Session 10

Classroom Case Studies, Grades K–2

This is the final session of the *Geometry* course! In this session, we will examine how geometry as a problem-solving process might look when applied to situations in your own classroom. This session is customized for three grade levels. Select the grade level most relevant to your teaching.

The session for grades K–2 begins below. Watch video program 10, for K–5 teachers, in this session. Go to page 229 for grades 3–5 and page 247 for grades 6–8.

Key Terms for This Session

Previously Introduced

- concave polygon
- congruent
- irregular polygon
- polygon
- quadrilateral
- rectangle
- regular polygon
- similar
- square
- trapezoid
- vertex

New in This Session

- van Hiele levels

Introduction

In the previous sessions, we explored geometry as a problem-solving process. You put yourself in the position of a mathematics learner, both to analyze your individual approach to solving problems and to get some insights into your own conception of geometric reasoning. It may have been difficult to separate your thinking as a mathematics learner from your thinking as a mathematics teacher. Not surprisingly, this is often the case! In this session, however, we will shift the focus to your own classroom and to the approaches your students might take to mathematical tasks involving geometry.

As in other sessions, you will be prompted to view short video segments throughout the session; you may also choose to watch the full-length video for this session. **[See Note 1]**

Learning Objectives

In this session, you will do the following:

- Explore the development of geometric reasoning at your grade level, including the van Hiele model of geometric learning
- Review mathematical tasks and their connection to the mathematical themes in the course
- Examine children's understanding of geometric concepts

Note 1. This session uses classroom case studies to examine how children in grades K-2 think about and work with geometry. If possible, work on this session with another teacher or a group of teachers. A group discussion will allow you to use your own classroom and the classrooms of fellow teachers as case studies to make additional observations.

Part A: Geometry as a Problem-Solving Process (25 min.)

The study of geometry can include both problem solving and connections to other areas of mathematics (arithmetic, algebra, etc.). Too often, classrooms focus almost exclusively on correctly identifying shapes and their properties by name. While mathematical language and clear communication are important in geometry, it is important to include other kinds of geometric problems as well so that geometry isn't reduced to mere nomenclature. **[See Note 2]**

When viewing the video segment, keep the following questions in mind:

a. How does the teacher incorporate geometric language into the lesson without making it the focus of the lesson? What is the purpose of having students describe the shapes rather than name them?

b. Where in the lesson are students learning new geometric content? What is that content?

c. Where in the lesson are students solving problems and thinking mathematically? How does the problem solving relate to the geometric content?

d. Thinking back to the big ideas of this course, what are some geometric ideas these students are likely to encounter through their investigation of this situation?

Video Segment (approximate time: 22:32-25:35): You can find this segment on the session video approximately 22 minutes and 32 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, second-grade students in Ms. Christiansen's class are working to describe different figures so that their fellow students can draw them just from the description. They use vocabulary they know to describe figures, learning names for new shapes as they work.

Problem A1. Answer the questions you reflected on as you watched the video:

a. How does the teacher incorporate geometric language into the lesson without making it the focus of the lesson? What is the purpose of having students describe the shapes rather than name them?

b. Where in the lesson are students learning new geometric content? What is that content?

c. Where in the lesson are students solving problems and thinking mathematically? How does the problem solving relate to the geometric content?

d. Thinking back to the big ideas of this course, what are some geometric ideas these students are likely to encounter through their investigation of this situation?

Problem A2. This lesson is not couched in a "real-world context." Students are sorting shapes and thinking about mathematical ideas in the abstract. What are the advantages and disadvantages of this kind of lesson? Are "mathematics only" lessons important in your classroom? What purpose do they, as opposed to contextualized lessons, serve? **[See Note 3]**

Note 2. Before examining specific problems at this grade level, you will watch with an eye toward geometric problem solving a teacher in her classroom. The purpose in viewing the video is not to reflect on the teacher's methods or teaching style, but to watch closely the way she brings out geometric ideas while engaging her students in a problem-solving task. Think about how the task meets students at their current level of geometric sophistication and also helps move them to the next level.

Note 3. This is a particularly good discussion to have with your colleagues. Everyone has different opinions and thoughts about the use of context in the mathematics classroom. Spend some time talking about not just what you think, but why you think it. Cite examples from your own experience instead of focusing on what you have heard others say.

Part A, cont'd.

