Session 6

The Pythagorean Theorem

Key Terms for This Session

Previously Introduced

- altitude
- perpendicular bisector
- right triangle
- side-angle-side (SAS) congruence

New in This Session

- converse
- coordinates
- hypotenuse
- Pythagorean theorem
- theorem

Introduction

In this session, you will look at a few proofs and several applications of one of the most famous theorems in mathematics: the Pythagorean theorem. Proof is an essential part of mathematics, and what separates it from other sciences. Mathematicians start from assumptions and definitions, then follow logical steps to draw conclusions. If the assumptions are correct and the steps are indeed logical, then the result can be trusted and used to prove further results. When a result has been proved, it becomes a theorem.

For information on required and/or optional materials for this session, **see Note 1.**

Learning Objectives

In this session, you will learn how to do the following:

- Examine different formal proofs of the Pythagorean theorem
- Examine some applications of the Pythagorean theorem, such as finding missing lengths
- Learn how to derive and use the distance formula

Note 1.

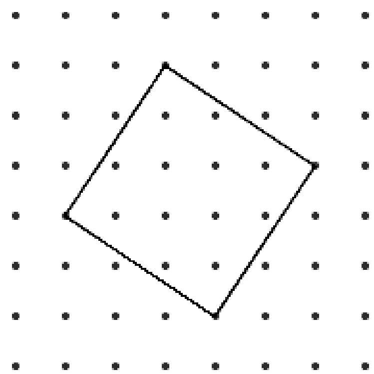
Materials Needed:

- scissors
- graph paper (at least 10 pages)

Part A: The Pythagorean Theorem (20 minutes)

Calculating Area

Here is a square drawn on dot paper:



Problem A1. Come up with a method to find the exact area of the square in square units. You can either use calculations or count the square units.

Squares Around a Right Triangle

Each of the three figures in Problem A2 shows a right triangle with squares built on the sides. Determine the exact area of all three squares for each figure.

Problem A2.

Figure 1:

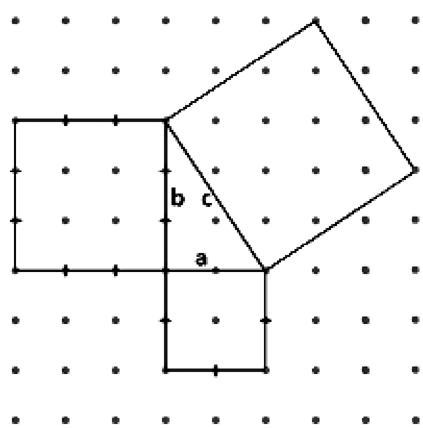


Figure 2:

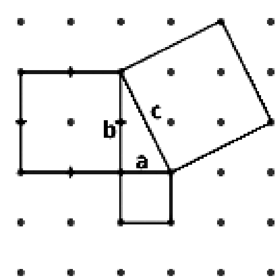
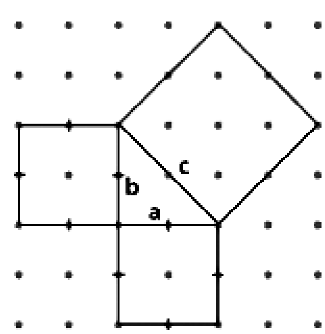


Figure 3:



Problems A1 and A2 are adapted from *IMPACT Mathematics, Course 1,* developed by Educational Development Center, Inc., pp. 536-537.

Part A, cont'd.

Problem A2, cont'd.

Figure #	Area of Square on Side *a*	Area of Square on Side *b*	Area of Square on Side *c*
Figure 1			
Figure 2			
Figure 3			

Problem A3. Look at your table, and come up with a relationship between the three squares that holds for all of the pictures.

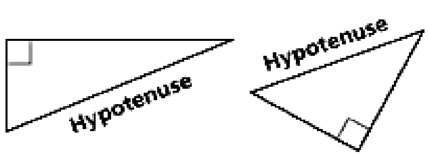
Video Segment (approximate time: 4:58-6:36): You can find this segment on the session video approximately 4 minutes and 58 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, the participants calculate the areas of the squares built on the right triangles. As they do the activity, a pattern begins to emerge, and they are able to formulate the relationship they discovered.

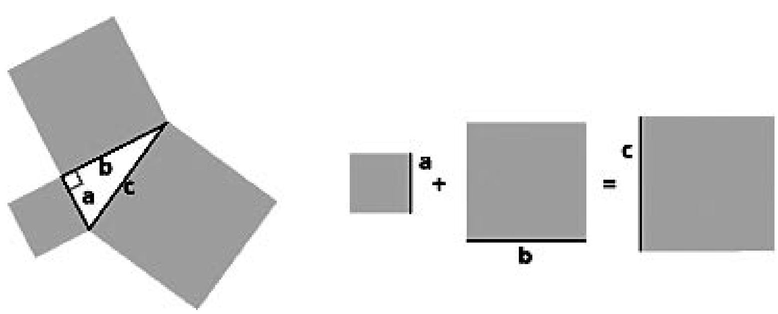
Was your method of calculating areas similar to or different from the one shown here? Can you come up with yet another way of doing this?

The Theorem

In a right triangle, the side opposite the right angle (side *c* in all of the pictures in Problem A2) is called the hypotenuse.

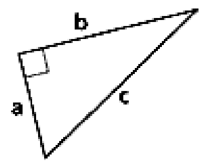


The problems you just solved illustrate the Pythagorean theorem: In a right triangle, the square built on the hypotenuse is equal in area to the sum of the squares built on the other two sides.



Part A, cont'd.

Today, most people think of the theorem as stating a relationship among three numbers, *a, b,* and *c,* which represent the lengths of the sides of a right triangle.



$a^2 + b^2 = c^2$

The Pythagorean theorem is named for Pythagoras, a Greek mathematician who lived from about 569-500 B.C.E., around the same time as Lao-Tse, Buddha, and Confucius. Pythagoras was the leader of a society that would likely be considered a cult by modern standards. They studied mathematics and numerology, were very superstitious about what they ate and how they lived, and were sworn to secrecy.

