

Workshop 2

Linear Functions and Inequalities

Overview

Description

This workshop presents two activities through which teachers can help students understand linear functions and solve linear equations and inequalities studied in Algebra 1. The activities use a variety of techniques to help students understand important mathematical concepts.

- Part I: Tom Reardon uses the Phone Bill Problem, gleaned from his own experience, to help his students deepen their understanding of linear functions and their application.
- Part II: Janel Green uses a hot dog-selling scheme as a vehicle to help her students learn how to solve linear equations and inequalities using three different methods: tables, graphs, and algebra.

Featured Textbooks

- Textual materials written by Tom Reardon
- Textual materials written by Janel Green

Featured Educators

- Tom Reardon, Austintown Fitch High School; Austintown, Ohio
- Janel Green, Walt Whitman High School; South Huntington, New York

Featured Commentators

- Diane Briars, Pittsburgh Public Schools; Pittsburgh, Pennsylvania
- Frances R. Curcio, Professor of Mathematics Education; Queens College of the City University of New York

Learning Objectives

In these activities, you will learn ways that students can:

- Analyze a data set to find patterns.
- Plot the data set on a graph with paper and pencil and with a graphing calculator.
- Find an equation that contains the data set.
- Understand the meaning of the slope and y -intercept in that equation.
- Use the equation to predict other values in the problem situation.
- Solve linear equations numerically (using a table), graphically, and algebraically.
- Solve linear inequalities numerically, graphically, and algebraically.

Workshop Session (On-Site)

Part I: Linear Functions

Getting Ready (15 minutes)

Look at a copy of the phone bill that Tom Reardon uses in the lesson you will watch (see Appendix). Predict how Tom will use the bill as a vehicle to help his students deepen their understanding of linear functions. What are some of the activities and questions you think he will introduce in his class?

Watching the Video (30 minutes)

Watch Part I: Linear Functions.

Going Further (15 minutes)

Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- Discuss Tom's strategy of giving students only a short period of time to do a task or come up with an answer to question. What are the benefits and drawbacks of this approach?
- What evidence did you see that Tom's students understood the connections between the multiple representations in this lesson?
- Discuss how Tom used the SMART Board as a pedagogical tool. Would you use it differently, and if so, how? In what ways could you achieve the same goals without a SMART Board?
- Do you think that Tom's students understood the fact that they could choose any two of the points in the data set and arrive at the same equation? What specific evidence supports your position?
- Discuss all of the mathematical ideas and content standards from your school or state that were addressed in this lesson. Generate some examples of other tasks you might use to address the same ideas and standards.

Part II: Solving Linear Equations and Inequalities

Getting Ready (15 minutes)

Janel's lesson is based on the profit equation $P = 0.5H - 450$, where P represents the profit, and H the number of hot dogs sold. How many hot dogs must be sold to produce a profit of at least \$100? Solve this problem using tables, graphs, and equations.

Watching the Video (30 minutes)

Watch Part II: Solving Linear Equations and Inequalities.

Workshop Session (On-Site), cont'd.

Going Further (15 minutes)

Select two or three of the questions listed below for discussion. You may want to discuss the others on Channel-Talk or reflect on them in your online journal.

- Janel said that she took a chance by letting students come up with the questions concerning the hot dog-selling scheme to begin the lesson. Discuss this strategy.
- Discuss Janel's strategy of having students write their thoughts on the board for all to see. What are some pros and cons of this strategy?
- Talk about Janel's use of technology in this lesson. What did you like? What might you do differently, and why?
- Janel used a seemingly simple problem as a vehicle to help her students learn how to solve equations and inequalities using tables, graphs, and algebra. What other concepts might benefit from a similar approach?
- Discuss the importance of having students learn to solve equations and inequalities using tables, graphs, *and* algebra. Share some ways that you help your students become proficient with multiple representations.
- Janel worked through the three ways to solve an equation with the entire class. Then she asked them to solve inequalities working in groups. In what ways were the students able to connect the idea of solving equations to solving inequalities? What steps do you think Janel should take to follow up on solving inequalities?
- Discuss the student presentations. Assess student learning based on what you saw in the video. Based on your assessment, what steps would you take to ensure that each student understands the concepts presented?

