

LEARNING

Some Insights Drawn

One day, each of Jennifer Roth's second graders received an envelope containing twelve shapes—several different kinds of triangles and quadrilaterals—cut from construction paper. Roth gave her students the following instructions: "Take a look at these shapes, and try to find some ways to sort them into groups.

Work alone for a few minutes, and then talk with your neighbor about what you've decided." As the children set to work, some could not wait to begin talking about the shapes with their partners, but others silently emptied their envelopes and began moving their shapes around on the table. After a few minutes, Roth began to listen in on students' conversations, frequently pausing to ask questions.

Danna and Cassie had grouped a square, a rhombus, and a rectangle whose sides were not exactly equal, but close. Danna explained, "We're thinking about how things are equal. See, these shapes are all equal." Cassie added, "Same length, like a square." Roth registered the girls' use of language—"shapes are all equal" meaning that the shapes are equilateral—then she nodded and moved on.

Emma commented that the parallelogram could go with the trapezoid. "See, it fits right here," she said as she lay one shape over the other to show how the angles were the same.

Stopping at Leah and Kevin's table, Roth saw that they had divided their shapes into two groups, "triangle" and "not triangle." Under the label "not triangle," they placed a square, a rectangle, and a parallelogram. Under "triangle" were three different triangles and a trapezoid. The children's misplacement of the trapezoid came as a surprise to Roth because in their previous mathematics lesson, they had had a lengthy discussion about the definition of *triangle*: "A triangle has three sides and three points." Roth asked Leah and Kevin why they had placed the trapezoid with the triangles. The children were hesitant and their explanations were muddled, but their main point was that it *felt* trianglelike. Pointing to the trapezoid, Leah said, "This one looks like a triangle with the top cut off."

Categorizing Shapes by "Gestalt" or by Properties

Although we, too, might have been surprised by Leah and Kevin's notion that a trapezoid should be classified as a triangle, research over the last few decades indicates that their thinking is not idiosyncratic. In fact, these children's observations are quite consistent with the findings of researchers. Children first come to identify shapes by their "gestalt," their overall look. In describing a shape, they often use visual prototypes; for example, triangles look like "witches' hats." They do not attend to geometric properties or to characteristic traits of the class of shapes represented. In their developing geometric understandings, children must experi-

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G E O M E T R Y

from Teacher Writing

ence a “shift” in their thinking—some researchers describe it as a jump to a new conceptual level—to recognize and characterize shapes by their properties. These levels are the first two of the five articulated by Pierre van Hiele and Dina van Hiele-Geldof. (For further discussion, see Clements and Battista [1992] and Crowley [1987].)

In Roth’s class, several children had, apparently, made this shift. Danna and Cassie had identified the property “sides of equal length” and were exploring which quadrilaterals share that property. Emma was beginning to look at angles when she remarked that the parallelogram and trapezoid in her envelope could be classified together because they “fit.” And Jake, another child, began to think about right angles by identifying which shapes could fit into the corner of his page.

Leah and Kevin had not yet made that shift. Even though the class had talked about triangles having “three sides and three points,” they still attended to the triangle-like quality of the trapezoid rather than to its properties. And although Leah and Kevin’s classmates were already moving forward to think in terms of properties, these two children would not be considered unusually slow second graders.

Lori Sanford, Roth’s colleague at another school, had a similar experience when she asked her second graders to think about triangles. And although they, too, had discussed the definition of *triangle*, when given a variety of drawn shapes, some children claimed that those shown in **figure 1a** were all triangles. In contrast, since their sense of triangle was of a shape resembling a witch’s hat, they claimed that those shown in **figure 1b** were *not* triangles.

These notions are common not only among second graders; evidence indicates that they frequently persist well beyond the elementary grades (Clements and Battista 1992).

Studying Children’s Thinking in Classroom Contexts

Research into children’s geometric understandings has fostered insight into such matters, but we do

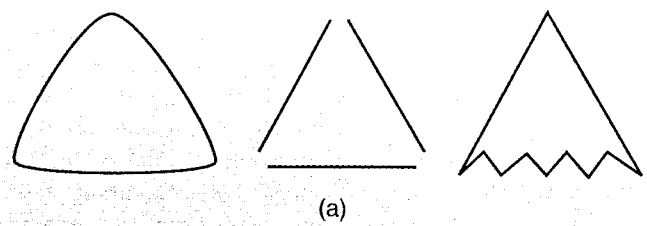
not know very much about what happens in classrooms when students work on geometric ideas over time. In fact, since for many years, very little geometry has been taught prior to high school, most of the research informs us of only the results of this neglect. Thus, as a field, we have much to learn: What are the ways in which children’s geometric understandings are expressed in classroom contexts? What kinds of lessons and discussions help children develop more sophisticated lines of thought? What does progress look like when ideas are explored over time?

