Session 3

Polygons

Key Terms for This Session

New in This Session

- concave polygon
- convex polygon
- irregular polygon
- isosceles trapezoid
- kite
- line symmetry/reflection symmetry
- parallelogram
- polygon
- quadrilateral
- rectangle
- regular polygon
- rhombus
- square
- trapezoid
- Venn diagram
- vertex

Introduction

In this session, you will use puzzles and a classification game to explore polygons. You will play with the definitions of various polygons to begin to internalize their meaning. You will also begin to look at mathematical definitions more formally and explore how a term can have different, but equivalent, definitions. **[See Note 1]**

Learning Objectives

In this session, you will learn to do the following:

- Broaden your thinking about polygons
- Classify polygons according to some of their features
- Understand how to divide polygons into triangles and the implications of that
- Begin to understand mathematical definitions

Note 1. The following materials are needed for anyone choosing to do the non-interactive activity in place of the Interactive Activity:

- Loops of string (at least three loops per group or individual working alone)

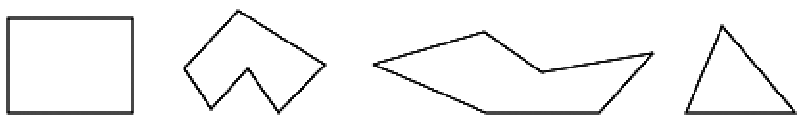
Part A: Hidden Polygons (20 minutes)

Identifying Polygons

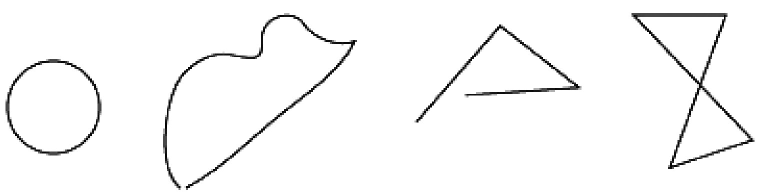
Polygons are two-dimensional geometric figures with these characteristics:

- They are made of straight line segments.
- Each segment touches exactly two other segments, one at each of its endpoints.
- They are closed—they divide the plane into two distinct regions, one "inside" and the other "outside" the polygon.

These shapes are polygons:



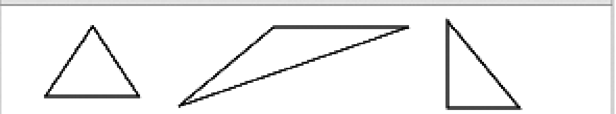
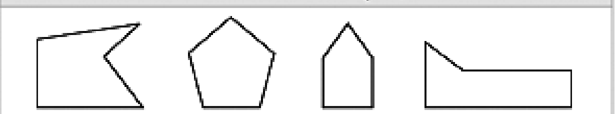
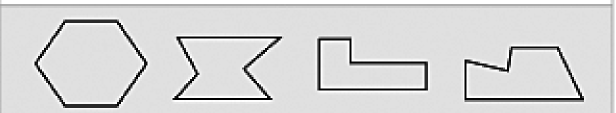
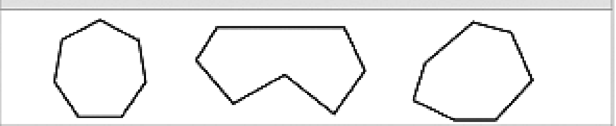
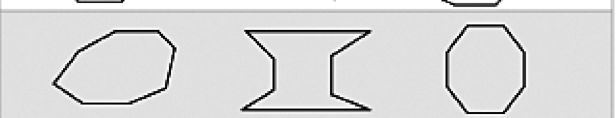
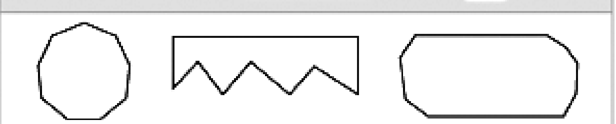
These shapes are not polygons:



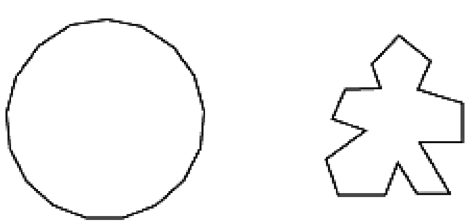
Problem A1. Look at the shapes above that are not polygons. Explain why each of these shapes does not fit the definition of a polygon.

Part A, cont'd.

Polygons can be classified according to the number of sides they have. **[See Note 2]**

Name	# of Sides	Examples
Triangle	3	
Quadrilateral	4	
Pentagon	5	
Hexagon	6	
Heptagon	7	
Octagon	8	
Nonagon	9	
Decagon	10	

Polygons with more than 10 sides are not usually given special names. A polygon with 11 sides is described as an 11-gon, a polygon with 12 sides as a 12-gon, and so on. Each of the polygons below is a 17-gon.



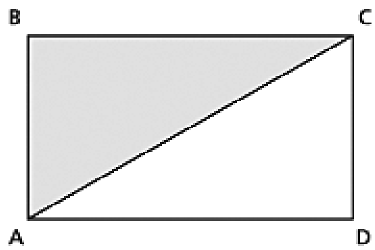
When people talk about a general polygon—one where you don't know the exact number of sides—they often refer to it as an *n*-gon.

Note 2. Discuss or reflect on why there is no name for a two-sided polygon. Namely, if two segments meet only at endpoints, the figure cannot be closed; therefore it cannot be a polygon.

Part A, cont'd.

Finding Polygons

Each corner of a polygon, where two sides meet, is called a vertex. The plural of vertex is vertices. Labeling vertices with capital letters makes it easy to refer to a polygon by name. For example, this figure contains two triangles and one quadrilateral:



To name one of the polygons in the figure, list its vertices in order as you move around it in either direction. One name for the shaded triangle is Triangle ABC. Other names are possible, including BCA and ACB. One name for the white triangle is Triangle ADC.

The quadrilateral in the figure could be named Quadrilateral ABCD, BCDA, DCBA, or DABC. All of these names list the vertices in order as you move around the quadrilateral. The name ACBD is not correct.

In the following activities, you will search for polygons in several figures. You'll calculate a score for each figure by adding the following:

a. 3 points for each triangle

b. 4 points for each quadrilateral

c. 5 points for each pentagon

d. 6 points for each hexagon

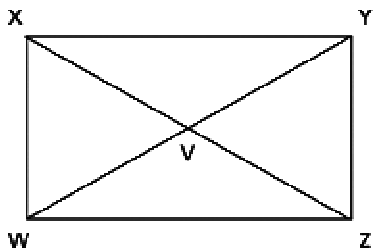
As you work, try to discover a systematic way to find and list all the polygons in a figure.

Be careful to give only one name for each polygon. You may want to record your work for each problem in a table like this one, which shows the result for this figure.

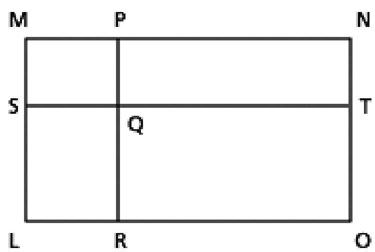
Polygon	**Names**	**Score**
Triangle	ABC, ADC	6
Quadrilateral	ABCD	4
Pentagon	None	–
Hexagon	None	–
Total Score		10

Part A, cont'd.

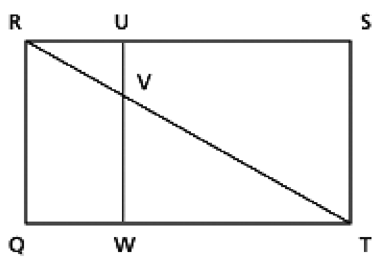
Problem A2. How many polygons can you find in the following figure? **[See Tip A2, page 71]**



Problem A3. How many polygons can you find in the following figure?