Between Sessions (On Your Own)

Homework Assignment

If possible, try one or both of the activities from the Workshop 2 video in your classroom and reflect in your journal about your students' understanding of the concept. As an alternative, modify an existing lesson plan on linear equations or solving linear equations and inequalities so that it incorporates the use of tables, graphs, and algebra. Write your goals for this activity in your journal.

Ongoing Activities

You may want to carry on these activities throughout the course of the workshop:

Keep a Journal

Read the Teaching Strategies for Workshop 2 and answer the journal prompts. Include thoughts, questions, and discoveries from the workshop itself and learning experiences that take place in your own classroom. You are encouraged to use the online journaling tool at www.learner.org/channel/workshops/algebra.

Web Site: www.learner.org/channel/workshops/algebra

To help students better understand mathematical concepts, investigate the Resources section to deepen your understanding of worthwhile mathematical tasks and the appropriate use of technology.

Share Ideas on Channel-Talkinsights@learner.org

Share your ideas about how the appropriate use of technology and the selection of worthwhile tasks helps students better understand mathematical concepts.

Video Teacher Reflections



Tom Reardon

Below are Tom Reardon's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

When watching Workshop 2, Part 1, what did you notice about your teaching strategies and student thinking?

It came across that I encourage good "mathematical" communication among my students. When I called on Dan as the "reporter" for his group, it was clear that Krystina was the one who did most of the thinking, but I tried to get Dan to explain what he learned from her. I thought this would be a good lesson for him.

I have always thought that my questioning techniques were a strength, and I think that came across here. I liked when I asked, "How would you convince me...?"

I was very impressed with the students' communication, and their thinking and reasoning. They provided good explanations to each other. They engaged themselves with the problem well. They made me very proud. They responded much better than they had ever done before. They showed that they could do and understand this level of mathematics, and sometimes they even looked like they were enjoying themselves doing so.

What were these students' expectations of themselves as mathematics learners at the beginning of the year? Did that change during the year?

At the beginning of the school year, these students had very low expectations of doing anything mathematically. And unfortunately, they were okay with that—even to the point of being proud of it. Many times they wouldn't even TRY to do the mathematics. This is a very basic algebra class. It has the lowest-achieving math students in our high school. Their motivation is low, and they have very poor basic math skills. At the beginning of the year, I couldn't get them to complete assignments or even take notes. Even if I told them I would collect their notes and grade them, several would still not turn in anything. It was very frustrating and disheartening. I kept thinking, "Where are these students going to fit into society with such little work ethic and low self-esteem?"

It took several months for me to win them over. I think the fact that they were asked to do this video made them feel very special, and they responded

extremely well. They had been holding back on me! They raised their level of expectation for themselves, and they responded very well. They surprised me. (When my wife watched this video, she said that these students didn't look like the students that I described to her—they didn't look like the lowest-achieving math students in the school, and that is a huge compliment to them.)

How is this class different from the other classes you teach? What are some things you enjoy about working with students at the basic algebra level, and with this group in particular?

I also teach Intermediate Algebra and Geometry, which is a pre-Algebra 2 course that I developed and for which I wrote the textbook, and I teach pre-calculus and AP Calculus. Those classes have the best math students in our school: highly self-motivated and with positive self-esteem.

When teaching basic algebra [to] this group, it is extremely satisfying to me when I/we have success—when I reach them, when what we do makes a difference, when they see a use for mathematics in the "real world," when they say, "That was cool how that worked out" or "I liked what we did in math class today." Those are special moments that we cherish.

One of the teaching strategies this workshop will highlight is "worthwhile tasks." Which tasks did you feel were most worthwhile for the students, and why?

The group interactions are very crucial to the success of the lessons in my class. Keeping them communicating about what the problem means and how to solve it makes the lesson appear to be more worthwhile to them.

I am also very big on getting my students to verify their solutions. I strongly emphasize that students know how to know when their answer is right, or at least reasonable. I like to tell them that in real life there is no "back of the book" where they can look up the correct answers.

Video Teacher Reflections, cont'd.

How did the technology you have in your classroom make a difference in the students' learning during this lesson?

I have been a big fan of APPROPRIATE use of technology for several years now. It is awesome.