Part B: Proving the Pythagorean Theorem (65 minutes)

What Is a Theorem?

A theorem in mathematics is a proven fact. A theorem about right triangles must be true for every right triangle; there can be no exceptions. Just showing that an idea works in several cases is not enough to make an idea into a theorem.

The Pythagorean theorem has been proven to work for every possible right triangle. Of course, you can't draw every right triangle on graph paper, make squares on the sides, and find their areas.

Many people have written proofs of the Pythagorean theorem. In fact, whole books exist that contain nothing but proofs of this one theorem! The proof that follows is probably from China, about 200 B.C.E. Rather than learning of it from the Pythagoreans, though, the author of the proof most likely developed the theorem independently.

Constructing a Proof

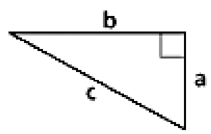
For the proof outlined below, follow the directions at each step, and answer the questions as you work. When you are finished, you will have constructed a proof of the Pythagorean theorem. **[See Note 2]**

Part A: The Theorem is adapted from *Connected Geometry,* developed by Educational Development Center, Inc., p. 197.

Note 2. Use graph paper to construct this proof. If you are working in a group, have everyone follow the steps as a whole group. You can choose to display a poster for Step 6 to help in the explanation.

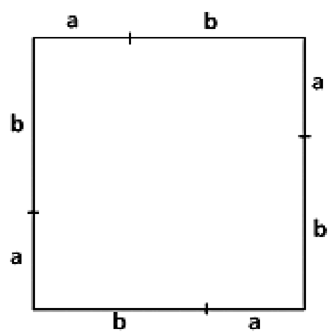
Part B, cont'd.

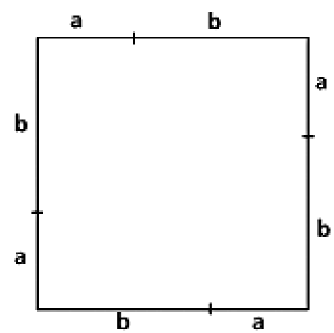
Step 1: Construct an arbitrary right triangle that is not isosceles. Label the short leg *a,* the long leg *b,* and the hypotenuse *c.*



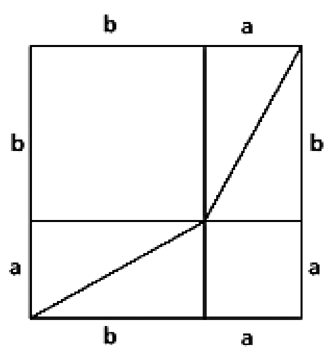
The proof still works if the triangle is isosceles—that is if $a = b$—but it is always better to work through a proof with an example that is not special in any way.

Step 2: Construct two squares whose sides have length $a + b$.





Step 3: Dissect one of the squares as shown below:

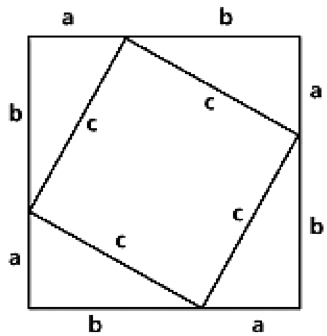


This dissection yields a square with side length *a* in one corner, a square with side length *b* in the opposite corner, and two rectangles, each cut along one diagonal into two right triangles.

Problem B1. Show that each of the four triangles you have just created is congruent to the original right triangle.

Part B, cont'd.

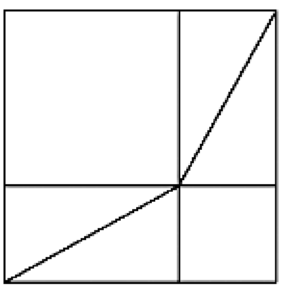
Step 4: Dissect the other square as shown below:



This dissection yields four triangles congruent to the original right triangle and a remaining piece in the center.

Problem B2. Show that the piece in the center is a square with side length *c*. **[See Tip B2, page 142]**

Step 5: The two original squares have the same area.



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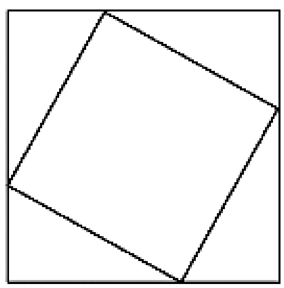
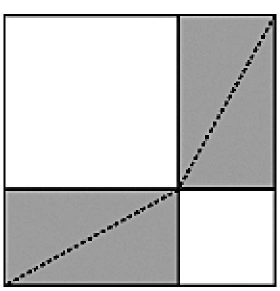


The eight triangles are congruent. So the four from the first square are equal in area to the four from the second square.

Shaded Area Here   Equals   Shaded Area Here



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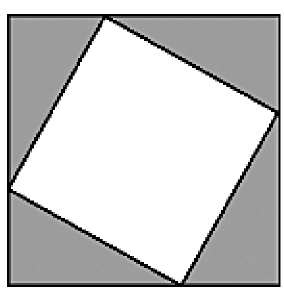
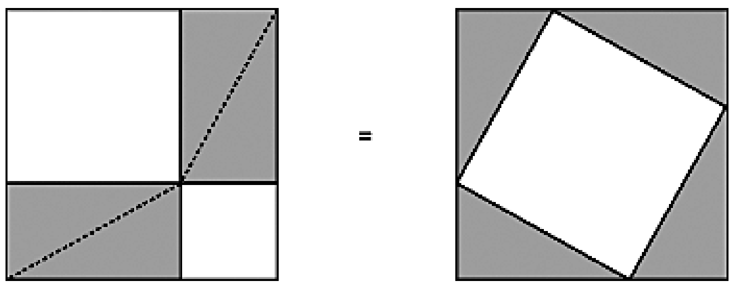


Part B, cont'd.