Problem A4. How many polygons can you find in the following figure?



Video Segment (approximate time: 6:46-7:42): You can find this segment on the session video approximately 6 minutes and 46 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, Ric and Michelle discuss their strategies for finding polygons in various figures. Watch this segment after you have completed Problems A2-A4, and compare your strategies with those of the onscreen participants.

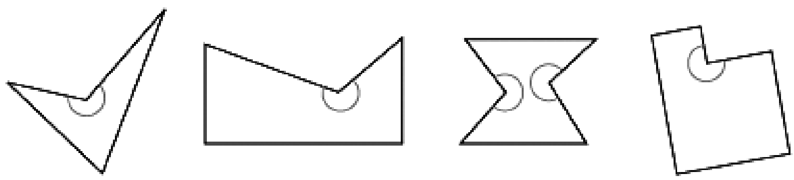
What kinds of strategies did Ric and Michelle use to find polygons in their figures? Did you use any other strategies?

Part B: Classifying Polygons (40 minutes)

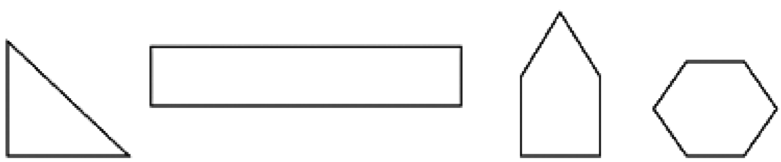
Properties of Polygons

Polygons can be divided into groups according to certain properties.

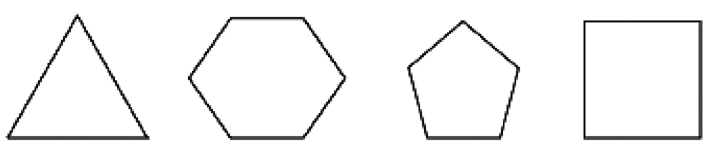
Concave polygons look like they are collapsed or have one or more angles dented in. Any polygon that has an angle measuring more than 180° is concave. These are concave polygons:



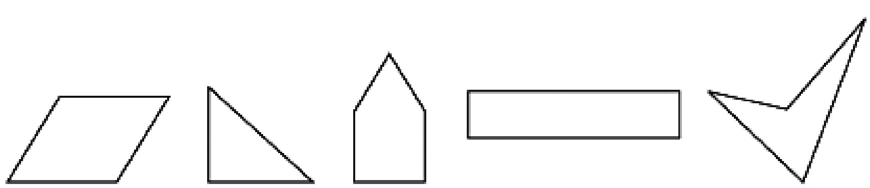
These polygons are not concave:



Regular polygons have sides that are all the same length and angles that are all the same size. These polygons are regular:



The polygons below are not regular. Such polygons are referred to as irregular. **[See Note 3]**

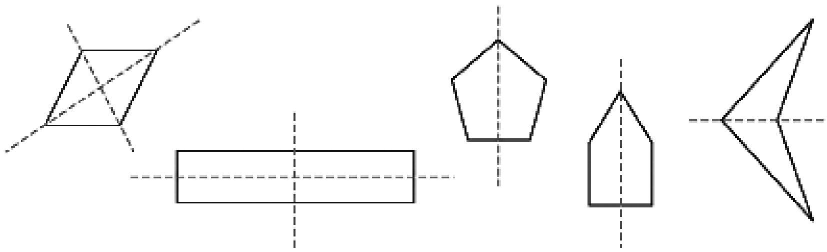


Note 3. For each shape, take a moment to discuss or reflect on how it fails to meet the definition of "regular." For example, in the quadrilateral on the left, all the sides have the same length. But because the angles are not all the same, it is not a regular polygon. In the rectangle, all the angles are the same, but the sides have different lengths. So it is not a regular polygon.

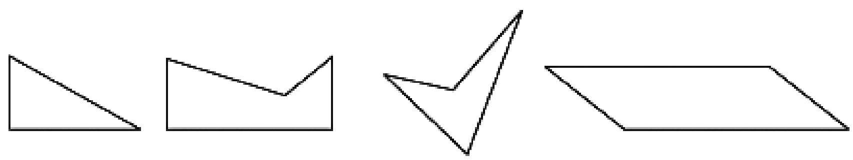
Part B, cont'd.

A polygon has line symmetry, or reflection symmetry, if you can fold it in half along a line so the two halves match exactly. The "folding line" is called the line of symmetry.

These polygons have line symmetry. The lines of symmetry are shown as dashed lines. Notice that two of the polygons have more than one line of symmetry.

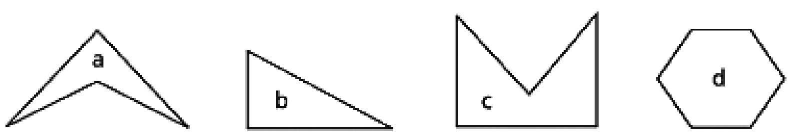


These polygons do not have line symmetry:

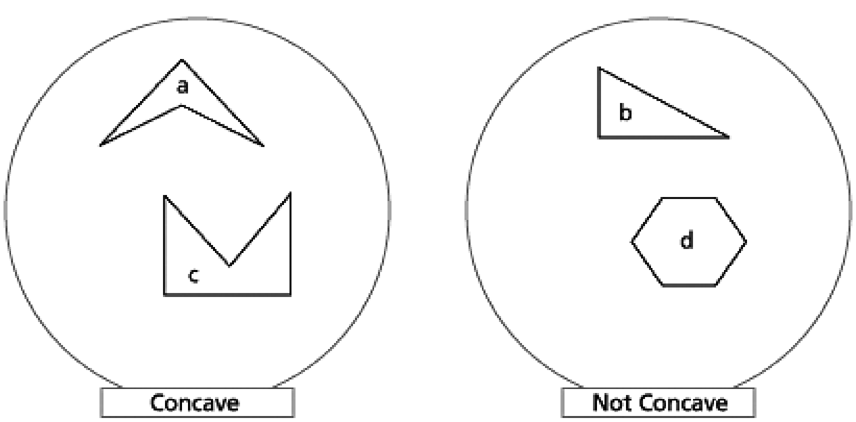


Grouping Polygons

Consider the polygons below:

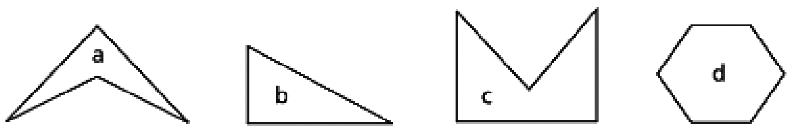


This diagram shows how these four polygons can be grouped into the categories Concave and Not Concave.



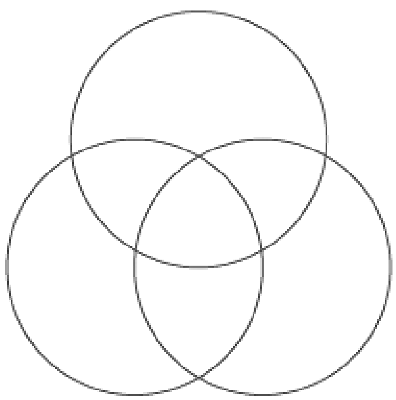
Part B, cont'd.

Problem B1. Make a diagram to show how these same four polygons can be grouped into the categories Line Symmetry and Not Concave. Use a circle to represent each category. **[See Note 4] [See Tip B1, page 71]**



Polygon-Classification Game

You will now play a polygon-classification game using Venn diagrams.



