11:08 When we were looking
at our rock,
144 01:07:11:10 01:07:12:22 I thought it was
golf-ball sized,
145 01:07:12:24 01:07:14:10 and she thought
it was egg-shaped.
146 01:07:14:12 01:07:17:15 And some people, you know,
might have harder
or smoother fingers,
147 01:07:17:17 01:07:19:20 might think it's smooth
or rough in texture.
148 01:07:19:22 01:07:21:19 So it's a personal
preference thing
149 01:07:21:21 01:07:24:11 that there really
isn't a standard
for, you know,
150 01:07:24:13 01:07:26:10 "What does this
feel like to you?"
151 01:07:26:12 01:07:29:02 CHAPIN:
Exactly, and there
isn't a standard unit
152 01:07:29:04 01:07:30:17 for texture.
153 01:07:30:19 01:07:35:01 There isn't a standard unit
of shape or even for color.
154 01:07:35:03 01:07:36:01 All right?
155 01:07:36:03 01:07:37:28 Well, what we're going to do is,
156 01:07:38:00 01:07:40:24 we actually are going
to try to measure our rock
157 01:07:40:26 01:07:44:02 in terms of some of these
characteristics or attributes.
158 01:07:44:04 01:07:47:21 We are going to look
at its mass or weight.

159 01:07:47:23 01:07:51:12 We are going to look
at its volume,
160 01:07:51:14 01:07:56:21 and we are going to also measure
its surface area.
161 01:07:56:23 01:08:01:21 Before you do each one, the
first thing I'd like you to do
162 01:08:01:23 01:08:06:02 is make an estimate of,
"How much does that weigh?"
163 01:08:06:04 01:08:08:19 "How great a volume
does it have?"
164 01:08:08:21 01:08:10:23 "What is its surface area?"
165 01:08:12:01 01:08:13:16 First, we should measure
surface area.
166 01:08:13:18 01:08:14:27 They gave us this
aluminum foil,
167 01:08:14:29 01:08:16:02 so why don't we wrap it...
168 01:08:16:04 01:08:17:02 All right.
169 01:08:17:04 01:08:18:02 Very tightly.
170 01:08:18:04 01:08:19:11 All right.
171 01:08:19:13 01:08:21:18 But we have all this...
well, if we make a mistake,
172 01:08:21:20 01:08:23:15 we should cut out
a couple of pieces first.
173 01:08:23:17 01:08:24:18 This way we
can try again,
174 01:08:24:20 01:08:25:24 because it doesn't
look like
175 01:08:25:26 01:08:27:16 it's going to take up
this whole thing.
176 01:08:27:18 01:08:29:17 So I'm just going
to cut out a piece of foil,
177 01:08:29:19 01:08:30:27 so we'll have more
for later.
178 01:08:33:04 01:08:34:02 Can you wrap that?
179 01:08:34:04 01:08:35:02 Yep.
180 01:08:40:11 01:08:42:20 If we have extra,
we can just cut it off.
181 01:08:42:22 01:08:43:20 All right.
182 01:08:46:22 01:08:47:20 It's overlapping.
183 01:08:47:22 01:08:48:20 That's okay.
184 01:08:48:22 01:08:50:00 All right.
185 01:08:50:02 01:08:52:02 We'll overlap a little bit,
and we'll cut some off,
186 01:08:52:04 01:08:53:06 and we'll estimate it out.
187 01:08:53:08 01:08:54:10 This is very hard.
188 01:08:54:12 01:08:55:10 ( *laughs* )
189 01:08:55:12 01:08:57:00 It's not as easy
as it looks.
190 01:08:57:02 01:08:59:07 All right,
just a second.
191 01:09:09:02 01:09:10:02 Uh-huh.
192 01:09:10:04 01:09:12:08 That's pretty good
right there.
193 01:09:13:29 01:09:17:06 CHAPIN:
We actually then took some
measurements on our rocks,