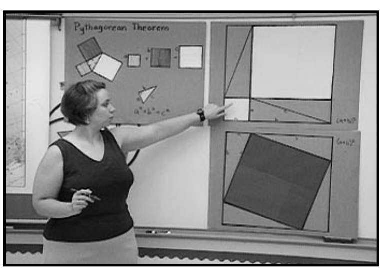
Step 6: Remove the four triangles from each square. What remains in the first square will have the same area as what remains in the second square.

Unshaded Area Here　　Equals　　Unshaded Area Here



The geometric equality—the Pythagorean theorem, as Euclid knew it—has been shown.

Our more modern algebraic interpretation, $a^2 + b^2 = c^2$, follows from the algebraic formulas for the areas of the squares. The areas of the two squares on the left are a^2 and b^2. The area of the square on the right is c^2. Geometric reasoning tells you that the areas on the left ($a^2 + b^2$) and right (c^2) are equal.



Video Segment (approximate time: 10:51-13:28): You can find this segment on the session video approximately 10 minutes and 51 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, the participants demonstrate why a^2, b^2, and c^2 are all squares with 90° angles at their vertices.

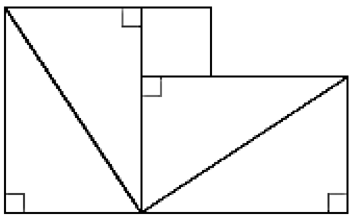
Was your method similar to the one shown here? If not, in what ways was it different?

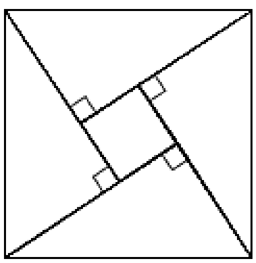
Part B, cont'd.

More Proofs

Below are two more proofs of the Pythagorean theorem, but only the pictures are given to you.

Problem B3. This is a famous one-word proof written by the Hindu mathematician Bhaskara Acharya (1114–1185). What is the reasoning behind his proof?

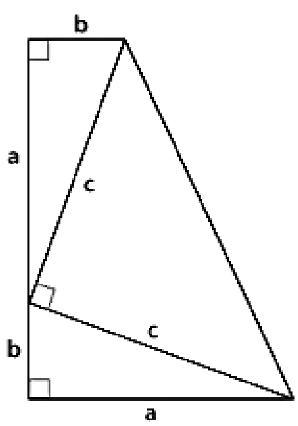




Behold!

[See Tip B3, page 142]

Problem B4. James Garfield (1831-1881) was the 20th president of the United States. Five years before becoming president, he discovered this proof of the Pythagorean theorem. What is the reasoning behind his proof?



[See Tip B4, page 142]

Part C: Applications of the Pythagorean Theorem (35 minutes)

Finding Missing Lengths

The Pythagorean theorem is useful for finding missing lengths. If you can find a right triangle in a picture, you can often find the length you need. All you need to know is that if $a^2 + b^2 = c^2$, then $\sqrt{a^2 + b^2} = \sqrt{c^2} = c$. That is, you can take the square root of both sides of the equation. Because you're working with lengths (so all the values are positive), you can find *c*.

Problem C1. Find the length of the diagonal of a square whose sides have the following lengths:

1 foot

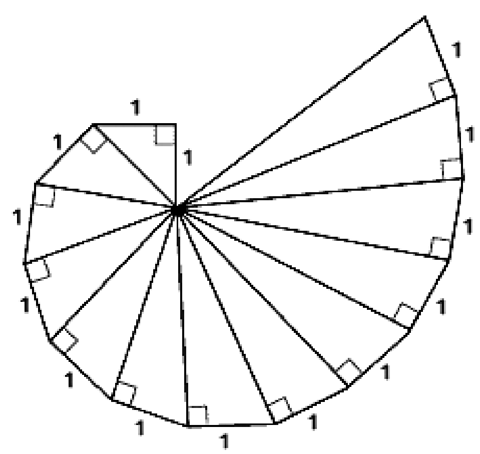
2 feet

10 feet

n feet

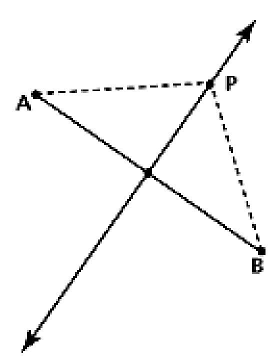
[See Tip C1, page 142]

Problem C2. Find the lengths of all the segments that are not labeled in the picture below. Describe a pattern in the lengths.



Problem C3. The perpendicular bisector of a segment is perpendicular to the segment and goes through the midpoint. Show that any point on the perpendicular bisector of a segment is the same distance from each of the endpoints.

The line is the perpendicular bisector of AB. Using the Pythagorean theorem, explain why PA and PB (the dashed segments) are the same length.



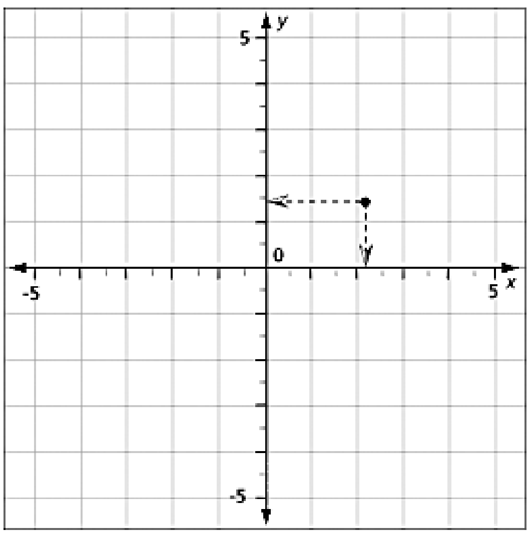
Part C, cont'd.