Venn diagrams use circles to represent relationships among sets of objects. They are named after John Venn (1834-1923) of England. Venn, a priest and historian, published two books on logic in the 1880s. Venn diagrams can be used to solve certain types of logic puzzles.

Problem B2. As a warm-up for the game, put each of the labels Regular, Concave, and Triangle next to one of the circles on the diagram. Place all the polygons in the correct regions of the diagram. You can find the polygons to be cut out for this activity on page 69 and the Venn diagram on page 70.

[See Tip B2, page 71]

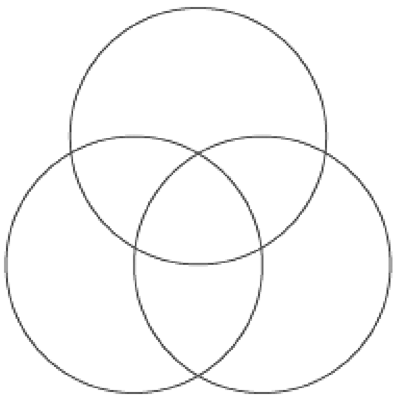
Note 4. This is best done as a full-group discussion with an overhead or at the board. Have someone draw the two regions, hold up cutouts of the shapes, and ask the group to decide where each would go. If you are working on an overhead, string is better for creating the regions than drawing because it is easily altered to "fix" the regions to overlap.

Part B, cont'd.

The following activity works best when done in groups. If you are working alone, consider asking a friend or colleague to work with you.

You will need a set of polygons, a set of category labels, and at least three loops of string for the polygon classification game. You will find the polygons and labels to be cut out on page 69.

Using string, set up a Venn diagram with three circles.



If you are working with a partner or in a small group, choose one member of your group to be the leader, and follow these rules to play the polygon classification game:

- The leader selects three category cards and looks at them without showing them to the other group members.
- The leader uses the cards to label the regions, placing one card face down next to each circle.
- The other group members take turns selecting a polygon, and the leader places the polygon in the correct region of the diagram.
- After a player's shape has been placed in the diagram, he or she may guess what the labels are. The first player to guess correctly wins.
- Take turns being the leader until each member of the group has had a chance.

If the leader misplaces one or more shapes, your group may have trouble guessing the labels. Encourage everyone to ask the leader to check placement of particular pieces that seem wrong. If your group is stuck, if possible, ask someone from another group to peek at the labels and help the leader move shapes if necessary.

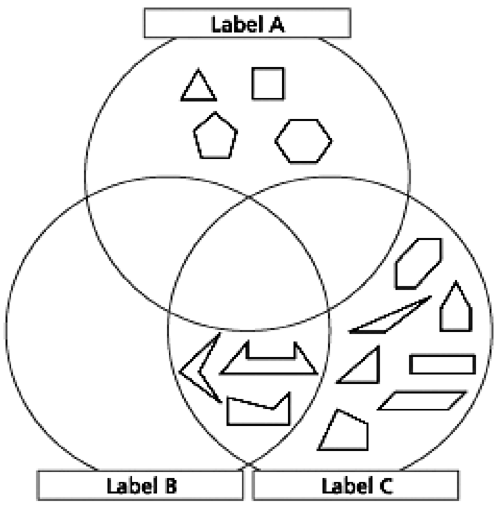
Try It Online! www.learner.org

This activity can be explored online as an Interactive Activity. Go to the *Geometry* Web site at www.learner.org/learningmath and find Session 3, Part B, Problem B3.

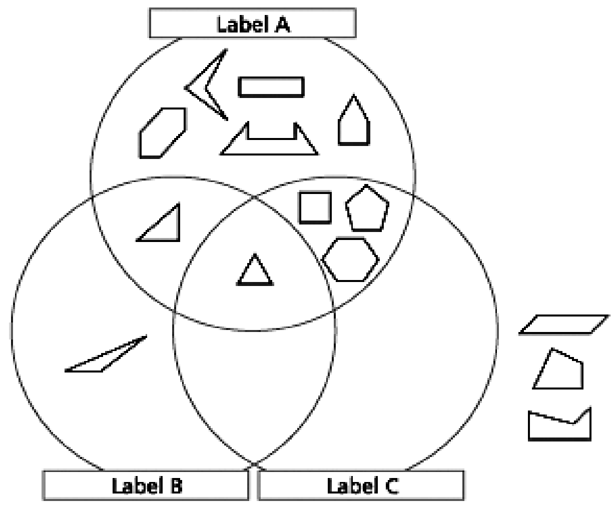
Part B, cont'd.

Problem B3. If you are working on your own or to help you get started, look at each of these Venn diagrams and see if you can determine how they should be labeled.

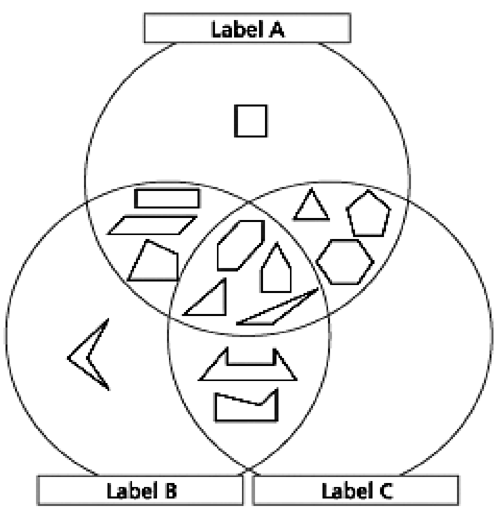
a.



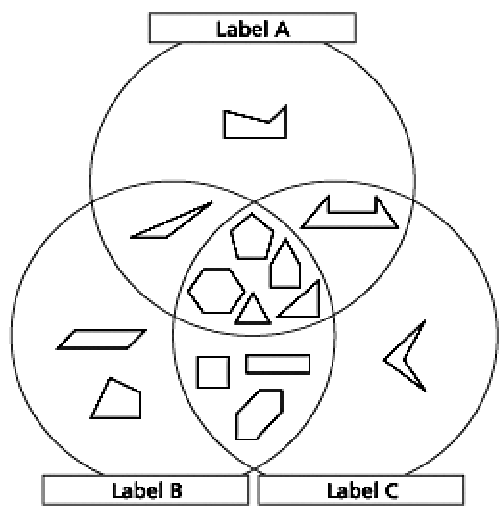
b.



c.



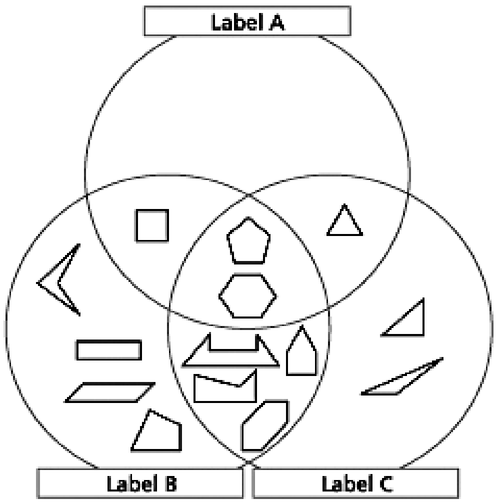
d.



Part B, cont'd.

Problem B3, cont'd.