194 01:09:17:08 01:09:20:01 and there were a couple of
things that were going on.
195 01:09:20:03 01:09:22:20 One was to look at surface area,
and to discover
196 01:09:22:22 01:09:27:03 that we could actually come up
with an approximate surface area
197 01:09:27:05 01:09:30:20 for each of these rocks
by covering them with foil
198 01:09:30:22 01:09:33:21 and then laying that foil out
on grid paper
199 01:09:33:23 01:09:35:25 and counting the square units.
200 01:09:35:27 01:09:37:20 MAN:
Make it nice
and flat first.
201 01:09:37:22 01:09:38:29 Teamwork.
202 01:09:48:16 01:09:51:18 This is really rough.
203 01:09:51:20 01:09:54:01 Whoop, there we go.
204 01:09:54:03 01:09:55:01 That's okay.
205 01:09:55:03 01:09:56:03 So, what we got?
206 01:09:56:05 01:09:57:12 So that's
our rough shape.
207 01:09:57:14 01:09:58:29 Let's kind of
darken that in.
208 01:10:00:18 01:10:03:27 CHAPIN:
Why would we want
to use square centimeters
209 01:10:03:29 01:10:06:24 and centimeter paper
for this measurement?
210 01:10:06:26 01:10:08:23 Well, when we were first given
the assignment,
211 01:10:08:25 01:10:10:17 we were asked to estimate
the surface area
212 01:10:10:19 01:10:12:03 and that's what
we estimated it in.
213 01:10:12:05 01:10:14:11 So we figured we would try
the centimeter paper first
214 01:10:14:13 01:10:16:14 to see if we could play it off
our own estimate.
215 01:10:16:16 01:10:17:15 Okay.
216 01:10:17:17 01:10:18:15 And then maybe go
217 01:10:18:17 01:10:19:25 to the smaller paper later.
218 01:10:19:27 01:10:22:07 To get a more accurate
reading of what it is.
219 01:10:22:09 01:10:24:05 CHAPIN:
Now, in using
these squares,
220 01:10:24:07 01:10:26:05 why is it essential
to use squares
221 01:10:26:07 01:10:27:23 for finding
the surface area
222 01:10:27:25 01:10:29:21 rather than just
a linear measure
223 01:10:29:23 01:10:31:11 like "so many
centimeters"?
224 01:10:31:13 01:10:32:16 For area?
225 01:10:32:18 01:10:34:19 Because it's

two-dimensional.
226 01:10:34:21 01:10:36:04 Absolutely.
227 01:10:36:06 01:10:39:29 Having this two-dimensional
shape or area of covering,
228 01:10:40:01 01:10:41:19 we have to use a unit
229 01:10:41:21 01:10:43:05 that's appropriate
for covering.
230 01:10:43:07 01:10:45:24 Right, if we just want
to know how long
the rock was,
231 01:10:45:26 01:10:47:12 then we could use,
like, a ruler.
232 01:10:47:14 01:10:49:09 Exactly.
233 01:10:49:11 01:10:50:09 All right.
234 01:10:50:11 01:10:51:09 All right, let's try.
235 01:10:51:11 01:10:52:08 Here we go.
236 01:10:55:20 01:10:56:19 Oh.
237 01:10:56:21 01:10:57:20 Oh, boy.
238 01:10:57:22 01:10:59:26 Yeah, it's not...
it's a lot less.
239 01:10:59:28 01:11:01:25 We were high
on our estimate.
240 01:11:04:06 01:11:09:01 It'd be close to halfway
between the 600 and 650, so...
241 01:11:09:03 01:11:11:11 about six and a quarter.
242 01:11:11:13 01:11:12:16 So 25...
243 01:11:12:18 01:11:14:29 So it went up about
25 milliliters.
244 01:11:15:01 01:11:17:01 We were way off
on that one.
245 01:11:17:03 01:11:20:14 CHAPIN:
We also then were interested
in finding the volume,
246 01:11:20:16 01:11:23:00 and... because we put
our rocks into water
247 01:11:23:02 01:11:24:18 and used displacement.
248 01:11:24:20 01:11:29:11 Now, displacement allows us to,
by the rising of the water...
249 01:11:29:13 01:11:31:22 or look at the quantity
of the water
250 01:11:31:24 01:11:34:06 that is displaced
when you put the rock in
251 01:11:34:08 01:11:35:19 and then to measure that
252 01:11:35:21 01:11:37:22 and use that as a measure
for volume.
253 01:11:37:24 01:11:39:21 And they were questioning,
254 01:11:39:23 01:11:43:12 if a rock displaced so many
milliliters of water,
255 01:11:43:14 01:11:45:10 how could that be volume?
256 01:11:45:12 01:11:49:12 The interesting fact
is that one milliliter is equal
257 01:11:49:14 01:11:51:23 to one cubic centimeter
of water.
258 01:11:51:25 01:11:53:01 Okay.
259 01:11:53:03 01:11:54:07 Okay, ready?
260 01:11:54:09 01:11:56:11 So now we had estimated...