The Distance Formula

"As long as algebra and geometry have been separated, their progress has been slow and their uses limited; but when these two sciences have been united, they have lent each mutual forces, and have marched together towards perfection."

—Joseph-Louis Lagrange (1736-1813)

French mathematician René Descartes (1596-1650) was the first to employ algebra in solving geometry problems. His key insight, and one that has affected the study of mathematics ever since, was that of the development of coordinate geometry (or Cartesian geometry, named for its creator).



By convention, the axes are shown in the horizontal and vertical positions. The horizontal axis is called the x-axis, and the vertical is called the y-axis. When you describe a point, you list the coordinates in order (x,y). There's nothing magical in these conventions; they just make it easy for everyone to understand each other.

The coordinates of the point are about (2.1,1.4).

To use the power of algebra to solve problems in geometry, you need the distance formula—a way to measure the distance between two points. Luckily, the distance formula is just a special case of the Pythagorean theorem, so you don't have to remember anything new.

Use the following activity to experiment with the distance formula and to answer Problem C4. Plot the given pair of points on graph paper and find the distance between them by applying what you know from the Pythagorean theorem.

Try It Online! www.learner.org

This problem can be explored online as an Interactive Activity. Go to the Geometry Web site at www.learner.org/learningmath and find Session 6, Part C.

Part C, cont'd.

Problem C4. Find the distances between the following:

a. A = (0,0) and B = (1,1)
b. A = (2,3) and B = (-1,-1)
c. A = (-3,-2) and B = (5,4)
d. A = (-3,4) and B = (1,-1)
e. A= (-2,-3) and B = (-3,5)
f. A = (-3,-1) and B = (0,0)

[See Tip C4, page 142]

Problem C5. Plot the given pair of points and find the distance between them.

a. I = (10,-7) and J = (2,-7)
b. K = (1,5) and L = (1,-15)
c. A = (x,a) and B = (x,b)
d. A = (a,y) and B = (b,y)

[See Tip C5, page 142]

Problem C6. Find the distance between the two points pictured: A = (x_1,y_1) and B = (x_2,y_2).

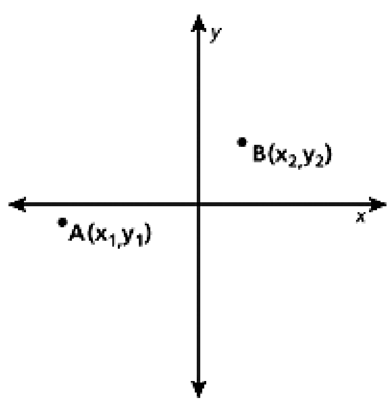


Diagram in Part C: The Distance Formula is adapted from *Connected Geometry,* developed by Educational Development Center, Inc., p. 355.

Homework

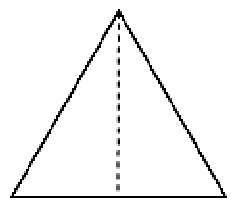
Problem H1. Find the height of an equilateral triangle with the following side lengths:

a. 1 centimeter

b. 2 centimeters

c. 10 centimeters

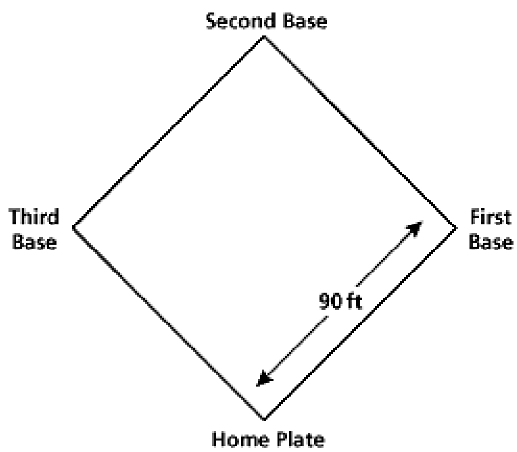
d. *n* centimeters



Problem H2.

a. A baseball diamond is really a square 90 feet on a side. How far is second base from home plate?

b. Baseball rules specify that the pitcher's mound must be 60 feet, six inches from home plate, in the direction of second base. Is the pitcher's mound in the center of the diamond? If not, is it closer to home plate or second base?



Converse

One common mistake in mathematics is assuming that if a statement is true, the converse of the statement is also true. To form the converse of a statement, you switch the "if" and "then" parts of the statement. Here's an example where the converse is clearly not true:

Statement: If you live in San Francisco, then you live in California.

Converse: If you live in California, then you live in San Francisco.

You can write the Pythagorean theorem as an "if-then" statement as well:

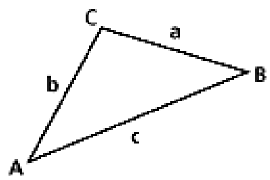
The Pythagorean theorem: If a triangle is a right triangle, then the square built on the hypotenuse is equal to the sum of the squares built on the other two sides.

Converse: If the square built on the hypotenuse is equal to the sum of the squares built on the other two sides, then you have a right triangle. (If $a^2 + b^2 = c^2$ for some triangle, then it must be a right triangle.)

Homework, cont'd.

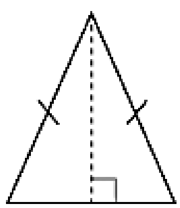
Problem H3. Below is an outline for a proof of the Pythagorean theorem's converse. Complete the proof by filling in the reasons.

Suppose you have some triangle ABC, where the lengths of the sides satisfy the relationship $a^2 + b^2 = c^2$.



a. Construct a right triangle with legs *a* and *b* (so there is a right angle between the sides with lengths *a* and *b*).
b. What is the length of the hypotenuse of your new triangle? Why?
c. Your new triangle and triangle ABC must be congruent. Why?
d. Triangle ABC must be a right triangle. Why?

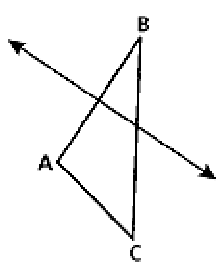
Problem H4. Show that the altitude to the base (non-equal side) of an isosceles triangle bisects the base.