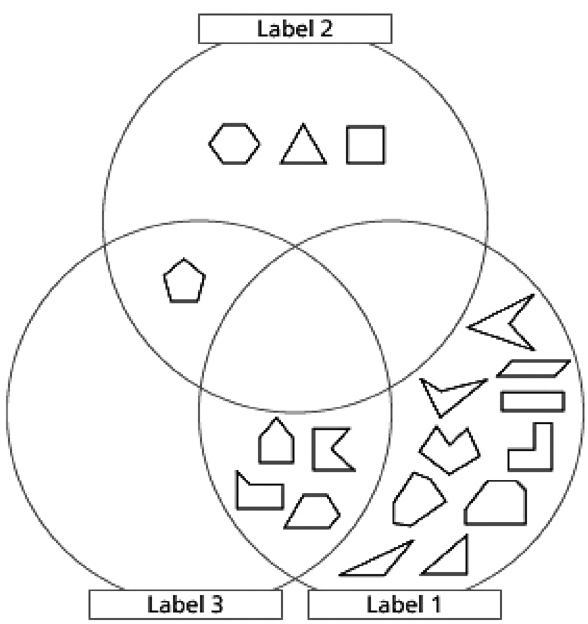
e.



More Venn Diagrams

Take It Further

Problem B4. Use the picture of a Venn diagram below:



a. Determine what the labels on this diagram must be.

b. Explain why there are no polygons in the overlap of the Label 1 circle and the Label 2 circle.

c. Explain why there are no polygons in the Label 3 circle that are not also in one of the other circles.

Problem B5. Create a diagram in which no polygons are placed in an overlapping region (that is, no polygon belongs to more than one category).

Problem B6. Create a diagram in which all of the polygons are placed either in the overlapping regions or outside the circles (that is, no polygon belongs to just one category).

Part C: Definitions and Proof (45 minutes)

Definitions

Definitions and proof both play essential roles in mathematics. Definition is crucial, because you need to know what something is (and what it's not) before you can make conjectures about it and prove these conjectures.

When mathematicians create a definition, they strive to be concise—to communicate a lot of information in a few words. This can make reading and understanding mathematical definitions difficult. This is further complicated by the fact that more than one definition may work for a given object. You may find yourself coming across an unfamiliar definition for a familiar object, and somehow you have to make sense of it.

A benefit of mathematical definitions is that you'll never find the circularity associated with dictionary definitions. For example:

Webster's attempt to define "dimension"

> **Dimension:** Any measurable extent, such as length, width, and depth.
>
> **Extent:** The space, amount, or degree to which a thing extends; size; length; breadth.
>
> **Measurement:** Extent, quantity, or size as determined by measuring.
>
> **Size:** That quality of a thing that determines how much space it occupies; dimensions; extent.
>
> **Length:** The measure of how long a thing is; the greatest of the two or three dimensions of anything; extent in space.

Here's a mathematical definition of a geometric property called convexity. The word may be familiar to you, but try to focus just on the definition provided for this activity.

> A figure is convex if, for every pair of points within the figure, the segment connecting the two points lies entirely within the figure.

Write and Reflect

Problem C1. Use the definition above to make sense of the notion of convex figures. What do they look like? Can you describe what they look like in your own words? Take whatever steps are necessary for you to understand the mathematical definition. Describe the steps you took to understand the definition. How did you make sense of it for yourself?

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Part C, cont'd.

Understanding Definitions

People go about understanding mathematical definitions in different ways; the steps they take may vary. Here's one way:

Step 1. Read the definition more than once.

Step 2. Identify what "things" the definition is talking about.

Step 3. Generate a test case.

Step 4. Determine if the example fits the definition.

Step 5. Find examples that do not fit the definition.

Step 6. Try to generalize the examples to create an image of the full concept. (In the case of convex figures, you might think about curved figures, three-dimensional figures, etc.)

Convex, like many other mathematical ideas, has several different, but equivalent, definitions. Not every proposed definition will work, though, and some definitions are better than others—they may be more clear, use fewer words, or be easier to test.

Problem C2. Which of these definitions work for convex polygons? A polygon is convex if and only if…

a. all diagonals lie in the interior of the polygon.
b. the perimeter is larger than the length of the longest diagonal.
c. every diagonal is longer than every side.
d. the perimeter of the polygon is the shortest path that encloses the entire shape.
e. the largest interior angle is adjacent to the longest side.
f. none of the lines that contain the sides of the polygon pass through its interior.
g. every interior angle is less than 180°.
h. the polygon is not concave.

Video Segment (approximate time: 12:48-16:12): You can find this segment on the session video approximately 12 minutes and 48 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, Ric and Michelle discuss whether a given statement about polygons works for convex polygons, then share their ideas with the whole group. Watch this video segment after you have completed Problem C2.

How did Ric and Michelle test to see if their given statement about polygons works for convex polygons?

Part C, cont'd.

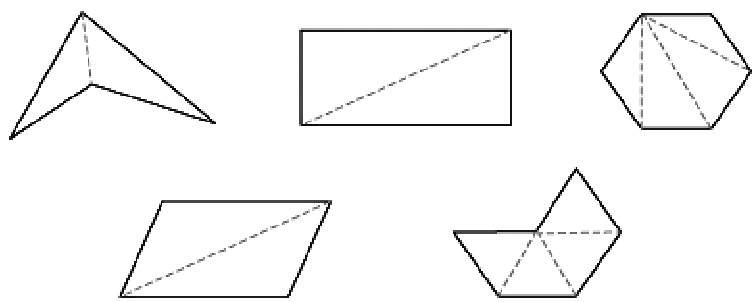
Dividing Polygons Into Triangles

You may know that the sum of the angles in a triangle is 180°. Can you prove it? In Session 4, we'll look at one mathematical argument for which the result is true. For now, we'll assume that it's true (based on some strong evidence) and look at some consequences of that fact. This is how mathematicians often work: They assume an intermediate result, called a lemma. If the lemma turns out to be useful in proving other results, they go back and try to prove that the lemma itself is true.

Video Segment (approximate time: 16:28-18:01): You can find this segment on the session video approximately 16 minutes and 28 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

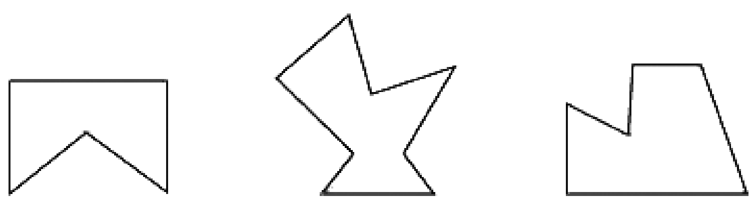
Watch this video segment for a quick demonstration that shows that the sum of angles in a triangle is 180°. Even though this is not a proof, it will be useful for building further conjectures about polygons.

Problem C3. Polygons with any number of sides can be divided up into triangles. Here are a few examples:



Draw several other examples of polygons divided into triangles for polygons of varying numbers of sides. Be sure not to use just regular polygons, and be sure not to use just convex polygons.

Problem C4. How would you divide the polygons below into triangles?



Take It Further

Problem C5. Describe a method so that, given any polygon, you are able to divide it into triangles.
[See Tip C5, page 71]

Part C, cont'd.

Triangles in Convex Polygons

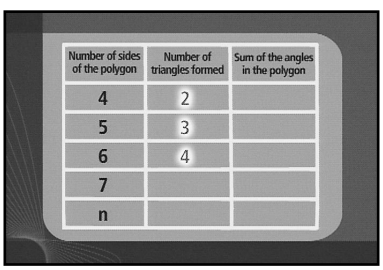
There are a few different methods that will work for dividing a polygon into triangles. One method is particularly convenient because, for polygons with the same number of sides, you get the same number of triangles. Here's an outline of the method for convex polygons (minor changes are necessary if you work with a concave polygon):

- Pick any vertex to start.
- Connect that vertex to every other vertex, except the two that are adjacent to it.