80 grams.
261 01:11:56:13 01:11:57:11 80 grams?
262 01:11:58:19 01:12:00:09 And 70 is
way too much, okay.
263 01:12:00:11 01:12:01:09 Way too much.
264 01:12:02:29 01:12:04:04 Okay.
265 01:12:04:06 01:12:06:18 That's 60.
266 01:12:06:20 01:12:09:15 We have to...
we have to make sure
that we account
267 01:12:09:17 01:12:11:13 for the two...
two grams.
268 01:12:11:15 01:12:12:13 Mm-hmm-- .2.
269 01:12:12:15 01:12:14:10 .2, that was...
it was...
270 01:12:14:12 01:12:15:12 heavy anyway.
271 01:12:15:14 01:12:17:05 So let's move it up.
272 01:12:22:16 01:12:24:18 All right.
273 01:12:24:20 01:12:25:21 And we take...
274 01:12:25:23 01:12:26:24 Let's just call it 61.
275 01:12:26:26 01:12:27:25 Okay, 61.
276 01:12:27:27 01:12:28:27 That's close enough.
277 01:12:28:29 01:12:30:04 So it's actually 61.
278 01:12:30:06 01:12:31:10 61 grams.
279 01:12:31:12 01:12:33:17 So let's double-check
against this balance.
280 01:12:33:19 01:12:34:19 Right.
281 01:12:34:21 01:12:36:15 CHAPIN:
And then finally I wanted
282 01:12:36:17 01:12:39:22 to introduce the participants
to different kinds of scales
283 01:12:39:24 01:12:42:27 to start talking
about mass versus weight,
284 01:12:42:29 01:12:46:19 and to actually determine
the mass of these rocks
285 01:12:46:21 01:12:48:09 using balance scales.
286 01:12:50:00 01:12:50:28 61.
287 01:12:56:04 01:12:57:12 It's off by
about a gram.
288 01:12:57:14 01:12:59:16 I think if you look
at it head on, it's right on.
289 01:12:59:18 01:13:00:18 Oh.
290 01:12:59:18 01:13:00:18 Yeah.
291 01:13:02:18 01:13:03:27 Right, it's right on.
292 01:13:03:29 01:13:05:01 So it is 61.
293 01:13:05:03 01:13:06:01 Boy, were we off.
294 01:13:06:03 01:13:07:20 I... yeah.
295 01:13:07:22 01:13:10:14 Well, who's got
the biggest rock?
296 01:13:10:16 01:13:13:29 ( *light laughter* )
297 01:13:14:01 01:13:16:28 What did you find out about
the surface area of your rock?
298 01:13:17:00 01:13:18:14 The shape of the rock
made it hard
299 01:13:18:16 01:13:20:00 to... to figure out
an estimate.

300 01:13:20:02 01:13:21:08 It had crevices.
301 01:13:21:10 01:13:23:19 This one dips in,
and each face of it...
302 01:13:23:21 01:13:25:26 We said there were
roughly six faces.
303 01:13:25:28 01:13:28:24 Well, I'm sure there...
there's more than that.
304 01:13:28:26 01:13:31:00 And each face is
shaped differently,
305 01:13:31:02 01:13:32:18 so it makes it
hard to...
306 01:13:32:20 01:13:35:06 to actually fully measure
that with the tinfoil.
307 01:13:35:08 01:13:38:19 You'd almost have to cut one for
each face and fit it on there
308 01:13:38:21 01:13:41:05 to get a really, really
accurate measure of it.
309 01:13:41:07 01:13:42:23 CHAPIN:
Okay.
310 01:13:42:25 01:13:45:02 What if we could, though?
311 01:13:45:04 01:13:48:07 What if we could really
squeeze in there
312 01:13:48:09 01:13:51:06 and get every little surface
covered?
313 01:13:51:08 01:13:57:09 Could we come up with exactly
the surface area of our rock?
314 01:13:57:11 01:13:59:22 We could measure something
with centimeters,
315 01:13:59:24 01:14:02:03 we can measure something
with millimeters,
316 01:14:02:05 01:14:04:18 we can measure something
with a smaller unit,
317 01:14:04:20 01:14:06:02 and... and there's always--
318 01:14:06:04 01:14:07:17 excuse me--
319 01:14:07:19 01:14:09:24 that danger of having
that little bit more
320 01:14:09:26 01:14:11:06 that we're not measuring.
321 01:14:11:08 01:14:14:19 It should get more and more
accurate as you go,
322 01:14:14:21 01:14:17:18 but there's always
that margin for error.
323 01:14:17:20 01:14:18:29 We sometimes call that,
324 01:14:19:01 01:14:21:15 the precision is getting
closer and closer
325 01:14:21:17 01:14:23:29 in terms of what the size
of our unit is.
326 01:14:24:01 01:14:26:04 So if we use very,
very small units,
327 01:14:26:06 01:14:28:07 we can get
a more precise measure.
328 01:14:28:09 01:14:31:10 And then the question
for us all to think about
329 01:14:31:12 01:14:33:20 as we keep going
through this course is,
330 01:14:33:22 01:14:36:07 can we ever get it
all the way down--