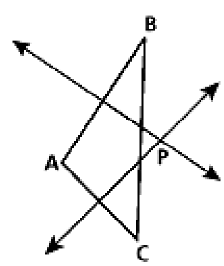
Concurrencies

Recall that in Session 1, it seemed (through experiments in folding paper) that the three perpendicular bisectors of any triangle were concurrent—they met at the same point, called the circumcenter of the triangle. We now have the tools to provide an explanation for this surprising fact.

All the points on the perpendicular bisector of AB are the same distance from A and B.



All the points on the perpendicular bisector of AC are the same distance from A and C.



Problem H5. Use the drawings and explanations above to describe point P, where the two perpendicular bisectors meet. Why must the perpendicular bisector of BC also go through point P?

Tips

Part B: Proving the Pythagorean Theorem

Tip B2. Try to show that the piece's angles must all be right angles.

Tip B3. Think about the first picture as two squares together. What are the new side lengths of the two squares?

Tip B4. Compute the area of the figure two ways—first by finding and adding the area of the three triangles and second by using the formula for the area of a trapezoid (1/2 · (sum of bases · height)). What must be true of the two areas you computed?

Part C: Applications of the Pythagorean Theorem

Tip C1. Draw a picture and look for right triangles.

Tip C4. Try constructing a right triangle whose hypotenuse is a line segment connecting points A and B, with *a* as its horizontal side (parallel to the x-axis). Then use the distance formula, the algebraic version of the Pythagorean theorem.

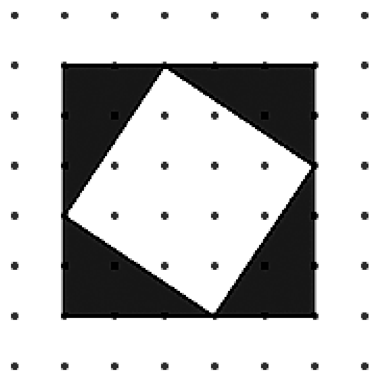
Tip C5. In parts (c) and (d), make sure your answer takes into consideration that you don't know whether length *a* or *b* is greater. Is there a way to be sure your answer is never negative?

Solutions

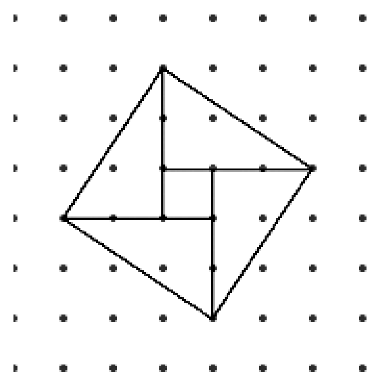
Part A: The Pythagorean Theorem

Problem A1. Draw a square with side lengths of 5 units around the given square so that the vertices of the given square are on the sides of the new square. The area of the new square is 25 square units (since its side length is 5), and its area is larger than the area of the original square by the area of the four right triangles whose legs have lengths of 2 and 3 units. So the area of the original square is 25 - 4 · (3 · 2 / 2) = 13 square units.

It looks like this:



Another method is the following:



Again, the area of the square is
4 · (2 · 3 / 2) + 1 · 1 = 13 square units.

Problem A2. Using the same method from A1, here are the areas:

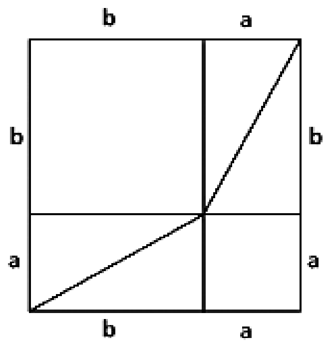
Figure #	Area of Square on Side *a*	Area of Square on Side *b*	Area of Square on Side *c*
Figure 1	4	9	13
Figure 2	1	4	5
Figure 3	4	4	8

Problem A3. The area of the square on side *c* is always the sum of the areas of the squares on the other two sides.

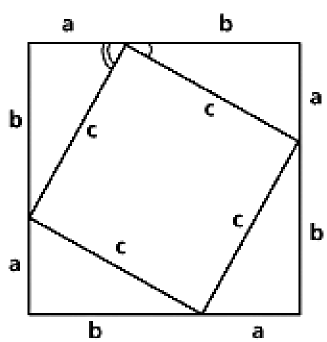
Solutions, cont'd.

Part B: Proving the Pythagorean Theorem

Problem B1. By construction, all four triangles are right triangles. The longer leg of each triangle has length *b* (since it equals the length of the side of the larger inscribed square), and the shorter leg of each triangle has length *a* (since it equals the length of the side of the smaller inscribed square). So all four triangles have two congruent sides as well as the angle between the two sides. So, by SAS congruence, the four triangles are indeed congruent to the original.

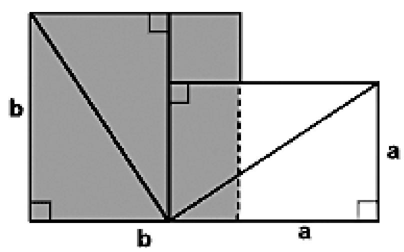


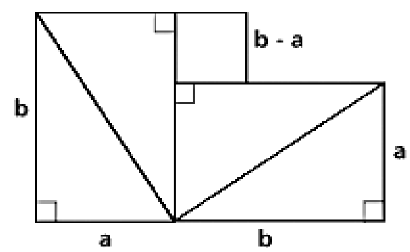
Problem B2. The sides of the piece in the center are all equal to *c,* since they are the longest sides of four triangles that are congruent to the original triangle (see the argument of Problem B1). Now we have to show that the angles are each 90°. Notice that three angles line up to make a straight line, or 180°: the small and the middle angles from the original right triangle (marked with one and two lines, respectively), and the one from the quadrilateral that lies between the two (see picture). Since the three angles of a triangle also add up to 180°, the angle in the quadrilateral must be the same as the largest angle in the triangle—a right angle. This is true for all four angles in the shape, and so we know that we have a quadrilateral with four congruent sides and four right angles, so it must be a square.

