

Annenberg/CPB Course Guide

Learning Math

Geometry

A 10-part video- and Web-based course for K-8 teachers

Produced by WGBH Educational Foundation

Learning Math: Geometry

is produced by
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About This Course

The *Learning Math* Courses

When teachers have a deep conceptual understanding of mathematics, it can help their students develop strong mathematical skills and knowledge. *Learning Math* is a series of five multimedia, college-level courses designed to teach mathematics content to elementary and middle school teachers. Organized around the content standards developed by the National Council of Teachers of Mathematics (NCTM), the courses will cover Number and Operations; Patterns, Functions, and Algebra; Geometry; Measurement; and Data Analysis, Statistics, and Probability.

Overall Objectives

- To help teachers better understand mathematics content
- To provide engaging explorations of mathematics using video, interactive activities, and problem solving
- To encourage teachers to view mathematics as more than rote sets of rules and procedures

Overview: *Geometry*

Geometry introduces geometric reasoning as a method for problem solving. In this course, you will explore the properties of geometric figures such as triangles, quadrilaterals, and other polygons. You will make constructions using pencil and paper, and also dynamic software, and you will practice using mathematical language to express ideas and justify your reasoning. Some important geometric ideas such as symmetry, similarity, and trigonometry will also be examined. Lastly, you will begin to explore the basis of formal mathematical proofs and solid geometry. The course material progresses from more visual, intuitive ways of solving problems to more formal explorations of geometric ideas, properties, and, finally, proofs.

The course consists of 10 sessions, each with a half hour of video programming, problem-solving activities provided online and in a print guide, and interactive activities and demonstrations on the Web. Although each session includes suggested times for how long it may take to complete all of the required activities, these times are approximate. Some activities may take longer. You should allow at least two and a half hours for each session. The 10th session explores ways to apply the concepts of geometry you've learned in K-8 classrooms. You should complete the sessions sequentially.

Session 1: What Is Geometry?

Explore the basics of geometric thinking using rich visualization problems and mathematical language. Use your intuition and visual tools for geometric construction. Reflect on the basic objects of geometry and their representation.

Session 2: Triangles and Quadrilaterals

Learn about the classifications of triangles, their different properties, and relationships between them. Examine concepts such as triangle inequality, triangle rigidity, and side-side-side congruence, and look at the conditions that cause them. Compare how these concepts apply to quadrilaterals. Explore properties of triangles and quadrilaterals through practical applications such as building structures.

Session 3: Polygons

Explore the properties of polygons through puzzles and games; then proceed into a more formal classification of polygons. Look at mathematical definitions more formally, and explore how terms can have different but equivalent definitions.

Session 4: Parallel Lines and Circles

Use dynamic geometry software to construct figures with given characteristics, such as segments that are perpendicular, parallel, or of equal length, and to examine the properties of parallel lines and circles. Look past formal definitions and discover the properties and relationships among geometric figures for yourself.

About This Course, cont'd.

Session 5: Dissections and Proof

Review and explore transformations such as translation, reflection, and rotation. Apply these ideas to solve more complex geometric problems. Use your knowledge of properties of figures to reason through, solve, and justify your solutions to problems. Analyze and prove the midline theorem.

Session 6: The Pythagorean Theorem

Continue to examine the idea of mathematical proof. Look at several geometric or algebraic proofs of one of the most famous theorems in mathematics: the Pythagorean theorem. Explore different applications of the Pythagorean theorem, such as the distance formula.

Session 7: Symmetry

Investigate symmetry, one of the most important ideas in mathematics. Explore geometric notions of symmetry by creating designs and examining their properties. Investigate line symmetry and rotation symmetry; then learn about frieze patterns.

Session 8: Similarity

Examine your intuitive notions of what makes a “good copy,” and then progress toward a more formal definition of similarity. Explore similar triangles, and look into some applications of similar triangles, including trigonometry.

Session 9: Solids

Explore various aspects of solid geometry. Examine Platonic solids and why there are a finite number of them. Investigate nets and cross sections for solids as a way of establishing the relationships between two-dimensional and three-dimensional geometry.

Session 10: Classroom Case Studies

Explore how the concepts developed in this course can be applied at different grade levels through case studies of K-2, 3-5, and 6-8 teachers (former course participants), all of whom have adapted their new knowledge to their classrooms. Select video 10 for K-5 teachers and videos 11 and 12 for 6-8 teachers.