331 01:14:36:09 01:14:39:09 precise, precise, precise--
down to an exactness?
332 01:14:39:11 01:14:43:21 And that's a key issue
around measurement.
333 01:14:43:23 01:14:45:27 We were thinking
about the difference
334 01:14:45:29 01:14:48:03 between the...
the level of accuracy
335 01:14:48:05 01:14:49:22 and level of approximation
336 01:14:49:24 01:14:52:25 when we were doing volume
versus the surface area,
337 01:14:52:27 01:14:56:02 and with the volume,
there was just the one reading.
338 01:14:56:04 01:14:58:07 We put the rock in,
and we read it,
339 01:14:58:09 01:15:00:14 and there was
a lot of estimating
340 01:15:00:16 01:15:02:17 in terms of trying to figure out
341 01:15:02:19 01:15:04:28 where in between
those lines it was.
342 01:15:05:00 01:15:07:19 On the other hand,
with the surface area,
343 01:15:07:21 01:15:12:00 there were many more chances
for human error to take a part,
344 01:15:12:02 01:15:16:00 and we felt that with this,
it was just the one reading
345 01:15:16:02 01:15:17:28 where human error could...
346 01:15:18:00 01:15:19:15 or human discretion
could come in.
347 01:15:19:17 01:15:21:08 You bring up
some interesting points,
348 01:15:21:10 01:15:23:29 that there are a number of
places where error can occur.
349 01:15:24:01 01:15:27:24 One, it can be in the accuracy
of our instrument.
350 01:15:27:26 01:15:31:00 Some of our instruments
are very precise
351 01:15:31:02 01:15:34:16 or enable us to have
a more accurate reading.
352 01:15:34:18 01:15:37:19 In terms of the...
the beakers here
353 01:15:37:21 01:15:40:18 are only 50 milliliter
gradations,
354 01:15:40:20 01:15:42:28 and that... it's hard then
355 01:15:43:00 01:15:46:09 to get some, you know,
close estimate.
356 01:15:46:11 01:15:48:22 But then we also have ourselves
to consider,
357 01:15:48:24 01:15:50:17 and that is we take
measurements,
358 01:15:50:19 01:15:53:14 we may be constantly
making error
359 01:15:53:16 01:15:57:12 that is going to contribute
to that as well.
360 01:16:01:19 01:16:04:15 If we quickly review
what we've done so far,

361 01:16:04:17 01:16:07:11 we now know that
measurement is the process
362 01:16:07:13 01:16:09:20 of quantifying properties
of an object
363 01:16:09:22 01:16:11:29 by comparison
with some standard unit.
364 01:16:12:01 01:16:15:18 And we've gone through the
procedures to actually measure.
365 01:16:15:20 01:16:17:24 We have to select an attribute.
366 01:16:17:26 01:16:20:10 We have to choose
an appropriate unit
367 01:16:20:12 01:16:22:19 so we can measure
that attribute,
368 01:16:22:21 01:16:26:00 and then we have to determine
the number of units.
369 01:16:26:02 01:16:28:13 What we're going
to look at next is,
370 01:16:28:15 01:16:31:11 what if we are actually
measuring something,
371 01:16:31:13 01:16:34:25 and we don't have a standard
unit of measure, okay?
372 01:16:34:27 01:16:37:15 Is it still possible to measure?
373 01:16:37:17 01:16:40:02 Can we gather some information?
374 01:16:40:04 01:16:44:12 And to do that, we are going
to use a tangram puzzle.
375 01:16:44:14 01:16:49:07 Now, tangram pieces
are in your container,
376 01:16:49:09 01:16:53:02 and they consist of seven shapes
377 01:16:53:04 01:16:58:07 that actually fit together
to make a square.
378 01:16:58:09 01:17:02:16 And we are going to use
three of these shapes.
379 01:17:02:18 01:17:07:07 There is a medium triangle
and two small triangles...
380 01:17:07:09 01:17:10:21 CHAPIN:
In our third activity,
we actually used tangram pieces
381 01:17:10:23 01:17:12:10 to build different polygons.
382 01:17:12:12 01:17:14:16 And then we evaluated
those polygons
383 01:17:14:18 01:17:17:13 in terms of which polygon
had the greatest area
384 01:17:17:15 01:17:19:21 and which had
the greatest perimeter.
385 01:17:19:23 01:17:21:20 Brought up
some interesting points.
386 01:17:21:22 01:17:24:12 First off, they actually
all have the same area,
387 01:17:24:14 01:17:27:23 because they are constructed
out of the same three shapes.
388 01:17:27:25 01:17:31:22 But it is not always so clear-
cut when you look at the shapes
389 01:17:31:24 01:17:34:01 that all of them
have the same area,
390 01:17:34:03 01:17:35:14 and you have
to reason through that.