The SMART Board is such a great teaching tool because:

1. It gets the students' attention.
2. The use of color and highlighting may seem subtle, but it is very effective, especially for the visual learner.
3. The virtual graphing calculator is an awesome teaching and learning tool. Students can see what buttons to push and see the immediate response of the calculator. I love how I can capture screen shots and place them in the electronic notes for reference later.
4. It did not come out on the video, but it would be great if somehow you could include how I am able to place my actual daily class notes on my Web site for students and parents to view and/or print out—especially when the student is absent.

The graphing calculators that we used are essential to the success of this lesson. Plotting the five points by hand would have been tricky. Graphing the equation $y = .24x + .85$ also would have been tricky by hand. We can do more real-life problems with nasty real-life numbers by utilizing graphing technology. The graphing calculator is a TOOL—it is not the goal or focus of the lesson.

Discuss the decisions you made while preparing and teaching this lesson. Address Diane Briars's question about introducing the term "step function."

My goals for this lesson were to have students understand what "slope" and "y-intercept" mean and how they can be used to model real-world data. I wanted students to see multiple representations: in words, tables, graphs, and equations. I wanted to reinforce solving linear equations.

Diane is correct in pointing out that this data is more realistically and accurately modeled with a step function because the data is discrete and not continuous. I didn't address that today because I really thought that introducing that would take away from the lesson. The lesson has enough good mathematical stuff in it already; I don't want to confuse the issue. I would not have been able to achieve my goals had I introduced the step function today.

If you were to teach this lesson again to a basic algebra class, what would you do differently?

I would use more praise phrases when the students give correct responses. I don't think that I do that often enough. I also need to have the students explain their answers more and not have me repeat what they say or [tell them] what I think they should have said.

Video Teacher Reflections, cont'd.



Janel Green

Below are Janel Green's responses to some of the comments and questions raised by other mathematics educators after they viewed the workshop video:

When watching Workshop 2, Part II, what did you notice about your teaching strategies and student thinking?

I took a chance and asked students to come up with their own questions. I chose to do this so that the students would feel more connected to the problem and appreciate the solutions more.

What mathematics background did your students have before this class? What are some things that you will do over the course of the school year to build their confidence?

Prior to this lesson, students had been exposed to graphing equations and writing inequalities. They had very little experience with the graphing calculator. This class has students at different levels, and, therefore, they came into it with different attitudes toward mathematics. I encourage all students to participate and try to make them feel comfortable about mathematics by relating mathematics to real-life situations.

What tasks did you feel were most worthwhile for the students, and why?

The writing assignment was a worthwhile task for two reasons. First, the students were given the opportunity to write problems that were meaningful to them. As a result, they were given a chance to direct part of the lesson. This gave them power and responsibility for their own learning. Second, the writing assignment was beneficial because it gave students time to think about what they wanted to say. Some students take longer than others to come up with ideas and writing allows all students to do so.

Another task that was worthwhile was part of cooperative learning. Cooperative learning was beneficial in this lesson because students saw the beauty in mathematics. Each member of each cooperative learning group had his or her own task. One student was in charge of graphing, another was in charge of an algebraic solution, and one was in charge of making a table. Together, they saw how one problem could be solved three different ways. They discovered math power together and on their own.

A final task that was worthwhile was the presentation. Presentations allow students to summarize what they have learned.

Do you agree with Dr. Curcio that was a good idea to introduce the idea of multiple representations in a familiar and relatively simple problem context?

I do agree with Dr. Curcio. The object of the lesson was to connect graphing, algebraic solutions, and making a table to solve problems. In order to meet the objective of this lesson, the problem presented had to be relatively simple. Otherwise, students might have gotten "hung up" on the problem and lost sight of the goal of connecting the three methods together.

By the end of the lesson, the students seem to have a clear understanding of how to solve an equality using the table, graph, and symbolic manipulation. What about their understanding of the inequality? What would be your next steps to solidify that understanding?

In a later lesson, I will introduce inequalities formally and show students how to graph and solve them.

If you were to teach this lesson again, what would you do differently? Why?

I would probably have a lesson on how to use the graphing calculator prior to this one, so that I wouldn't have to take time during this lesson to teach the calculator and so that students would have a better understanding of the calculator's "windows" function.