Especially because instructional programs in the United States are only now beginning to include substantial units on geometry in the elementary grades, teachers are positioned to offer important information regarding these questions. After all, teachers are with their students every day, attending to the development of their ideas and working to figure out how to help their students move forward.

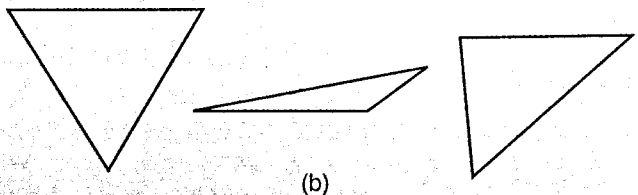
To this end, Teaching to the Big Ideas (TBI), a four-year (1993–1997) professional-development

FIGURE 1

Some second graders saw these figures as triangles.



Some second graders did not believe that these shapes are triangles.



and research project, was designed to bring together the insights of teachers and researchers to address the big ideas of mathematics that emerge in classroom contexts as teachers open up their teaching to students' ideas (Schifter, Russell, and Bastable, in press). A collaborative effort among the Center for the Development of Teaching at the Education Development Center, SummerMath for Teachers at Mount Holyoke College, the Mathematics Center at TERC, and fourteen school systems in Massachusetts, the TBI project engaged thirty-six elementary school teachers in an intensive program. Through summer institutes, biweekly after-school meetings, and biweekly classroom visits, participants explored mathematics content and reflected on the nature of mathematics, how it is learned, and the implications for their own instruction of new insights into these large issues. As teachers transformed their instruction, the project investigated the student thinking that emerged in their classrooms. During the fourth year of the project, teachers led seminars, based on a professional-development curriculum produced by the project (Schifter, Bastable, and Russell 1998), for colleagues in their school system.

A mechanism that the project developed for its investigations is "episode writing." Twice in the first year and as a regular monthly assignment in the second and third years, teachers wrote about episodes from their own teaching. The assignment was to write a two- to five-page narrative that captured some aspect of the mathematical thinking of one or more students, using transcriptions of classroom dialogue or samples of students' written work. The intention was that through the set of episodes, representing thirty-six classrooms from kindergarten through grade 6, the project could identify common cognitive issues and track how ideas develop through the grades. The full text of a sample episode has been included as an appendix.

The foregoing classroom scenes come from this body of writings. They illustrate how general research findings about the development of geometric understanding in the early grades appear in the particulars of the classroom. Other episodes, drawn from other classrooms, extend our understandings of issues that arise as children continue their study of the properties of geometric shapes into higher elementary grades.

Teachers Illustrate the Challenges

Judy Bishop's fourth graders had many of the same ideas as Sanford's second graders. When they, too, denied that shapes like those in figure 1b were triangles, Bishop set aside time to stretch their under-

standing. If triangles are defined as shapes with "three corners and three sides [and] the corners have to meet and the lines must be straight," students learned that triangles can be tilted, flipped, stretched, and squashed and still remain triangles.

Once the children understood, they went on to other shapes. Defining a square, Bishop reported, became a bigger problem. The children started off well enough—a square has four corners and "even sides," that is, all sides have the same length. But how flexible were her students in applying their definition? To check, Bishop drew the shape in figure 2 on the chalkboard. "Yes," the students agreed. "It's a square tipped on its side." As with triangles, the orientation of a square did not matter—it could be rotated, "tipped," or "tilted" and yet remain a square.

Next Bishop tried another shape: an elongated rhombus, shown in figure 3. But to her consternation, all the students in her class agreed that this shape, too, was a square. "It just happens to be a square with 'sharp' corners," they said. "A regular square has 'dull' corners." This shape was a "squished square."

Flustered, Bishop asked each student to draw a square. When they had all produced "regular, dull-cornered squares," she asked them, "Why do you think each person has drawn a 'regular' square?" "That's how we learned about squares in the lower grades," they explained. "But now we know that there are different kinds of squares."

Considered from the perspective of her students,

FIGURE 2

Judy Bishop's fourth graders saw this shape as a square.

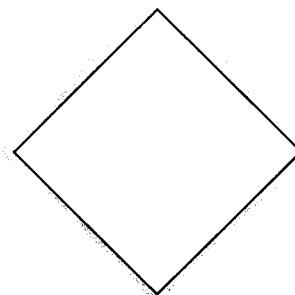


FIGURE 3

The fourth graders called this shape a square, too.