391 01:17:35:16 01:17:37:04 If it has
the largest area,
392 01:17:37:06 01:17:40:11 it doesn't necessarily
mean it's the largest
perimeter.
393 01:17:40:13 01:17:41:24 Well, let's...
394 01:17:41:26 01:17:45:15 So what... which...
you know, what is the shape
395 01:17:45:17 01:17:49:10 with the greatest
or the largest area?
396 01:17:49:12 01:17:51:06 WOMAN:
See, I'm thinking
they're all the s...
397 01:17:51:08 01:17:52:28 I have this thought
they're all the same
398 01:17:53:00 01:17:54:22 because we used
the same exact pieces.
399 01:17:54:24 01:17:55:22 Mm-hmm.
400 01:17:55:24 01:17:56:29 But...
401 01:17:57:01 01:17:59:13 And they all take up
the same amount of space.
402 01:17:59:15 01:18:02:20 The area of a triangle
is base times height
divided by two.
403 01:18:02:22 01:18:07:00 Is that going to be
the same as... as
the area of a square...
404 01:18:07:02 01:18:08:26 which is length
times width,
405 01:18:08:28 01:18:11:14 or the rectangle,
length times width?
406 01:18:13:02 01:18:14:26 CHAPIN:
This one looks bigger.
407 01:18:14:28 01:18:16:17 It does look
bigger, but...
408 01:18:16:19 01:18:17:22 WOMAN:
How can it be bigger?
409 01:18:17:24 01:18:18:25 We used the same pieces.
410 01:18:18:27 01:18:20:10 CHAPIN:
Did you overlap
411 01:18:20:12 01:18:21:17 any of the pieces?
412 01:18:21:19 01:18:22:25 WOMAN:
No.
413 01:18:22:27 01:18:24:25 Are all the pieces
used completely
414 01:18:24:27 01:18:26:07 in each of your shapes?
415 01:18:26:09 01:18:27:28 Completely in each
of our shapes.
416 01:18:28:00 01:18:29:20 Yeah, they're all
used completely.
417 01:18:29:22 01:18:31:22 So why wouldn't the area
all be the same?
418 01:18:31:24 01:18:34:05 I'm doing... I'm doing
the seeing-is-believing,
419 01:18:34:07 01:18:36:02 like, if we use
three pieces, then...