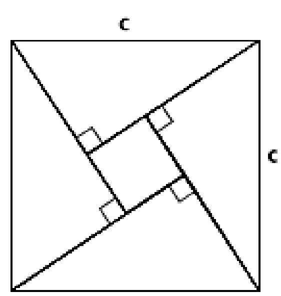


Solutions, cont'd.

Problem B3. The larger (shaded) square has side *b,* so its area is b^2 (see picture below). The smaller square has side *a,* so its area is a^2. The pieces that make up all of this are four right triangles with sides *a* and *b,* and a small square with side *b* - *a*. (It is the difference between the side *b* square and the smaller side of the triangle.) The same five pieces make up the large square with area c^2 in the third picture. So $a^2 + b^2 = c^2$.







Problem B4. The picture shows a trapezoid made up of three triangles. The areas of the triangles are *ab* / 2 (two of them), and c^2 / 2. On the other hand, using the formula for the area of the trapezoid, the overall area is (*a* + *b*)(*a* + *b*) / 2. So:

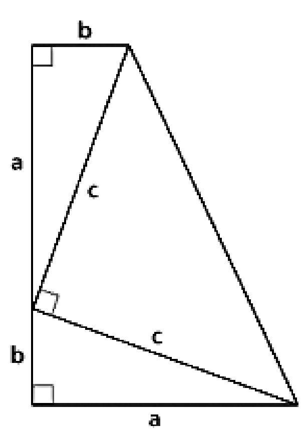
$ab / 2 + ab / 2 + c^2 / 2 = (a + b)^2 / 2$

Multiplying both sides by 2 and expanding the right-hand side, we get the following:

$2ab + c^2 = a^2 + 2ab + b^2$

or

$a^2 + b^2 = c^2$



Solutions, cont'd.

Part C: Applications of the Pythagorean Theorem

Problem C1. In all four parts of the problem, we draw a diagonal of length *d* inside the square of side length *a,* dividing the square into two congruent right triangles (see picture). Using the Pythagorean theorem, we get $a^2 + a^2 = d^2$, or $d = \sqrt{2a^2} = a\sqrt{2}$**.**

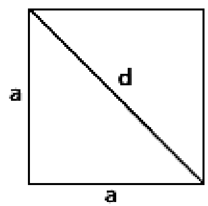
So we get the following:

a. $\sqrt{2}$ feet

b. $2\sqrt{2}$ feet

c. $10\sqrt{2}$ feet

d. $n\sqrt{2}$ feet



Problem C2. All the segments are right triangles, so we can apply the Pythagorean theorem. The smallest one has both legs of length 1, so its hypotenuse has length $\sqrt{2}$ (as in Problem C1, part (a)). The next smallest triangle has legs of length 1 and $\sqrt{2}$, so its hypotenuse has length $\sqrt{a^2 + b^2} = \sqrt{1^2 + \sqrt{2}^2} = \sqrt{3}$. Similarly, the length of the hypotenuse of the next smallest triangle is $\sqrt{4} = 2$. Following the pattern, the length of the hypotenuse of the *n*th triangle is $\sqrt{n + 1}$, so the last hypotenuse has length $\sqrt{14}$.

Problem C3. Let O be the point of intersection of the line segment AB and its perpendicular bisector. The lengths AO and OB are congruent since O bisects AB. The angles AOP and POB are both right angles since the bisector is perpendicular. So, by using the Pythagorean theorem, we get:

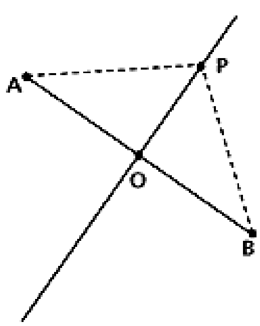
$AP^2 = AO^2 + OP^2$

$AP = \sqrt{AO^2 + OP^2}$

By a similar argument we can say that:

$BP^2 = \sqrt{BO^2 + OP^2}$

We know, however, that AO and BO are the same length, since O is the midpoint of segment AB. This means that the two values under the square root are the same, and AP must be the same length as BP.

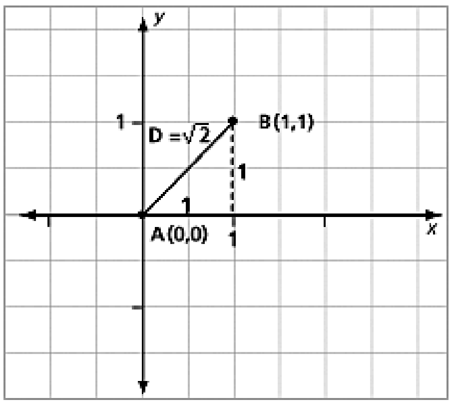


This can also be proven with SAS triangle congruence, which you first saw in the Session 2 Homework. It states that if two triangles have two sides equal in length and the angles between those sides are equal in their degree measure, then the two triangles are congruent. We know that the lengths AO and OB are congruent since O bisects AB. We also know that they share side OP. The angle between those two sides in each triangle is 90º since the bisector is perpendicular to AB. Thus, the two triangles are congruent, and so AP and BP are the same length.

Solutions, cont'd.