Course Components

Each *Learning Math* course consists of 10 two-and-a-half-hour sessions. The first nine sessions are devoted to mathematics content; the 10th session covers classroom applications. Concepts are developed within and across the sessions and the sessions increase in difficulty as they progress. Each session includes reading, problem solving, and group or individual activities that are available on the Web and in print, and a half-hour of video viewing, available on the Web,* on the Annenberg/CPB Channel,** or on videocassette.*** There are additional problems and readings to complete for homework.

* Broadband access is required to view the video on the Web; see Tech Tips, page 5.

** The schedule for broadcasts on the Annenberg/CPB Channel can be found on the course Web site.

*** Purchase videocassettes at www.learner.org or by calling 1-800-LEARNER.

Visit the *Geometry* Web site at www.learner.org/learningmath.

About This Course, cont'd.

The following components are in each course:

Key Terms

Key mathematical terms relevant to each session are listed at the beginning of that session. These terms are divided into two parts: terms that are new in that session and terms that were introduced in a previous session. Definitions for key terms may be found in the glossary in the Appendix of this guide.

Notes

Notes can be used by facilitators, study groups, or individuals working alone. They provide extended information about the topics presented in the course, including help for dealing with stumbling blocks that may come up and recommendations for different ways to approach the content.

Problems

Each session contains mathematical problems to be solved individually or by groups. Problems build upon previous concepts and increase in difficulty as the course progresses.

Take It Further

The problems marked "Take It Further" are optional and are not counted as part of the two-and-a-half hour time-frame for each session. These problems are designed for individuals who would like to explore a topic in greater depth. They are often more difficult than the other problems in the session, and they may introduce new information or concepts not previously discussed.

Interactive Activities

Each session in the course includes at least one interactive activity on the course Web site. These activities help you learn new mathematics content or reinforce existing knowledge through hands-on exploration directly on the Web. The interactive activities require the Flash plug-in, which you can download for free from Macromedia's Web site (see Tech Tips, page 5). There are also non-Flash versions of each activity that don't require the Flash plug-in and can be completed offline. If you are working with only the guide, the interactive activities have been adjusted and are included in the guide.

Tips

Tips are available for problems you may find more difficult or need help in getting started. Tips may be found at the end of each session in this guide.

Solutions

A solution is provided for every problem in *Learning Math*, with the exception of a few open-ended questions. When solving a problem with multiple parts, consider writing down your answers to all of the parts on paper first before checking the solution, because the answers to each part of the problem will be visible at once on the solution page. Solutions may be found at the end of each session in this guide.

The following sequence of activities will give you a sense of what you will do as a student using *Learning Math*:

1. Watch the **session video** in its entirety. You can watch the video before you begin the session to become comfortable with the material, or you can view the video after completing the session (so as not to view answers to problems).
2. Do problems sequentially. If you are having difficulty, refer to the **Tip**. If you want a challenge, try a **Take It Further**.
3. Check the **Solutions** at the end of each session.
4. You may want to read **Notes** as you go along to establish a deeper context for the content.
5. Watch **video segments** strategically placed throughout the session, either online by clicking on the "Play Video" button or on videocassette by fast-forwarding to the image and approximate time code that appears in the guide. Zero your VCR clock when the Annenberg/CPB logo appears at the beginning of the program to locate the image using the time code.
6. Do **homework problems** and **readings** (available as PDF files online) at the end of each session to reinforce your learning.

About This Course, cont'd.

Session Videos

Each *Learning Math* session includes viewing a video that is available on the course Web site, on videotape, or on the Annenberg/CPB Channel. The videos feature K-8 teachers working on the *Learning Math* course materials in a workshop with a facilitator. The videos for the nine content sessions show onscreen participants as they are introduced to the concepts, work through the problems, sometimes struggle to reach an understanding, and then reflect on what they have learned. At the end of most videos there is an example of how the content from the session is applied in a “real world” situation. The videos for the 10th session show participants from the videotaped workshops as they apply the content that they have learned back in their own classrooms. You may choose to watch each of these videos before or after you work on the associated course session.

Video Segments

Each session includes short excerpts from the associated video, which you watch (or review) and reflect on to see how the onscreen participants grapple with the same or similar problems and concepts you encounter in the course. Instructions are given to find the segments on videotape. The segments are also available on the course Web site, if you are watching the complete programs online or on the Annenberg/CPB Channel.