Join the Discussion **www.learner.org**

Post your answer to Problem A2 on an email discussion list; then read and respond to answers posted by others. Go to the *Geometry* Web site at www.learner.org/learningmath and find Channel Talk.

Problem A3. Ms. Christiansen's lesson is similar to one from Session 1, Part B of this course in which you had to build designs from pattern blocks based on descriptions of those designs. Discuss the ways in which Ms. Christiansen's lesson was similar to and different from the one in this course. What makes this more appropriate for second-grade students?

Part B: Developing Geometric Reasoning (40 min.)

Introducing van Hiele Levels

The National Council of Teachers of Mathematics (NCTM, 2000) identifies geometry as a strand in its *Principles and Standards for School Mathematics*.* In grades pre-K through 12, instructional programs should enable all students to do the following:

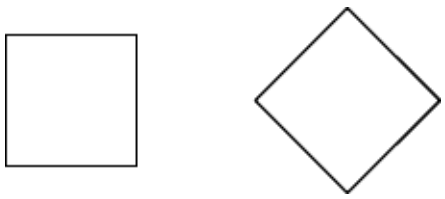
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Apply transformations and use symmetry to analyze mathematical situations
- Use visualization, spatial reasoning, and geometric modeling to solve problems

In grades pre-K-2 classrooms, students are expected to do the following:

- Recognize, name, build, draw, compare, and sort two- and three-dimensional shapes
- Describe attributes and parts of two- and three-dimensional shapes
- Investigate and predict the results of putting together and taking apart two- and three-dimensional shapes
- Recognize and apply slides, flips, and turns
- Recognize and create shapes that have symmetry
- Create mental images of geometric shapes using spatial memory and spatial visualization
- Recognize and represent shapes from different perspectives

Dutch educators Pierre van Hiele and Dina van Hiele-Geldof developed a theory of five levels of geometric thought. It is just a theory, but a useful one for thinking about activities that are appropriate for your students and prepare them to move to the next level, and for designing activities for students who may be at different levels.

Level 0: Visualization. The objects of thought at level 0 are shapes and what they look like. Students have an overall impression of the visual characteristics of a shape, but are not explicit in their thinking. The appearance of the shape is what's important. Students may think that a rotated square is a "diamond" and not a "square" because it looks different from their visual image of square. *(Early elementary school and, for some, late elementary school)*



* *Principles and Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 2000). Standards on Geometry: Grades K-2, 41, 96. Reproduced with permission from the publisher.

Part B, cont'd.

Level 1: Analysis. The objects of thought here are "classes" of shapes rather than individual shapes. Students are able to think about, for example, what makes a rectangle a rectangle. What are the defining characteristics? They can separate that from irrelevant information like the size and the orientation. They begin to understand that if a shape belongs to a class like "square," it has all the properties of that class (perpendicular diagonals, congruent sides, right angles, lines of symmetry, etc.). *(Late elementary school and, for some, middle school)*

Level 2: Informal Deduction. The objects of thought here are the properties of shapes. Students begin "if-then" thinking; for example, "If it's a rectangle, then it has all right angles." Students can begin to think about the minimum information necessary to define figures; for example, a quadrilateral with four congruent sides and one right angle must be a square. Observations go beyond the properties into mathematical arguments about the properties. Students can engage in an intuitive level of "proof." *(Middle school and, for some, high school)*

Level 3: Deduction. The objects of thought here are the relationships among properties of geometric objects. Students can explore relationships, produce conjectures, and start to decide if the conjectures are true. The structure of axioms, definitions, theorems, etc., begins to develop. Students are able to work with abstract statements and draw conclusions based more on logic than intuition. *(This is the goal of most 10th-grade geometry courses, but many students are not developmentally ready for it.)*

Level 4: Rigor. The objects of thought are deductive axiomatic systems for geometry. For example, students may compare and contrast different axiomatic systems in geometry that produce our familiar Euclidean plane geometry, finite geometries, the geometry on the surface of a sphere, etc. **[See Note 4]**

For more information on the van Hiele levels and how to work with students within each level, read the article "Geometric Thinking and Geometric Concepts" by John A. Van de Walle from *Elementary and Middle School Mathematics*. This reading is available as downloadable PDF files on the *Geometry* Web site. Go to www.learner.org/learningmath.

> Van de Walle, John A. (2001). Geometric Thinking and Geometric Concepts. In *Elementary and Middle School Mathematics: Teaching Developmentally, 4th ed.* (pp. 342-349). Boston: Allyn & Bacon.