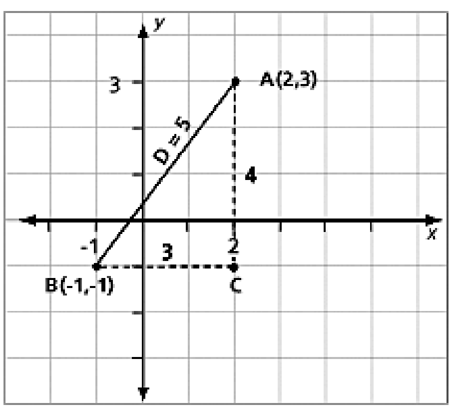
Problem C4.

a. The distance we want is the length of the line segment AB, which happens to be a hypotenuse of an isosceles right triangle whose legs have a length of 1 unit. Using the Pythagorean theorem, the distance we get is $\sqrt{1^2 + 1^2} = \sqrt{2}$.



b. From the point A = (2,3), draw a line perpendicular to the x-axis. Similarly, from B = (-1,-1), draw a line perpendicular to the y-axis. We will call the point where the two lines intersect point C. Then ABC is a right triangle whose hypotenuse AB is the distance between A and B. Using the Pythagorean theorem, we can calculate the distance as follows:

$D = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

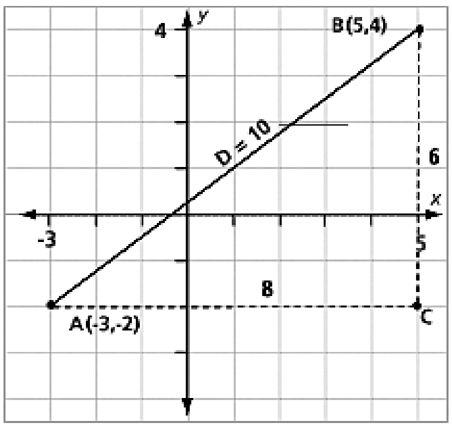


Solutions, cont'd.

Problem C4, cont'd.

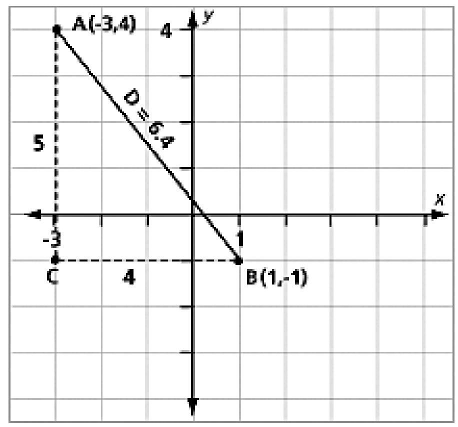
c. Using the Pythagorean theorem, we can calculate the distance as follows:

$D = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$



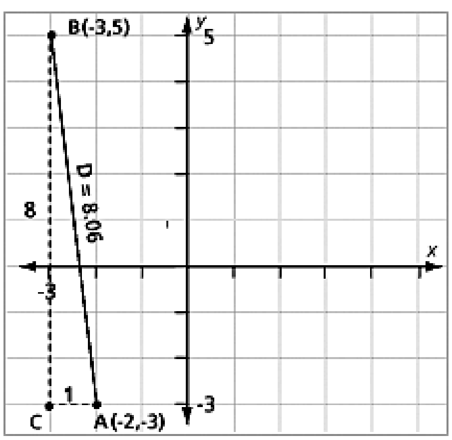
d. Using the Pythagorean theorem, we can calculate the distance as follows:

$D = \sqrt{5^2 + 4^2} = \sqrt{41} \approx 6.4$



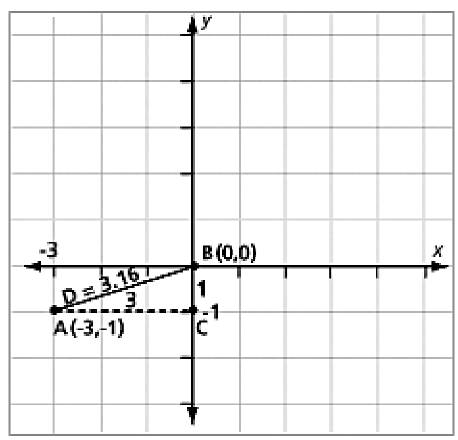
e. Using the Pythagorean theorem, we can calculate the distance as follows:

$D = \sqrt{1^2 + 8^2} = \sqrt{65} \approx 8.06$



f. Using the Pythagorean theorem, we can calculate the distance as follows:

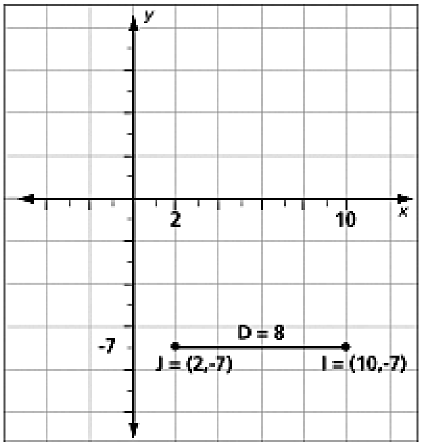
$D = \sqrt{3^2 + 1^2} = \sqrt{10} \approx 3.16$



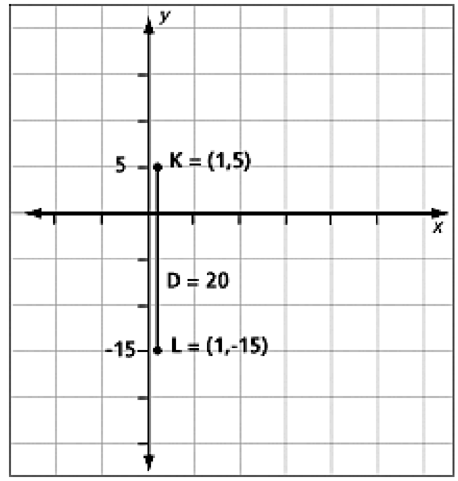
Solutions, cont'd.

Problem C5.

