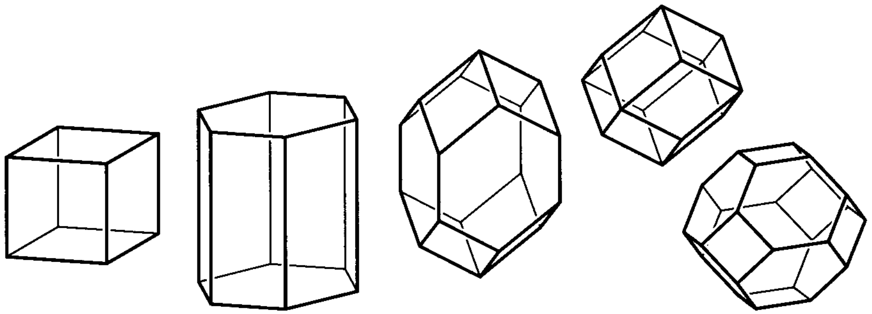

FIGURE 19. There are five combinatorial types of Dirichlet domains for three-dimensional lattices. These five shapes are much less well known than the Platonic solids but are at least as important!

houses, courtyards, and other structures. Young children are likely to have trouble building octahedra, but they can use small cubes to build larger cubes. The smallest composite cube is made up of 8 smaller ones; the next larger is made up of 27; by guessing how this series continues the child gains some understanding of volume. Older children—of any age—can also learn a lot from playing with cubes.

Cubes are the prototypical three-dimensional tile, and many structures, both mathematical and real, are based on it. It is worthwhile to try to build polyhedra out of cubes. For example, try building a regular octahedron by sticking sugar cubes together with glue. The larger you make your sugar octahedron (if it isn't too messy), the closer the stepped faces approximate smooth ones. Building polyhedra from cubes is thus a sophisticated lesson in volume measurement. It is instructive that H.S.M. Coxeter, in his classic work *Regular Polytopes,*[5] refers to the cube of any dimension as the "measure polytope." (The word "polytope" refers to the higher-dimensional analogues of polygons and polyhedra.)

Dissection

An important problem in many fields is how to divide a region into compartments of various shapes. An architect or designer partitions the interior of a building into rooms to serve certain purposes. We all fret over the most efficient way to pack a suitcase or the trunk of a car. A complex living object, such as a plant or a human being, has grown from a single cell that, in the early stages of growth, divided into "daughter" cells that grew and divided again. The study of how dividing cells organize themselves into tissues and then into organs is

one of the most exciting frontiers of biology. Some of the issues relate to the geometry of dissection, compartmentalization, and subdivision.

There are many interesting mathematical problems dealing with dissection. One of the most famous theorems in this field says that any polygon can be divided into a finite number of pieces and reassembled to form a congruent copy of any other polygon of the same area. Elementary school children enjoy the challenge of creating shapes with tangrams or other polygonal tiles; imagine the many challenging problems and puzzles that could be devised for older children related to this dissection theorem. More advanced students can discover that the analogous theorem for polyhedra is false; this is another fascinating and important result.

Another intriguing dissection problem is the creation of "rep-tiles," tiles that can be fitted together to form replicas of themselves (Figure 20). Alternatively, we can create such tiles by subdividing one into smaller congruent copies of itself. To create a tiling by rep-tiles, think of the daughter tiles growing to the size of the original one and then subdividing again. Repeating this process over and over again, we create a tiling that is self-similar in a certain sense; many of these tilings have no lattice structure. Is the tiling of Figure 20 lattice or nonlattice?

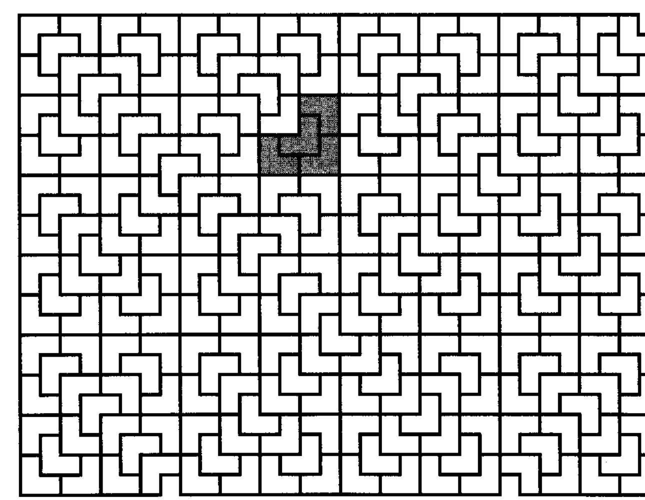

FIGURE 20. "Rep-tiles" are tiles that can be fitted together to form replicas of themselves. They build tilings that are self-similar and that, like this one, may have no lattice structure. Such tilings are of great interest today because they share many strange properties with some newly discovered crystalline materials.

(This is not easy to answer!) Tilings without lattices are of great interest today among mathematicians and solid-state scientists because they share many strange properties with some recently discovered crystalline materials called quasicrystals.

Senechal, Marjorie (1990). Shape. In *On the Shoulders of Giants: New Approaches to Numeracy*. Edited by Lynn Arthur Steen (pp. 148-160). Washington, D.C.: National Academy Press.