

# **Workshop 7**

## **Direct and Inverse Variation**

# Overview

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## Description

This workshop presents two activities through which teachers can give students experience in studying natural events by using simulations. Students simulate oil spills on land and water, and develop mathematical models of these phenomena. Through these models, they investigate direct and inverse variations.

- Part I: Peggy Lynn and her students simulate oil spills on land. The ratio of the number of drops of oil to the resulting surface area of the spill is used to introduce direct variation.
- Part II: Peggy Lynn and her students simulate oil spills in water. Students investigate the relationship between the volume of cylinders and the area of the base. The concept of inverse variation is developed in this context.

## Featured Textbook

- "Oil: Black Gold," in *SIMMS Integrated Mathematics: A Modeling Approach Using Technology; Level 1, Volume 2*; pp. 157-172. Simon & Schuster Custom Publishing, 1996.

## Featured Educator

- Peggy Lynn, West Yellowstone High School; West Yellowstone, Montana

## Featured Commentator

- Beatrice Moore-Harris, Associate Mathematics Manager, Project Grad Houston; Houston, Texas

## Learning Objectives

In these activities, you will learn how to help students:

- Develop and graph direct and inverse proportions.
- Develop mathematical models of real-world events.
- Use mathematical models to make predictions about data sets.

# Workshop Session (On-Site)

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## Part I: Direct Variation

### Getting Ready (15 minutes)

Write down everything you know about direct variation. Identify situations that are modeled by direct variation functions and share this information with your group.

### Watching the Video (30 minutes)

Watch Part I: Direct Variation.

### Going Further (15 minutes)

Select two or three of the questions listed below for discussion during the session. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- What strategies from the video might you incorporate in your classroom? Discuss how you might incorporate them.
- What should a teacher be looking and listening for while students are working in groups? Discuss the order in which Peggy selected students to present their work.
- How did Peggy capitalize on the student group that used a linear function that wasn't a direct variation as a model? Discuss how she modified the lesson as a result of their work.
- In a direct variation, multiplying the  $x$ -coordinate by a scale factor increases the  $y$ -coordinate by the same scale factor. Sometimes students assume that this is true for all linear functions. How did Peggy help students deal with this misconception?
- What are some possible next steps to this lesson? How might you move beyond the lesson in the video?

## Part II: Inverse Variation

### Getting Ready (15 minutes)

- List several examples of problem situations that could be modeled by direct variation functions and inverse variation functions.
- Divide into small groups, share your examples, and collectively produce more examples.

### Watching the Video (30 minutes)

Watch Part II: Inverse Variation.

### Going Further (15 minutes)

Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- Discuss whether or not it is worthwhile to have "messy numbers" when using real data in the teaching of mathematics.
- Discuss the questioning techniques Peggy used in this workshop. How do they enhance student learning?
- Discuss how the teacher introduces vocabulary words and formulas at the end of the lesson rather than the beginning. Why does she do this? What are the advantages of each method?
- What are some possible next steps to this lesson? How might you move beyond the lesson in the video with your students?
- Design another experiment for students that would be modeled by an inverse variation function.

# Between Sessions (On Your Own)

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## Homework Assignment

Read Peggy Lynn's thoughts and reflections in the Video Teacher Reflection section and discuss them on Channel-Talk. The reflections consist of Peggy's responses to a set of questions generated by mathematics educators who had viewed her video lessons. She wrote these responses after viewing the video herself.

## Ongoing Activities

You may want to carry on these activities throughout the course of the workshop.

### Keep a Journal

Read the Teaching Strategies for Workshop 7 and answer the journal prompts. Include thoughts, questions, and discoveries from the workshop itself and learning experiences that take place in your own classroom. You are encouraged to use the online journaling tool at [www.learner.org/channel/workshops/algebra](http://www.learner.org/channel/workshops/algebra).

### Web Site: [www.learner.org/channel/workshops/algebra](http://www.learner.org/channel/workshops/algebra)

Look at the Resources section for more ideas on questioning, lesson planning, and ways to teach the concepts of direct variation and inverse variation.

### Share Ideas on [Channel-Talkinsights@learner.org](mailto:Channel-Talkinsights@learner.org)

Share your thoughts about how you can improve your questioning techniques and lesson planning.

# Video Teacher Reflections

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## **Peggy Lynn**

Below are Peggy Lynn's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

### ***What are your thoughts after watching the video?***

Observing my class as an outsider looking in was very educational for me. My first response was "I love these activities!" It reaffirmed my dedication to using context to teach mathematics—in other words, it works! And the materials for these activities are easy to find, easy to set up, and easy to clean up. It was fun to watch how comfortable my students were with expressing their ideas and using hand tools (graphing scatterplots), as well as technology, to explore their results. I also feel using their own data helps them to realize that not every scatterplot is perfect, and that not every slope is an integer.

There would be more guided practice and independent practice (class examples and homework) in a normal class period than appeared on the video. I would also like to emphasize the use of a "warm-up" before every class to review the previous lesson and prerequisite skills needed for the lesson that day.

### ***Discuss your approach to questioning in each lesson.***

I think it is critical for students to explain their thinking—for correct and incorrect responses. My job is to guide them through the process as they construct the mathematical concepts desired. Asking how and why, not just what, during discussions is helpful to get students to communicate their thoughts. When a student answers a question incorrectly, I may suggest the question that their response would have answered correctly instead of just replying, "Wrong." It takes time to get students to be comfortable with making educated guesses—to be risk takers. There is a lot of power in figuring it out for yourself.

### ***Are there places where you wish you could have changed a question or a follow-up question?***

During the summary of both lessons, instead of just writing the ideas on the board, I wish I had allowed the students to contribute the key concepts.

### ***How well does this lesson follow the lesson plan that you wrote for it?***

I have used these lessons several times, so there are few surprises at this point. I do modify the chart for gathering data from what the textbook suggests so it lends itself better to the discussion I wish to have happen.

### ***Where did things go differently than you expected?***

- The question about whether or not to include  $(0,0)$  when finding the mean ratio of area/volume (drops) from the particular group that asked was unexpected. But it gave me the opportunity to discuss (briefly) division by zero.
- Including a  $y$ -intercept other than zero in their best-fit line created a great opportunity to compare general equations of lines to the special subset of direct proportions. And their attention was better since it was "their equation" rather than one I made up.

### ***What have you done prior to this lesson to prepare the students to engage in this type of activity?***

At the beginning of the year I read through the directions for an activity with them and discussed in detail the expectations for the activity. I also reminded them it is much like doing a lab in science. So by now my students have a lot of experience working in groups, reading directions for an exploration, and being expected to explain their reasoning. We also spent time in other modules doing a lot of graphing, by hand and using technology. Areas and volumes, and converting metric units, have also been previously studied.

### ***What are some other examples of how you collaborate with the science teacher?***

The biggest collaboration the science teacher and I make has to do with using the same vocabulary—or relating the vocabulary commonly used in the math room with the vocabulary commonly used in the science room. We also use some of the same contexts. In biology, she uses Punnett squares to study genetics. In math, I start with Punnett squares and expand to using tree diagrams to study the probability inherent in genetics. In physics, vectors are used to describe forces and speed. In math, when studying triangle trigonometry, I use forces and directional speeds as applications. And also in physics, a lot of formulas are needed. I use physics formulas when practicing solving an equation for a particular variable, and when working with algebraic substitution.

# Notes

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