| | | | |
|---|---|---|---|
| 420 | 01:18:36:04 | 01:18:37:02 | Right, right. |
| 421 | 01:18:37:04 | 01:18:38:16 | each should be the same. |
| 422 | 01:18:38:18 | 01:18:40:03 | The perimeter would be different |
| 423 | 01:18:40:05 | 01:18:41:08 | because that's the part |
| 424 | 01:18:41:10 | 01:18:43:09 | that you're talking about the longer side |
| 425 | 01:18:43:11 | 01:18:45:16 | versus the shorter side of the medium-size triangle. |
| 426 | 01:18:45:18 | 01:18:46:20 | I would agree with you. |
| 427 | 01:18:46:22 | 01:18:48:03 | I think that makes a lot of sense. |
| 428 | 01:18:48:05 | 01:18:50:02 | We're trying to figure out the area of each |
| 429 | 01:18:50:04 | 01:18:51:27 | to figure out which area's the greatest |
| 430 | 01:18:51:29 | 01:18:53:09 | and which area's the smallest. |
| 431 | 01:18:53:11 | 01:18:54:25 | But it just kind of dawned on me |
| 432 | 01:18:54:27 | 01:18:57:14 | that, wouldn't all the areas have to be exactly the same |
| 433 | 01:18:57:16 | 01:18:59:29 | because we used the exact same shape all three times? |
| 434 | 01:19:00:03 | 01:19:02:07 | So you mean, no matter how we arrange the shapes, |
| 435 | 01:19:02:09 | 01:19:03:22 | it's going to have the same area? |
| 436 | 01:19:03:24 | 01:19:04:29 | Right, I mean, here's our... |
| 437 | 01:19:05:01 | 01:19:06:18 | see if I can make our square again. |
| 438 | 01:19:06:20 | 01:19:08:05 | Of course, I won't be able to now. |
| 439 | 01:19:08:07 | 01:19:09:11 | There's our square. |
| 440 | 01:19:08:07 | 01:19:09:11 | Yeah. |
| 441 | 01:19:09:13 | 01:19:10:22 | So we arrange them like that, |
| 442 | 01:19:10:24 | 01:19:12:04 | and they take up as much space |
| 443 | 01:19:12:06 | 01:19:14:12 | as the red triangle and the two green triangles. |
| 444 | 01:19:14:14 | 01:19:16:02 | If we do this, don't they still take up |
| 445 | 01:19:16:04 | 01:19:17:11 | the exact same amount of space? |
| 446 | 01:19:17:13 | 01:19:18:22 | Same thing. |
| 447 | 01:19:17:13 | 01:19:18:22 | So let's prove it. |
| 448 | 01:19:18:24 | 01:19:19:18 | All right. |
| 449 | 01:19:19:20 | 01:19:20:22 | There's our square, right? |

| | | | |
|---|---|---|---|
| 450 | 01:19:20:24 | 01:19:22:20 | How long did we say this side was, |
| 451 | 01:19:22:22 | 01:19:24:01 | according to our thing? |
| 452 | 01:19:24:03 | 01:19:25:04 | Um, well... |
| 453 | 01:19:25:06 | 01:19:26:18 | According to our measurements, |
| 454 | 01:19:26:20 | 01:19:28:27 | we said it was the square root of two units. |
| 455 | 01:19:28:29 | 01:19:30:02 | How did we get that? |
| 456 | 01:19:30:04 | 01:19:31:23 | This side of the triangle right here... |
| 457 | 01:19:31:25 | 01:19:32:23 | Yeah. |
| 458 | 01:19:32:25 | 01:19:33:29 | This short side is one unit. |
| 459 | 01:19:34:01 | 01:19:35:13 | We'll call it one. |
| 460 | 01:19:35:15 | 01:19:38:14 | And then we both agreed that the triangle is isosceles, |
| 461 | 01:19:38:16 | 01:19:41:06 | so this triangle is also... this side is also. |
| 462 | 01:19:41:08 | 01:19:43:26 | We have a one and a one. |
| 463 | 01:19:43:28 | 01:19:47:17 | Either using Pythagorean Theorem-- |
| 464 | 01:19:47:19 | 01:19:50:01 | one-squared plus one-squared is two. |
| 465 | 01:19:50:03 | 01:19:52:03 | BOTH: And then square-root it. |
| 466 | 01:19:52:05 | 01:19:55:19 | Or just remembering what we remember from geometry-- |
| 467 | 01:19:55:21 | 01:19:58:04 | the hypotenuse of a right triangle, |
| 468 | 01:19:58:06 | 01:20:00:01 | of a 45-45-90 right triangle, |
| 469 | 01:20:00:03 | 01:20:02:24 | is the leg times the square root of two. |
| 470 | 01:20:02:26 | 01:20:04:00 | Okay. |
| 471 | 01:20:04:02 | 01:20:05:15 | One times the square root of two... |
| 472 | 01:20:05:17 | 01:20:06:25 | Which is the square root of two. |
| 473 | 01:20:06:27 | 01:20:08:06 | Is the square root of two. |
| 474 | 01:20:06:27 | 01:20:08:06 | Okay. |
| 475 | 01:20:08:08 | 01:20:09:28 | And it held true for this triangle, too, |
| 476 | 01:20:10:00 | 01:20:11:29 | because we said that this triangle's hypotenuse |
| 477 | 01:20:12:01 | 01:20:14:04 | was the same length as the bigger triangle's leg. |
| 478 | 01:20:14:06 | 01:20:16:04 | Right, so that has to be the square root of two. |
| 479 | 01:20:16:06 | 01:20:17:19 | So they're the square root of two. |