Problem C6. Use the method above or your own method, and fill in the table below. Remember that we are assuming that there are 180° in a triangle.

Number of Sides of the Polygon	Number of Triangles Formed	Sum of the Angles in the Polygon
3	1	180°
4		
5		
6		
7		
n		

Problem C7. Write a convincing mathematical argument to explain why your result for the sum of the angles in an *n*-gon is correct. **[See Note 5]**



Video Segment (approximate time: 19:32-20:31): You can find this segment on the session video approximately 19 minutes and 32 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, the participants discuss how to determine the number of triangles that can be formed in a polygon and the sum of the angles in that polygon. Watch this video segment after you have completed Problem C7.

What formula do the participants come up with to determine the sum of the angles in any polygon?

Note 5. Take the time to summarize the argument as a group or reflect deeply if working alone. Don't forget that we are assuming the angle sum of a triangle, but have an argument (not really a formal proof) that you can make (n - 2) triangles.

Homework

A quadrilateral is a polygon that has exactly four sides. Here are some definitions of special types of quadrilaterals:

- A trapezoid has one pair of opposite sides parallel. **[See Note 6]**
- An isosceles trapezoid is a trapezoid that has congruent base angles. (The base angles are the two angles at either end of one of the parallel sides.) **[See Note 7]**
- A parallelogram has two pairs of opposite sides parallel.
- A rhombus has all four sides congruent (the same length).
- A rectangle has three right angles.
- A square is a rhombus with one right angle.
- A kite has two pairs of adjacent sides congruent (the same length).

Problem H1. Go through the process of understanding a definition for each of the quadrilaterals named above. Does the given definition define the figures as you know them? For each type of quadrilateral, try to list alternative definitions, or at least several properties of the figures. (Think about sides, angles, diagonals, and so on.)

Problem H2. Fill in the chart below with yes, no, or maybe in each cell.

Type of Quadrilateral	Do diagonals bisect each other?	Are diagonals congruent?	Are diagonals perpendicular?
Trapezoid			
Isosceles Trapezoid			
Parallelogram			
Rhombus			
Rectangle			
Square			
Kite			

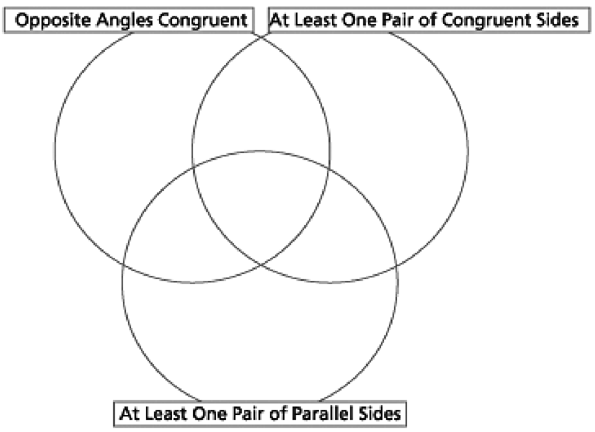
Note 6. Some people define trapezoid as "at least one pair of parallel sides" and others, as used here, as "exactly one pair of parallel sides." It's a matter of taste; there are no strong reasons to do it one way instead of the other, hence the differing opinions.

Note 7. Another, more common definition of an isosceles trapezoid is a trapezoid with the nonparallel sides congruent.

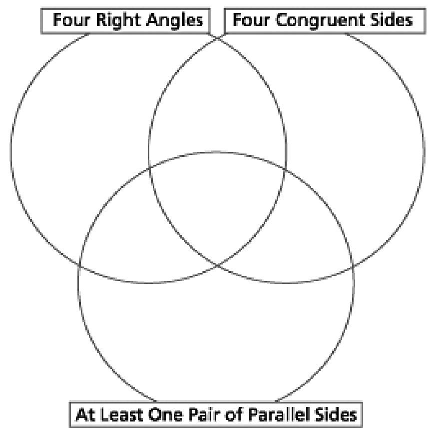
Homework, cont'd.

Problem H3. Put the following quadrilaterals in the appropriate spaces in each Venn diagram to indicate properties these types of quadrilaterals must have: trapezoid, isosceles trapezoid, parallelogram, rhombus, rectangle, square, and kite.

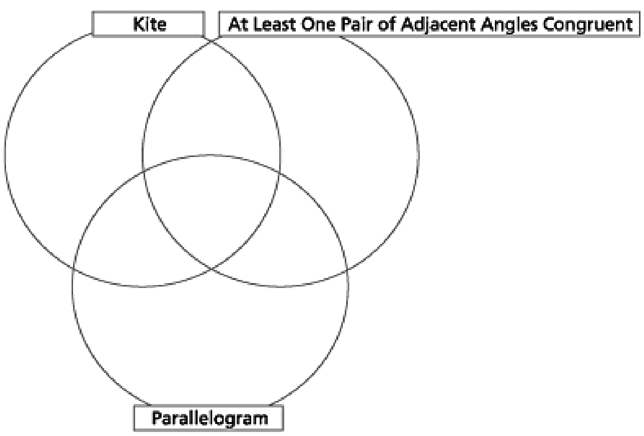
a.



b.



c.



Homework, cont'd.

Mathematicians often define things in a way that makes other work more convenient. For example, the definition for a prime number specifically excludes the number 1 as a prime. (A prime number is an integer whose only factors are itself and 1. A prime number has exactly two factors.) Why is the number 1 not a prime, when it fits the rest of the definition very well? Because a lot of proofs work based on the "fundamental theorem of arithmetic": Every number can be uniquely factored into primes (where a different order of the primes does not count as factoring differently). Suppose the number 1 were a prime. Then you could have all of these factorizations for the number 6:

$6 = 3 \cdot 2$

$6 = 3 \cdot 2 \cdot 1$

$6 = 3 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$

Recall our definition of a polygon: Polygons are two-dimensional geometric figures with these characteristics:

- They are made of straight line segments.
- Each segment touches exactly two other segments, one at each of its endpoints.
- Polygons are closed—they divide the plane into two distinct regions, one "inside" and the other "outside" the polygon.

Problem H4. Explain why our definition does not allow for "flat" polygons like the one shown here. What part(s) of the definition fails?

"Flat Triangle" ABC, with one 180° angle and two 0° angles.

Problem H5. Write down some reasons why we would not want to consider figures like the one above polygons.

Suggested Reading

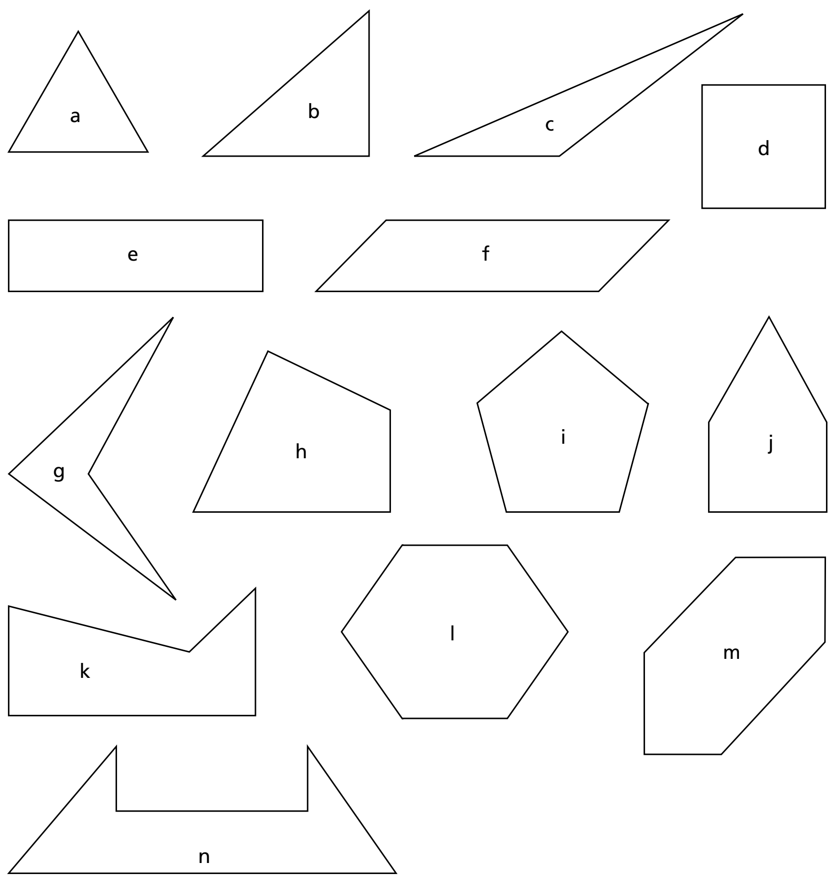
This reading is available as a downloadable PDF file on the *Geometry* Web site. Go to www.learner.org/learningmath.