Homework

Each session includes approximately 45 minutes of homework problems and/or reflective writing assignments that reinforce the session’s content.

Readings

Readings from journals and books are cited at the end of some sessions. They are available on the *Geometry* Web site as downloadable PDF files. Go to www.learner.org/learningmath. If you do not have access to the Internet, call 1-800-LEARNER to obtain a free set of readings.

Using the Videos, Guide, and Web Site

Each *Learning Math* course includes sequentially organized problems, video viewing, interactive activities, readings, and homework. The multimedia elements of the course create an exciting environment for probing mathematics content. The course can be taken entirely on the Web, followed in this print guide, or completed using a combination of Web and print. You can watch the videos online, on the Annenberg/CPB Channel, or on videocassette. If you are watching the programs on the Channel, we recommend taping them so you can look at short video segments when prompted in each session. These segments punctuate concepts developed in the course and create a “virtual” community of learners.

Ways To Take *Learning Math*

Learning Math was flexibly designed for a variety of users and situations. You may choose to work through the sessions on your own, in a study group, or as part of a facilitated, face-to-face, graduate-level course for credit.

Channel-Talk

Join an email discussion group and converse with other teachers taking this course. Go to the course Web site at www.learner.org/learningmath and select Channel-Talk.

Registration and Credit

Go to www.learner.org/4gradcredit for details on receiving graduate credit for *Learning Math*.

About This Course, cont'd.

Taking Multiple *Learning Math* Courses

The five *Learning Math* courses are designed to be independent of one another. You can take just one course, several courses, or all five courses in the order that fits your needs or the needs of your group. The courses also complement one another, with some topics discussed in more than one course but approached differently depending on the focus of that course. Taking several of these courses will increase your own conceptual understanding and ultimately that of your students.

Facilitating the Course

You can prepare for facilitating the course by reading through each session and its "Notes" section prior to meeting with your group. Reading through the material will help you become clear about the activities, plan how much time you need to spend on each one, and pull together necessary materials. The course is designed for use by an individual, but the Notes suggest ways for groups to work through the sessions.

Tech Tips

To use all or portions of the *Learning Math* course online, we recommend the following:

Internet Access

A minimum 56K modem connection is required, but an ISDN or high-speed connection is recommended. The slower your connection, the longer it will take to load larger features, such as the Flash activities.

To view the video programs and video segments online, a broadband connection to the Internet (DSL, cable modem, or LAN connection to a T1 line or greater) is required.

Web Browser

You will need Netscape 4.6 (or a higher version) or Internet Explorer 5.0 (or a higher version). Javascript should be enabled, if your browser allows you to disable it. Text fonts and colors, and many features, such as hidden Tips, may not be displayed correctly in older browsers.

Plug-Ins

These plug-ins will allow you to get the most out of the courses. Links to sites where you can download these programs for free are provided on the course Web site.

- Shockwave Flash 5 (or a higher version) for using the Interactive Activities
- Windows Media Player 7 (or a higher version) for watching videos
- Adobe Acrobat Reader for viewing the Readings in the Homework sections

Printing Web Pages

If you are having trouble printing some of the course content pages, you may try doing one or more of the following (from the "Print Preview," "Print...," or "Page Setup..." menu):

- Turn on "Shrink to Fit" mode (IE 5 only)
- Print the page in "Landscape" mode
- Reduce the scale of the printer output

Who's Who

Content Developer/Facilitator

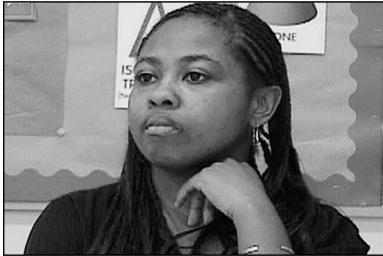


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Michelle Manes earned a Bachelor's degree in mathematics from UC Berkeley and a Master's of Education from Boston University. She has taught mathematics at all levels, from third grade through college. She is a primary author on the *Connected Geometry*, *Impact*, and *Mathematical Methods* curricula. She has special interests in mathematics history, women and girls studying mathematics, and Deaf education. She is currently working towards a Ph.D. in mathematics at Brown University.

Who's Who, cont'd.

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