Analyzing With van Hiele Levels

In this course, we have primarily worked across levels 2-4. You may feel that the activities we've done are not appropriate for the level of your students, and you're probably right. The goal for this session is for you to think about problems and activities that are at your students' level, and how to help them prepare for the next level of thinking.

> Students in pre-K-2 generally fall at level 0 (visualization). This level describes students who reason about shapes primarily on the basis of visual considerations of the whole without explicit regard to the properties of the components (Burger and Shaughnessy, 1986). One goal of the schooling of these students is to move them to level 1 (analysis), where they can informally analyze component parts and attributes. In the primary grades, students build the foundation for understanding shapes, both two- and three-dimensional. They learn what shapes look like, the features that distinguish shapes from one another, and ways to describe shapes.
>
> *Navigating Through Geometry in Prekindergarten-Grade 2*, p. 9*
> Reston, VA: National Council of Teachers of Mathematics, 2001

* Findell, Carol R.; Small, Marian; Cavanagh, Mary; Dacey, Linda; Greenes, Carole E.; and Sheffield, Linda Jensen. *Navigating Through Geometry in Prekindergarten-Grade 2*. (Reston, VA: National Council of Teachers of Mathematics, 2001). Reproduced with permission from the publisher.

Note 4. If you are working with a group of colleagues, take some time to discuss your own students. Where in the van Hiele levels do you see them functioning comfortably? (There will be a range, of course, because not all students are the same.) Try to cite evidence from you classrooms: With which tasks do students find success? With which tasks do they struggle?

Part B, cont'd.

Video Segment (approximate time: 22:32-25:35): You can find this segment on the session video approximately 22 minutes and 32 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

Watch this clip from Ms. Christiansen's class again, and think about how both the lesson and the teacher are encouraging students to move to that next level of geometric reasoning. **[See Note 5]**

Problem B1. Where in the video do you see evidence of the following?

- (Level 0 thinking) Students thinking about particular shapes and not their properties
- (Level 1 thinking) Students thinking about classes of shapes rather than the individual shapes. Do students seem concerned with orientation or size of the figures?

Problem B2. In Session 3, Part A, you worked on the problem of finding hidden polygons. Recall your own experience in this activity as an adult mathematics learner. During the activity, when did you have to use level 1 thinking? (How did you think about properties of figures to help you find them?)

Problem B3.

a. What do you think were the key pieces of geometry content in this activity? What knowledge did you learn, solidify, or connect with better?

b. What do you think were the key thinking and reasoning skills in this activity? How did the reasoning and geometric content tie together?

Problem B4. Now think about K–2 students and how this hidden polygon activity might work with them. What must students know and be comfortable with to get the most out of this activity? What are potential stumbling blocks for them?

Join the Discussion **www.learner.org**

Post your answer to Problem B4 on an email discussion list; then read and respond to answers posted by others. Go to the *Geometry* Web site at www.learner.org/learningmath and find Channel Talk.

Problem B5. What might students misunderstand or find confusing in the lesson? How could you alter the lesson or prepare them beforehand to help avoid these misunderstandings?

Note 5. Again, remember that the focus of the video case study is not to examine teaching practice, but to focus on the students and their thought processes.

Part C: Activities That Illustrate Geometric Reasoning (55 min.)

Cutting Corners Activity

In this part, you'll look at some activities designed for students in grades K-2. As you read each activity, answer these questions:

a. What is the geometry content in this activity?

b. What skills do students need to work through this activity? What skills will this activity help them develop for later work?

c. What level of geometric thinking is expected of students in the activity? Does it ask students to bridge levels?

d. What other questions might extend students' thinking about the activity?

e. Describe a lesson that you could develop based on the content of this activity. **[See Note 6]**

Activity Summary

Students explore ways to cut rectangles to make other shapes, including smaller rectangles, triangles, and shapes of the same size.

Materials Needed:

- one pair of scissors for each student
- six sheets of 8 1/2" x 11" paper
- paste
- six sheets of 12" x 15" colored construction paper for each student
- a straightedge for each group of students
- a pencil for each student

Begin Activity

Hold up a rectangular piece of paper and ask students to identify the shape. Also ask questions such as, "How many sides does a rectangle have? How many vertices does a rectangle have? What is special about the angles of a rectangle?" As students answer, point out the sides and vertices to reinforce the vocabulary and review right angles.