Senechal, Marjorie (1990). Shape. In *On the Shoulders of Giants: New Approaches to Numeracy.* Edited by Lynn Arthur Steen (pp. 139-148). Washington, D.C.: National Academy Press.

Activity Cut-Outs

Part B: Polygon-Classification Game

Cut out these polygons and category labels for the Polygon-Classification Game in Part B.

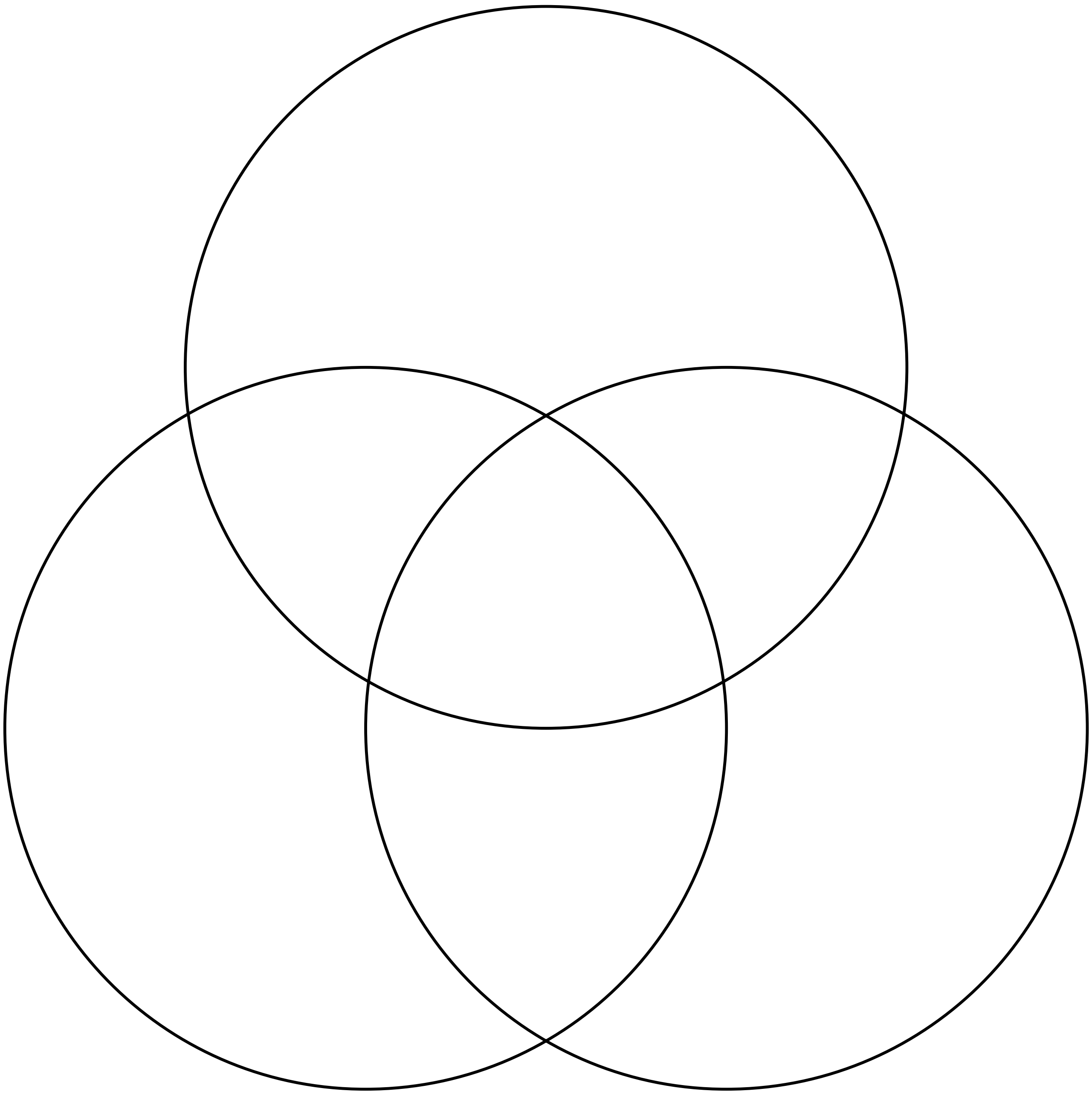


Regular	Concave	Triangle
Irregular	Not Concave	Not Triangle
Quadrilateral	Pentagon	Hexagon
Not Quadrilateral	Not Pentagon	Not Hexagon
Line Symmetry	No Line Symmetry	

Activity Cut-Outs, cont'd.

Part B: Polygon-Classification Game

Venn Diagram



Tips

Part A: Hidden Polygons

Tip A2. Here's a sample strategy for counting: Count triangles and quadrilaterals, and then look for their "complements." So YXWZV is everything in the shape except triangle YVZ. Another strategy would be to choose one vertex (e.g., X). Count all of the triangles that contain X. Then count all of the quadrilaterals that contain X, and so on. Next, count all of the triangles that contain vertex Y but not X, and so on.

Part B: Classifying Polygons

Tip B1. The circles need to overlap. Why? What needs to go in the middle?

Tip B2. As you're placing polygons in the labeled circles, think which properties each polygon has and then check whether that property is shared with any other of the labels. It might also be helpful to create a grid listing all the polygons and labels and filling it in for easier viewing of the shared properties among polygons.

Part C: Definitions and Proof

Tip C5. Make sure your method works for regular and irregular polygons, and also for convex and concave polygons.

Solutions

Part A: Hidden Polygons

Problem A1. The first two shapes are not polygons because they are not made of straight line segments. The third shape is not a polygon because it is not closed, while the fourth shape divides the plane into three regions, rather than two.