480 01:20:17:21 01:20:19:09 Now, using
our theory again,
481 01:20:19:11 01:20:21:18 the square root
of two-squared is two,
482 01:20:21:20 01:20:23:28 the square root
of two-squared is two.
483 01:20:24:00 01:20:24:28 Two plus two is four.
484 01:20:25:00 01:20:26:04 The square root
of that...
485 01:20:26:06 01:20:28:00 Square-root it to get
the hypotenuse of two.
486 01:20:28:02 01:20:29:11 Okay.
487 01:20:29:13 01:20:31:24 And then we can prove
that by going backwards,
488 01:20:31:26 01:20:34:08 by saying that we said
that this side was one.
489 01:20:34:10 01:20:36:06 We can put two
of the ones together.
490 01:20:36:08 01:20:37:15 So that makes that two.
491 01:20:37:17 01:20:40:05 And make our two and prove
ourselves kind of right
492 01:20:40:07 01:20:41:15 by going back to the beginning.
493 01:20:41:17 01:20:42:15 Okay, all right.
494 01:20:42:17 01:20:43:23 So back to
our parallelogram.
495 01:20:43:25 01:20:45:25 We agree that this
bottom side is two.
496 01:20:45:27 01:20:47:07 The hypotenuse is two.
497 01:20:47:09 01:20:50:04 And parallelogram has
the same formula
as a rectangle--
498 01:20:50:06 01:20:51:08 length...
base times height.
499 01:20:51:10 01:20:52:22 And here's our height.
500 01:20:52:24 01:20:55:11 Our perpendicular
was kind of drawn in
when we traced it.
501 01:20:55:13 01:20:56:16 All right.
502 01:20:56:18 01:20:58:06 It's a leg of
a small triangle.
503 01:20:58:08 01:20:59:06 So that's one.
504 01:20:59:08 01:21:00:08 So two times one.
505 01:21:00:10 01:21:02:27 That's also
two units squared.
506 01:21:02:29 01:21:04:13 Two square units.
507 01:21:04:15 01:21:07:18 A triangle--
one-half base times height.
508 01:21:07:20 01:21:09:20 We can call this the base.
509 01:21:09:22 01:21:10:20 And that's two.
510 01:21:10:22 01:21:11:20 Two units.
511 01:21:11:22 01:21:12:20 Our height.
512 01:21:12:22 01:21:13:22 Two.
513 01:21:13:24 01:21:15:13 Two units--
two times two is four.
514 01:21:15:15 01:21:16:14 Divided by two.
515 01:21:16:16 01:21:18:02 Cut in half--

two square units.

516 01:21:18:04 01:21:19:18 All right.

517 01:21:19:20 01:21:22:10 NARRATOR:
Next, participants
evaluate polygons

518 01:21:22:12 01:21:24:26 to determine which
has the greatest perimeter.

519 01:21:24:28 01:21:27:10 Without the use
of measuring tools,

520 01:21:27:12 01:21:30:08 they begin by choosing
a non-standard unit of measure,

521 01:21:30:10 01:21:31:21 like the side of a tangram.

522 01:21:31:23 01:21:35:05 What did you get
for a square for
the perimeter?

523 01:21:35:07 01:21:38:20 Well, I used... again,
using this as a unit,

524 01:21:38:22 01:21:41:26 I got four units...
four... four of this unit.

525 01:21:41:28 01:21:45:01 So are you using
approximate or
is that exact?

526 01:21:45:03 01:21:46:22 I don't know
if that's exact.

527 01:21:46:24 01:21:48:13 I don't know
if it's exact.

528 01:21:48:15 01:21:50:20 So you're... you're
approximating right now

529 01:21:50:22 01:21:52:04 on the parallelogram.

530 01:21:52:06 01:21:54:10 CHAPIN:
Different groups looked
at different units.

531 01:21:54:12 01:21:56:28 Some used the short side
of the smallest triangle.

532 01:21:57:00 01:22:01:18 Others used the side of the
medium triangle as their unit

533 01:22:01:20 01:22:04:20 and then did comparisons
with nonstandard units.

534 01:22:04:22 01:22:06:00 WOMAN:
And it worked out nicely

535 01:22:06:02 01:22:07:19 because the top one
was three-quarters.

536 01:22:07:21 01:22:10:12 Three-quarters, so
that's a good estimate
for you, okay.

537 01:22:10:14 01:22:13:26 Well, what did we find out
about the area of these shapes?