Ask, "How can I cut this rectangle to get two smaller rectangles whose shape and size are the same?" Take several suggestions from different students and follow their methods to cut several sheets of paper. Have the class verify that the resulting pieces are rectangles, and have them compare the rectangles' sizes and shapes. You can introduce the word "congruent" to describe two figures that fit perfectly on top of each other.

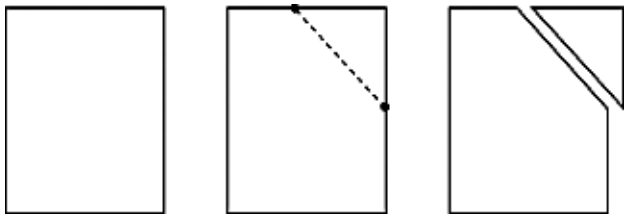
Note 6. It's difficult to identify the important content and how students might approach an activity without actually doing the mathematics yourself. Allow yourself time to work through the mathematics, even briefly, before going on to answering the other questions.

Part C, cont'd.

Using a new sheet of paper, call on students to tell where the paper should be cut to make two triangles. Have students compare a couple of different methods, as well as the sizes and shapes of the triangles they create. Also ask students to identify sides and vertices of the triangles to reinforce the vocabulary.

Main Activity

Hold up a fresh sheet of paper and say: "I will make one straight cut. I will start here (point to one side) and stop here (point to an adjacent side). Tell me about the shapes you think I will get." After the students describe the shapes, cut off a corner of the rectangular piece of paper, leaving a small triangle and a large pentagon. Have the students identify the number of sides and vertices of each shape, and repeat the word "pentagon" with students as that shape is discussed.



Review the names, numbers of sides, and number of vertices for several figures, including circle, triangle, square, rectangle, trapezoid, and pentagon. Give students scissors, paste, sheets of paper, and large construction paper. Tell them that their task is to create shapes by drawing a line from one side or vertex of the white paper to another side or vertex with a straightedge and then cut along the line. The students can then paste the resulting two pieces on the construction paper and record the names of the shapes. Have them repeat the activity six times, each time trying to create two new shapes.

Extension

Begin with an equilateral triangle or a trapezoid instead of a rectangle. Have the students follow the same procedure. (1) Identify the shapes that can be made with one straight cut from one side or vertex of the original shape to another side or vertex; and (2) compare the sizes and shapes of the cut figures to identify those that are congruent.

Problem C1. Answer the following questions about the Cutting Corners activity:

a. What is the geometry content in this activity?

b. What skills do students need to work through this activity? What skills will this activity help them develop for later work?

c. What level of geometric thinking is expected of students in the activity? Does it ask students to bridge levels?

d. What other questions might extend students' thinking about the activity?

e. Describe a lesson that you could develop based on the content of this activity.

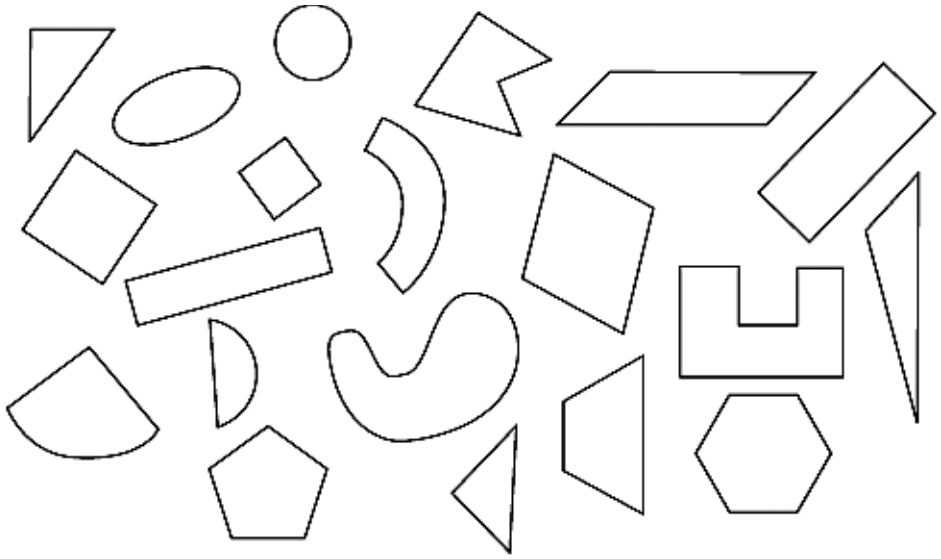
Part C, cont'd.