Problem A2. There are 13 polygons. They are as follows:

- Four small triangles, each defined by one side of the rectangle and two halves of the diagonals (e.g., XYV)
- Four pentagons, each a complement of one of the small triangles (e.g., VYZWX)
- Four large triangles, each defined by two sides of the rectangle and one of the diagonals (e.g., triangle XZW)
- The rectangle XYZW

Score: $(8 \cdot 3) + (1 \cdot 4) + (4 \cdot 5) = 48$ points

Problem A3. There are 13 polygons. They are as follows:

- Four small rectangles, all of which share the vertex Q (e.g., SMPQ)
- Four hexagons, each a complement of one of the small rectangles (e.g., PNOLSQ)
- Four larger rectangles, each defined by two small rectangles sharing one side (e.g., MNTS)
- The rectangle MNOL

Score: $(9 \cdot 4) + (4 \cdot 6) = 60$ points

Problem A4. There are 13 polygons. They are as follows:

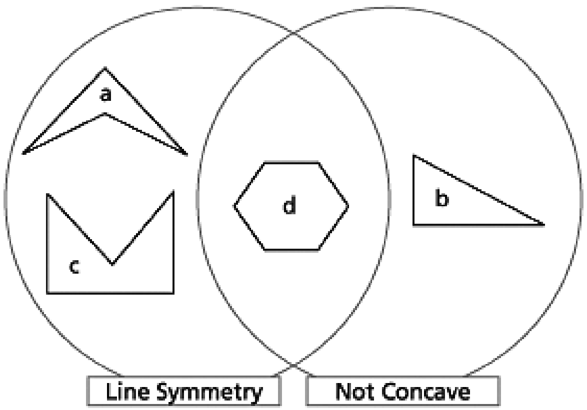
- Two small triangles (RUV and TWV)
- Their two complements (hexagons VUSTQR and VWQRST)
- Two quadrilaterals (RQWV and TSUV)
- Their two complements (pentagons VRSTW and VTQRU)
- Two larger triangles (RQT and TSR)
- Three rectangles (RUWQ, USTW, and QRST)

Score: $(4 \cdot 3) + (5 \cdot 4) + (2 \cdot 5) + (2 \cdot 6) = 54$ points

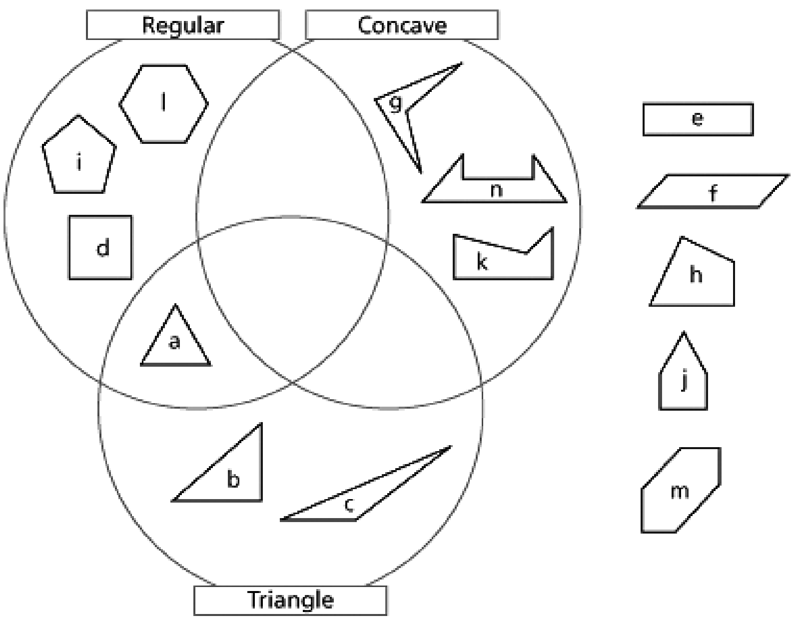
Solutions, cont'd.

Part B: Classifying Polygons

Problem B1.



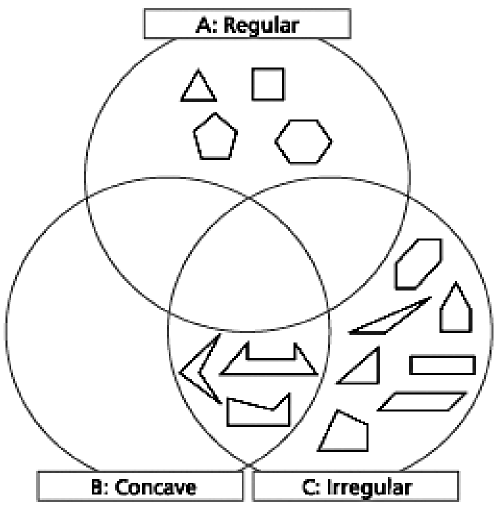
Problem B2. See the diagram below. Note that shapes e, f, h, j, and m are neither triangular, regular, nor concave, so they belong outside all three circles of the diagram.



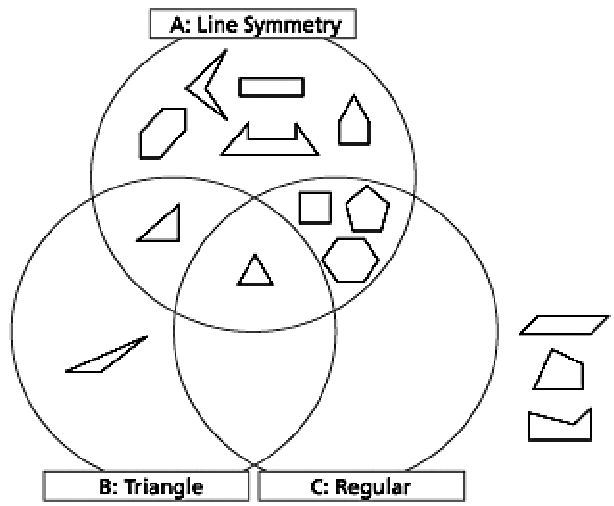
Solutions, cont'd.

Problem B3.

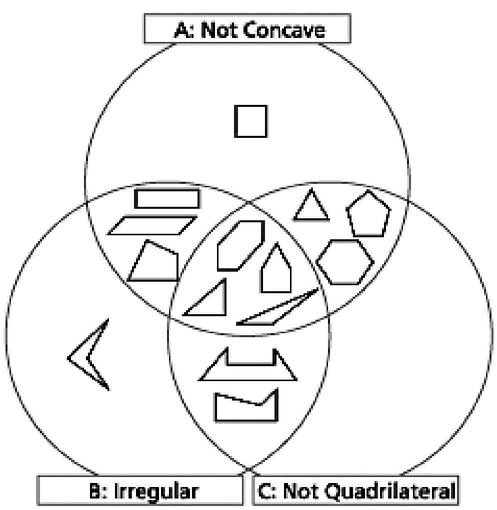
a.



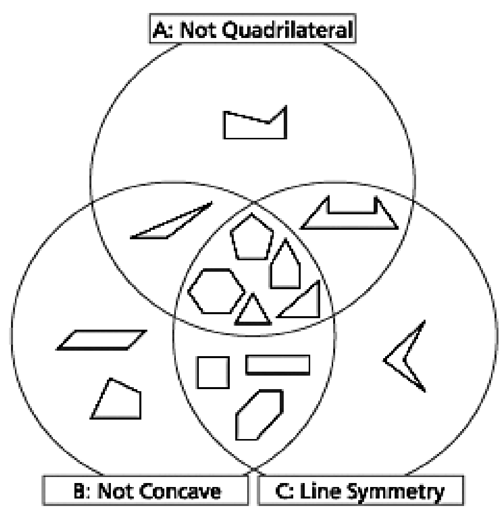
b.



c.



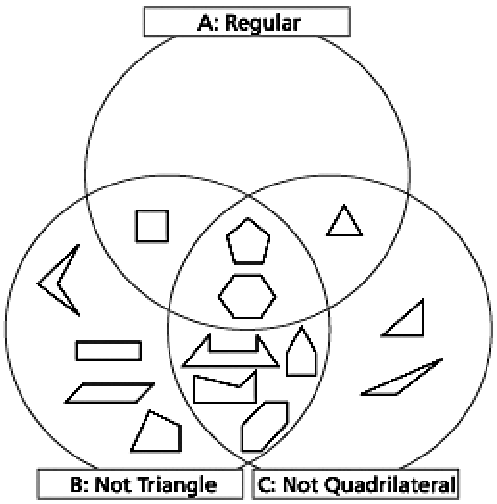
d.