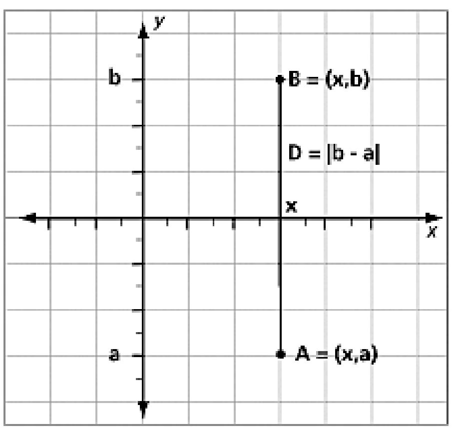
a. The distance is 8 units. See picture.



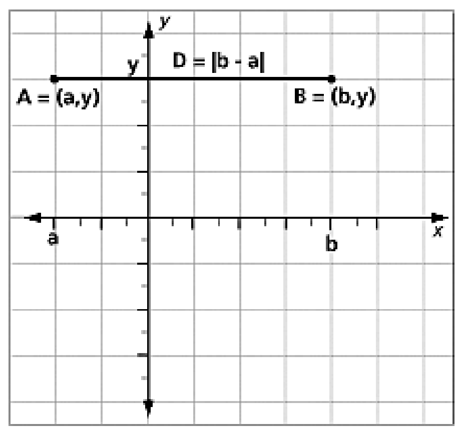
b. The distance is 20 units. See picture.



c. The distance is |*b* - *a*| units, where the vertical bars indicate absolute value. See picture.



d. The distance is |*b* - *a*| units. See picture.

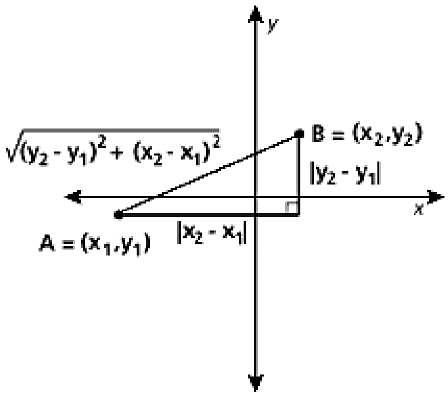


Solutions, cont'd.

Problem C6.

Repeating the procedure from Problem C4, we see that the distance is as follows:

$\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$



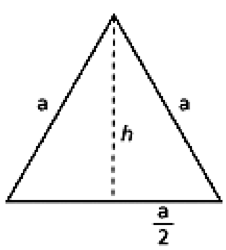
Since you get an answer that is non-negative whenever you square a number, it's standard to write $(x_2 - x_1)^2$ rather than $|x_2 - x_1|^2$. The two expressions always give the same answer.

Homework

Problem H1. If the length of each side is *a,* the height divides the triangle into two right triangles with one leg having length $a / 2$ (since the height bisects the side to which it is perpendicular), and the hypotenuse having length *a*. Let *h* be the length of the other leg—i.e., the length of the height. Using the Pythagorean theorem, we have $a^2 = (a / 2)^2 + h^2$.

So $h^2 = a^2 - (a / 2)^2 = a^2 - a^2/4 = 3\,a^2/4$.

So $h = \sqrt{3a^2 / 4} = (a\sqrt{3}) / 2$.



Applying this formula, we have the following:

a. $\sqrt{3}$ / 2

b. $\sqrt{3}$

c. $5\sqrt{3}$

d. $n\sqrt{3}$ / 2

Solutions, cont'd.

Problem H2.

a. The distance between home plate and second base is the hypotenuse of a right triangle whose legs have length 90 feet. Using the Pythagorean theorem, the distance is $\sqrt{90^2 + 90^2} = \sqrt{16{,}200}$, or approximately 127.3 feet.

b. If the pitcher's mound were in the center of the diamond, its distance from home plate would be half the total distance between home plate and second base. This distance is $45\sqrt{2}$. To the nearest inch, this is 63 feet, eight inches. This means that the pitcher's mound is three feet, two inches forward of the center of the baseball diamond.

Problem H3.

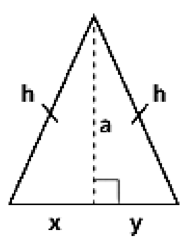
a. Follow the instructions.

b. Using the Pythagorean theorem, the length of the hypotenuse is $\sqrt{a^2 + b^2}$. By assumption, the sides of the original triangle satisfy the relationship $a^2 + b^2 = c^2$, so the length of the hypotenuse is $\sqrt{c^2} = c$.

c. The sides *a* and *b* are congruent by construction, and the third side is *c* (as shown in part (b) of the problem). So the two triangles have corresponding sides that are congruent, and by SSS congruence, they are congruent themselves.

d. Since congruent triangles have congruent corresponding angles, so the angle between the sides of length *a* and *b* must be a right angle for both triangles, and the second triangle is a right triangle by construction, the original triangle must also have a right angle.

Problem H4. By definition, the altitude forms a right angle with the base, so it divides the original triangle into two right triangles. The two right triangles share one leg (the altitude) of length *a* and have hypotenuses of same length, *h,* since the original triangle is an isosceles triangle. Suppose that the altitude divides the base into two line segments of length *x* and *y,* respectively. Applying the Pythagorean theorem to the two right triangles, we have $h^2 = a^2 + x^2$ and $h^2 = a^2 + y^2$. So we must have $a^2 + x^2 = a^2 + y^2$, or $x^2 = y^2$, and $x = y$ (since both *x* and *y* are positive, as they represent distances). So the altitude divides the base into two equal lengths; i.e., it bisects the base.



Solutions, cont'd.

Problem H5. Since it lies on the perpendicular bisectors of AB and AC, the point P is the same distance away from A and B and from A and C. In other words, it is the same distance away from A, B, and C. The perpendicular bisector of BC contains all points that are the same distance away from B and C, so it must contain P as well. So the perpendicular bisector of BC must go through P. In this case, the converse is also true and easily verified.

Now think of a circle whose center is P and whose radius is the length of PA. Since P is the same distance away from A, B, and C, this means that PA, PB, and PC will all have the same length. The circle with center P that passes through A will also pass through B and C. This circle is called the circumcircle of triangle ABC, and P is called the circumcenter.