Shape-Sorting Activities

Here are several short activities designed for students in grades K-2. Answer questions (a)-(e) about these activities.

Shape Sort #1

Ask students to sort the shapes below into groups that are alike in some way.



Problem C2.

a. What is the geometry content in this activity?

b. What skills do students need to work through this activity? What skills will this activity help them develop for later work?

c. What level of geometric thinking is expected of students in the activity? Does it ask students to bridge levels?

d. What other questions might extend students' thinking about the activity?

e. Describe a lesson that you could develop based on the content of this activity.

Part C, cont'd.

Shape Sort #2

Ask students to sort shapes by naming properties, not by naming the shapes. When two or more properties are combined, have them sort by one property at a time. Example: "Find all of the shapes that have four sides." (Find these.) "Now find those that also have all right angles." (This group should include squares as well as non-square rectangles.) After sorting, discuss what the name of the shapes is. Also try sorting by the same combination of properties but in a different order.

Problem C3.

a. What is the geometry content in this activity?

b. What skills do students need to work through this activity? What skills will this activity help them develop for later work?

c. What level of geometric thinking is expected of students in the activity? Does it ask students to bridge levels?

d. What other questions might extend students' thinking about the activity?

e. Describe a lesson that you could develop based on the content of this activity.

Homework

Solutions are not provided for these homework problems, since answers will vary depending on individual experiences.

Problem H1. Interview a teacher in the grade level above you. Show the teacher the Cutting Corners activity in this session, and ask him or her the following questions:

a. How does the content of this activity prepare students for geometric thinking in your grade?

b. Why do you think this content is important?

c. How could this activity be extended for students in your grade?

Problem H2. Look at a lesson or activity in your own mathematics program for your grade level that you think has potential for developing students' geometric reasoning. If you were to use this lesson or activity now, after taking this course, how might you modify or extend it to bring out more of the important ideas about geometry?

Solutions

Part A: Geometry as a Problem-Solving Process

Problem A1.

a. Answers will vary. Some ideas: The challenge of feeling a shape but not looking at it, and of describing its properties without using its name simultaneously, requires students to practice with vocabulary like "sides" and "vertices" but also keeps the lesson from seeming like a vocabulary lesson. Also, the fact that students have different roles—feeling and describing, listening and drawing—keeps the focus on communication rather than just on the words.

b. We see students learn new terms like "trapezoid," but with meaning and context and relating it to other things they know. (It's half a hexagon.)

c. Throughout the lesson, students work on solving a problem that involves discovering what the properties of a shape are. In other words, they think about what makes a square a square, a triangle a triangle, etc.

d. One of the big ideas that students encounter in this lesson is the idea of classification. In this lesson, we see how students begin to develop a sense of classification of polygons based on their properties. Classification is an idea that progresses through grade levels.

Problem A2. People have very different, and often very strong, opinions about the use of context in mathematics classrooms. It is important to present students with a variety of lessons. Students can be engaged by problems that are not context-based, as well as by those with real-world connections.

Problem A3. In this lesson, students describe just one shape rather than a design made of several shapes. Also, students feel the shapes without looking and draw the shapes that are described. This gives them the opportunity to touch, draw, describe, and listen to descriptions of the shapes.

Part B: Developing Geometric Reasoning

Problem B1. Answers will vary. Some possible responses:

- (Level 0 thinking) The students' natural response was to name the particular shape they were holding rather than to focus on properties. Also, they relate the shapes to others they know, thinking of a trapezoid as half a hexagon.
- (Level 1 thinking) The activity forces students to feel a shape, turning it around in their hands, and learn that it is the same shape no matter what its position or orientation. They are trying to focus on properties and how they can determine the class of shape from those properties.

Problem B2. Answers will vary. Some examples of level 1 thinking required by the activity include the identification of non-standard kinds of polygons (concave, asymmetric, etc.). Also, level 1 thinking requires knowing if you've already counted one of the shapes, even if you find it a different way or list the vertices in a different order.