Solutions, cont'd.

Problem B3, cont'd.

e.



Problem B4.

a. Label 1 is Irregular, Label 2 is Regular, and Label 3 is Pentagon.

b. The overlap would have to contain polygons that are simultaneously regular and irregular. No such polygons exist!

c. Every pentagon is either regular or irregular. For there to be a polygon in the Label 3 circle but not in any others, there would have to be a pentagon which is neither regular nor irregular. This is impossible.

Problem B5. Answers will vary. One possibility is to label the regions Triangle, Quadrilateral, and Pentagon.

Problem B6. Answers will vary. One possibility is Not Triangle, Not Quadrilateral, and Not Hexagon. Even if a shape is one of those three things, it's sure to not be the other two, so every shape is in the overlap of two circles, and some (pentagons) are in the overlap of all three.

Part C: Definitions and Proof

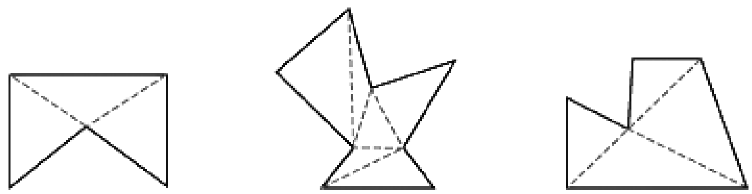
Problem C1. Answers will vary. One possible way of describing convex figures is that they are figures that are not dented, or that a rubber band stretched around the figure will touch the figure entirely.

Problem C2. Definitions (a), (d), (f), (g), and (h) are all equivalent descriptions of convex polygons. Statement (b) is true for all polygons, so it would apply to concave ones as well (triangle inequality!). Statement (c) is not true for all convex polygons. For example, try drawing a convex quadrilateral with one very long side; this side will probably be longer than the shortest diagonal of the quadrilateral. Statement (e) certainly does not describe convex polygons. (Consider an obtuse triangle which is convex but whose longest side is opposite to its largest angle.)

Problem C3. Answers will vary. You can find some more examples in Problem C4.

Solutions, cont'd.

Problem C4.



The answers may vary for the two figures on the right.

Problem C5. If an n-sided polygon is convex, we can pick a vertex and connect it to all other vertices, thereby creating n - 2 triangles. If the polygon is concave, we pick a vertex corresponding to an interior angle greater than 180° and connect that vertex to all the vertices so that all the resulting diagonals are in the interior of the polygon. Repeat the process if necessary. This has the effect of subdividing the original polygon into some number of convex polygons. We then divide each of the convex polygons into triangles. In the end, we will end up with n - 2 triangles.

Problem C6.

Number of Sides of the Polygon	Number of Triangles Formed	Sum of the Angles in the Polygon
3	1	180°
4	2	360°
5	3	540°
6	4	720°
7	5	900°
n	n - 2	(n - 2) · 180°

Problem C7. Assuming that the interior of an n-sided polygon can be divided into n - 2 triangles, observe that all of the angles of the triangles actually make up the interior angles of the polygon. This is true since the vertices of all the triangles coincide with the vertices of the polygon. Therefore, since each triangle contributes 180° to the overall sum of the angles, the sum is (n - 2) • 180°.

Homework

Problem H1. Answers will vary.

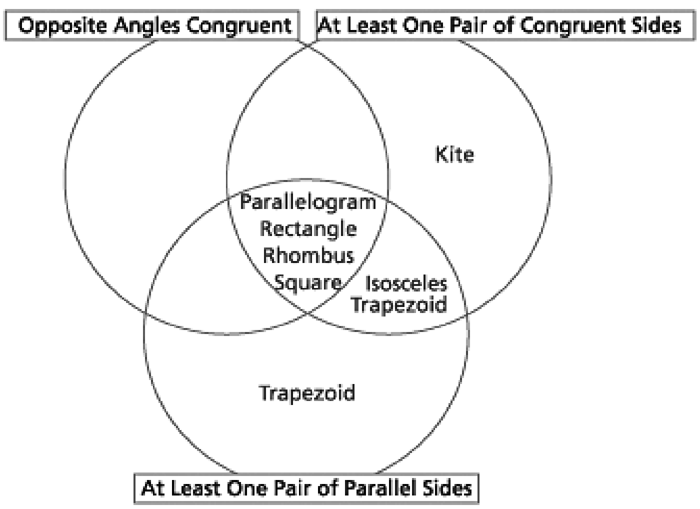
Solutions, cont'd.

Problem H2.

Type of Quadrilateral	Do diagonals bisect each other?	Are diagonals congruent?	Are diagonals perpendicular?
Trapezoid	No	Maybe	Maybe
Isosceles Trapezoid	No	Yes	Maybe
Parallelogram	Yes	Maybe	Maybe
Rhombus	Yes	Maybe	Yes
Rectangle	Yes	Yes	Maybe
Square	Yes	Yes	Yes
Kite	Maybe	Maybe	Yes

Problem H3.

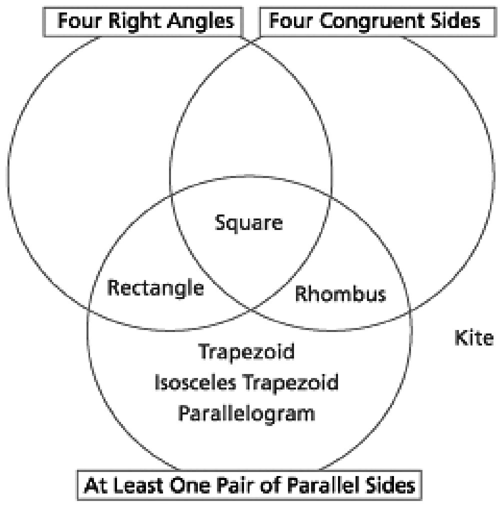
a.



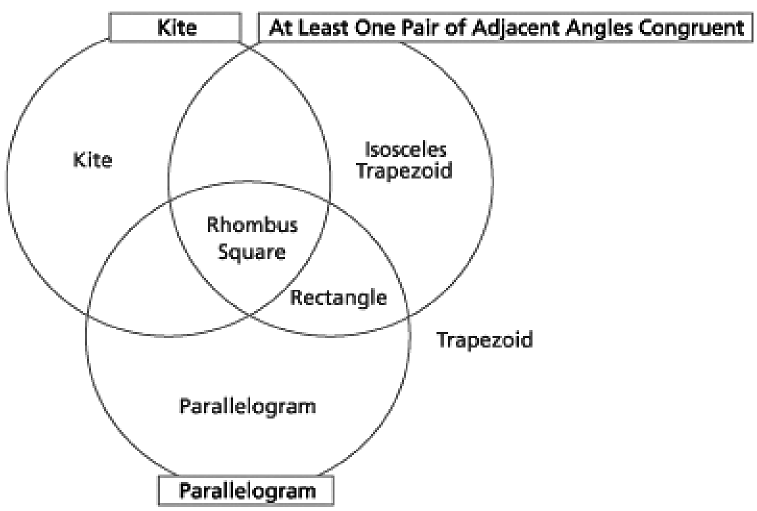
Solutions, cont'd.

Problem H3, cont'd.

b.



c.



Problem H4. Our definition requires that polygons divide the plane into an "interior" and an "exterior" region, which is not the case with a "flat" polygon. Also, the segment AB coincides with AC along its entire length; they do not just meet at an endpoint. (Likewise for BC and AC.)

Problem H5. Answers may vary, but one reason that "flat" polygons would not be considered "polygons" is that they would be indistinguishable from one another or from a line segment.