538 01:22:13:28 01:22:16:27 We found out that...
that they're all the same,

539 01:22:16:29 01:22:19:05 that we're using
all three pieces--

540 01:22:19:07 01:22:21:08 the medium and the two smalls--

541 01:22:21:10 01:22:25:06 and we're just rearranging them
in different configurations,

542 01:22:25:08 01:22:28:15 but that they all take up
the same amount of space.

543 01:22:28:17 01:22:30:02 Great.
544 01:22:30:04 01:22:33:12 Now, I noticed that as we then
also looked at perimeter,
545 01:22:33:14 01:22:36:20 that people had some different
approaches
546 01:22:36:22 01:22:39:00 of ways that they made sense
547 01:22:39:02 01:22:42:13 of which shape had the greatest
or the least perimeter.
548 01:22:42:15 01:22:44:20 Why don't you come on up, Lori?
549 01:22:49:03 01:22:52:03 LORI:
I chose to use a different side
550 01:22:52:05 01:22:53:14 of the triangle--
551 01:22:53:16 01:22:55:23 use the same triangle
but a different side.
552 01:22:55:25 01:22:57:28 In relationship
to the bigger triangle,
553 01:22:58:00 01:23:01:09 we found that the base
was the same length
554 01:23:01:11 01:23:04:06 as two of the sides
of this triangle.
555 01:23:04:08 01:23:07:11 So I chose to use this
as a unit of one.
556 01:23:07:13 01:23:10:03 And I'm going
to use the trapezoid.
557 01:23:10:05 01:23:12:19 And because we were only asked
558 01:23:12:21 01:23:15:26 to order them
from least to greatest
559 01:23:15:28 01:23:19:13 without finding out
the exact measurement,
560 01:23:19:15 01:23:22:29 I... I felt that this would be
an accurate way to do it.
561 01:23:23:01 01:23:30:01 We held it up and marked off
that this would be one,
562 01:23:30:03 01:23:33:03 and just moved it over,
563 01:23:33:05 01:23:37:12 and that this would be one.
564 01:23:37:14 01:23:41:25 And that this right here was
about... it's less than half,
565 01:23:41:27 01:23:43:25 so we called it a quarter.
566 01:23:43:27 01:23:48:01 So this would be one, one,
and one-quarter
567 01:23:48:03 01:23:50:21 of our... our unit of one.
568 01:23:50:23 01:23:53:20 This would be equal to one...
569 01:23:53:22 01:23:57:09 and this also
would be equal to one.
570 01:23:57:11 01:23:58:26 When we got up here,
571 01:23:58:28 01:24:02:10 we found that this was more
than half, less than one,
572 01:24:02:12 01:24:04:10 so we called it three-quarters.
573 01:24:04:12 01:24:07:19 And how nice that we have
the one-quarter down here.
574 01:24:07:21 01:24:09:07 ( *laughter* )
575 01:24:09:09 01:24:11:06 Worked out well.
576 01:24:11:08 01:24:14:04 So when we add it up, it's one,
two, three, four, five.
577 01:24:14:06 01:24:15:26 So we found that this was five.
578 01:24:15:28 01:24:17:10 And some of the other...

579 01:24:17:12 01:24:19:22 we found that these
also equaled five,
580 01:24:19:24 01:24:23:00 and that this one...
this one was four and a half,
581 01:24:23:02 01:24:27:08 using that same... same method
of measuring and marking off.
582 01:24:27:10 01:24:29:16 CHAPIN:
Great. Thank you.
583 01:24:29:18 01:24:31:10 Let's recap what
we've gone over today.
584 01:24:31:12 01:24:34:06 First, we have thought carefully
about, What is a measurement?
585 01:24:34:08 01:24:37:11 That it is the process
of quantifying properties
586 01:24:37:13 01:24:41:08 an... of an object by comparison
with a standard unit.
587 01:24:41:10 01:24:46:27 We've looked at the procedures
that are necessary to measure.
588 01:24:46:29 01:24:50:10 And we've started
to think about the difference
589 01:24:50:12 01:24:52:05 between precision and accuracy.
590 01:24:52:07 01:24:54:25 How do we get measures
that are more precise,
591 01:24:54:27 01:24:58:10 and what affects the accuracy
of our measurements?
592 01:24:58:12 01:25:01:22 So we will continue
to investigate these things
593 01:25:01:24 01:25:04:09 in future sessions this week.
594 01:25:07:00 01:25:12:29 Captioned by
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