Problem B3. Answers will vary. Some possible answers:

a. Key pieces of geometry are names, properties (number of sides and vertices), and naming of polygons. In addition, you need to be flexible in your thinking about polygons, recognizing irregular and concave polygons in addition to more regular and familiar ones.

b. Reasoning includes algorithmically finding every polygon, determining if you have found duplicates, and determining when you are done.

Problem B4. Answers will vary. Students will probably get a broader view of polygons, including familiarity with irregular and differently oriented polygons. They may struggle with the naming of polygons and with finding all of them in the more complicated figures.

Problem B5. Answers will vary. Some ideas: Alter the activity to have less of a focus on the naming of vertices; include opportunities to draw, color, or cut out the shapes. If students have seen lots of examples of irregular polygons and have had opportunities to cut up and put polygons together to form others, they will have more success with an activity like hidden polygons.

Part C: Problems That Illustrate Geometric Reasoning

Problem C1.

a. Answers will vary. Some of the goals:
   - To visualize and describe smaller shapes that will be created by cutting larger shapes
   - To identify the number of sides and vertices of simple two-dimensional shapes
   - To identify figures that are congruent (the same shape and size)

b. Prerequisite knowledge includes identifying and naming squares, triangles, and non-square rectangles, as well as identifying the sides and vertices of figures.

c. In this activity, students work across levels. They create and identify particular shapes. But they also visualize, make predictions, and think about properties and how to create them (especially when students try to create six different outcomes). This means they are working at level 0 and level 1.

d. The activity prepares students for other dissection-related activities (like the one in Session 5, Part B) and for thinking about extreme cases (which creates the most sides, cutting vertex to vertex or side to side; and does it matter which sides?). It also prepares students for thinking about congruence and similarity.

e. Possible extension: Begin with an equilateral triangle or a trapezoid instead of a rectangle. Have the students follow the same procedure. (1) Identify the shapes that can be made with one straight cut from one side or vertex of the original shape to another side or vertex, and (2) compare the sizes and shapes of the cut figures to identify those that are congruent.

Solutions, cont'd.

Problem C2.

a. This activity encourages students to think about different properties that figures can have, including sorting by straight sides and curved sides, concave and convex ("dented" and not), right angles and no right angles, and so on.

b. This activity has no prerequisite skills, since students are not asked to name the shapes or to sort them with regard to any particular attribute. Allowing students to think creatively about sorting and properties can lead to more structured activities like the ones on Venn diagrams in Session 3, Part B.

c. This activity is primarily level 0, as students are focused on particular shapes and not families. But categorizing by properties helps move them towards level 1 thinking.

d. To extend students' thinking, it is important to ask them to verbalize what the shapes in each of their sorted categories have in common. You can then ask them to draw new shapes that belong to either of their categories.

e. Ideas for lessons will vary. One way to do this may be to begin with sorting drawings of everyday objects as a whole class. For example, hold up several drawings of different types of shoes, different types of clothing, and different types of food. Tell students you want to make three groups, putting together all of the things that are alike. When that is done, tell students they will receive an envelope of shapes. Their goal is to put the shapes into two categories, so that everything in the same category has something in common. When students have completed the task, have different students share how they sorted the shapes.

Problem C3.

a. The goal of this activity is for students to identify shapes by their properties.

b. To work through this activity, students need to know the particular vocabulary you choose to focus on. Possible vocabulary words include, "right angle, parallel, 90^o." This will help them develop the skill of moving between the familiar name for an object (e.g., "square") and the properties of that object that might be currently relevant (e.g., "four sides that are the same length.)

c. This activity is much like Problem C2, but moving students more towards level 1. When students name the shapes that remain after a sorting and sort by the same information in a different order, it helps them shift their attention from the particular shapes to the classes of shapes they represent. They may also use some "if-then" thinking: For example, if a shape is a rectangle, it will have four 90^o angles.

d. To extend students' thinking, you can ask them to choose a property to sort by. Ask a student to look at one of the shapes in front of him and to think quietly about something that is true of that shape. Then, have the student announce just the property. Everyone must sort by that student's property.

e. A lesson could be very much like the one in Problem C2. For example, start with everyday objects (clothing, animals, furniture, etc.) and ask students to sort by properties. For example, if you asked for things with legs, you could choose many of the animals, and perhaps some of the furniture (chairs, tables, etc.) After a couple of examples, give students their shapes and a few minutes to explore them and become familiar with them. Start with easier properties, like the number of sides or angles, all straight sides or curves, etc. Then move into the more challenging properties.