FIGURE 12. The principle of the kaleidoscope is discovered by playing with two hinged pocket mirrors. The objects appear repeated in infinitely varied patterns, but as the angle between the mirrors is changed, some patterns reveal greater symmetry (and beauty) than others.

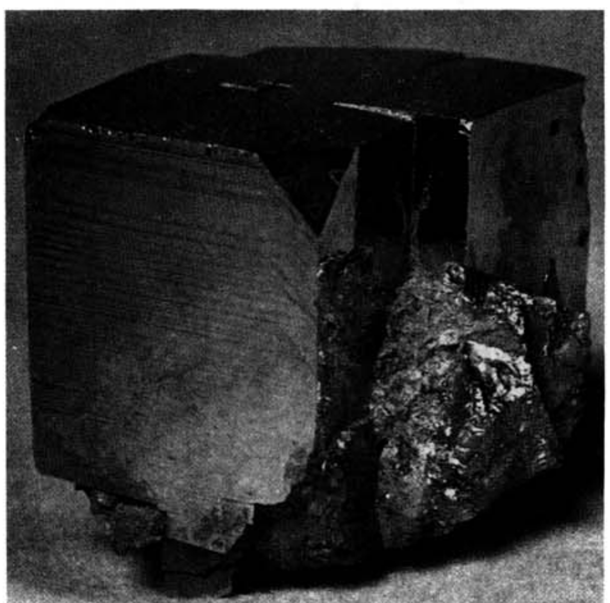

FIGURE 13. A pyrite crystal. The lines on the cube's faces indicate that the crystal's internal structure lacks some of the symmetries of the cube.

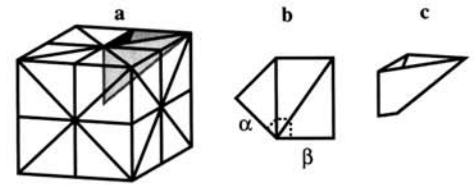

FIGURE 14. A cubic kaleidoscope can be made by placing mirrors or reflecting mylar on the inside of three sides of one of the tetrahedral sectors into which the cube is divided by its mirror planes (a). The net for these three walls is shown in (b); it consists of half a square and a rectangle whose base is the length of the square's edge and whose height is the length of the square's diagonal. Cut along the dotted lines, and then tape the edges α and β together (c). With the cut end down and parallel to a table, look at a piece of newspaper or other decorated material through the tetrahedron. You will see a decorated cube! By moving the tetrahedron along the plane surface, you will see a changing pattern on the cube.

The symmetry of three-dimensional figures appears to be more intricate, but actually the principles are the same as in the two-dimensional case. For example, the symmetry of the cube includes reflections in two kinds of mirror planes and rotations about three kinds of axes. Younger students can learn a great deal about the symmetry of the cube by trying to decorate it in ways consistent with its symmetry. Older students can be challenged by the task of changing this symmetry by decoration.

Such decorations appear in nature, where they provide clues to the structure of hidden patterns. For example, the pyrite crystal in Figure 13 appears at first glance to be an ordinary cube, but closer inspection reveals striations on the cube's faces. These striations are consistent with some, but not all, of the symmetries of the cube. The reason for the striations, it turns out, is that the arrangement of atoms inside the crystal is less symmetrical than its external cubic form suggests. Consequently, the pyrite crystal is a cube with texture, or a decorated cube.

One of the more exciting and instructive exercises for older students is to make a cubic kaleidoscope. The cube is divided by its mirror planes into 48 congruent tetrahedra. If a model of one of these tetrahedra is lined with mirrors or some reflecting paper such as mylar, with the triangle belonging to the cube space removed and the opposite vertex snipped off, an entire cube is generated by the reflections. Reflecting mylar pasted onto cardboard or heavy paper will work well; only three of the four tetrahedral walls should be constructed so that you will be able to see inside. Figure 14 shows how to construct such a kaleidoscope.[18]

Using Symmetry

If all we learn about symmetry is to identify it, we miss the whole point. Symmetry is an effect, not a cause.[19] Why are so many natural structures symmetrical? For example, what atomic forces ensure that