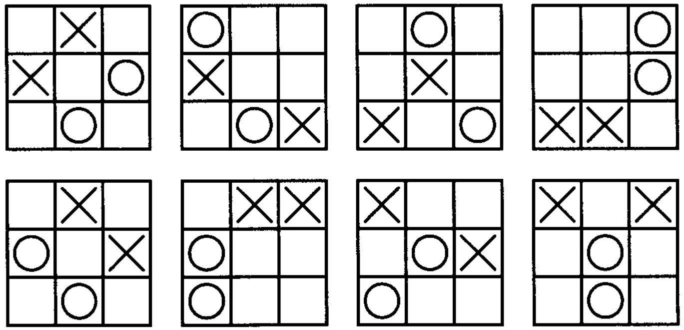

FIGURE 5. In torus tic-tac-toe the opposite sides of the board are identified—that is, considered to be the same. It is as if the board were rolled into a cylinder, which was then bent around to form an inner-tube shape that mathematicians call a torus. Can you tell which of these positions are equivalent in the torus-shaped game?

properties of shapes remain intact under such transformations. For example, the numbers of edges and vertices of a polygon are not altered if we stretch or bend the polygon. Thus the three hexagons of Figure 7 are all hexagons, even though they are neither congruent nor similar: a hexagon is any closed loop made of six line segments. Being a hexagon is a combinatorial property of a polygon.

Roughly speaking, the combinatorial properties of a shape are the things we can count and the way they are fitted together. Thus from the combinatorial point of view, the shapes in Figure 3a are equivalent, since each has 6 faces, 8 vertices, and 12 edges connected to each other in the same way. Network problems often involve combinatorial problems. For example, if we want to design a linking system for the computers in a building, we are concerned first with finding the possible arrangements of links and nodes that can provide the connections we want, and only then need we consider how long the cables will have to be.

Topology. Topological equivalence is even more general than combinatorial equivalence. From the standpoint of topology, all polygons are loops and all convex polyhedra are alike. Piaget argued that topological concepts occur prior to metric ones in child development; a child may recognize a loop before distinguishing among kinds of loops, such as circles and triangles. Being a loop, as opposed to a knot, is a topological property of shape.

Topology in school is often described as "rubber sheet geometry." It yields many excellent examples that can enlarge a child's concept of the flexibility of shape. In rubber sheet geometry, the shapes of Figure 3c are indistinguishable because each can be deformed into the other. Knots, of the boy and girl scout variety, are an excellent subject for hands-on study.[12] Children can learn to play tic-tac-toe on a torus and

other delightful games that require geometrical mental gymnastics (Figure 5).

With complexity of structure, topological classification necessarily becomes more sophisticated. Here computer visualization can be a useful tool. Older students can appreciate the concept of orientation, which characterizes the difference between a cylinder and a Mobius band (orientable and non-orientable), and the concept of genus, which characterizes the topological difference between a sphere and a torus (genus zero and genus one). Understanding such concepts enriches greatly the study of science and design as well as mathematics.

Naming

Shapes need names. One of the most fundamental uses of language is to assign names to things. Naming is a primitive concept that is echoed in our myths as well as in many contemporary religious practices. Naming is the first step toward knowing, whether it is the name of a person or the name of a shape. We cannot think about shapes (or anything else for that matter) or explain our ideas to others if we do not use names. Learning technical names is sometimes disparaged as a rote activity, but such objections miss the point. Technical names are usually not arbitrary; they encode the conceptual framework in which we organize the things we are naming.

For example, in English-speaking countries, last names indicate the family and first names designate an individual in a family. Thus Mary Jones is a person named Mary who is a member of the Jones family. The names of shapes serve similar functions: a tetrahedron is a member of the polyhedron family, a representative of the subfamily of those polyhedra that have four faces (see Figure 6). When we use the word

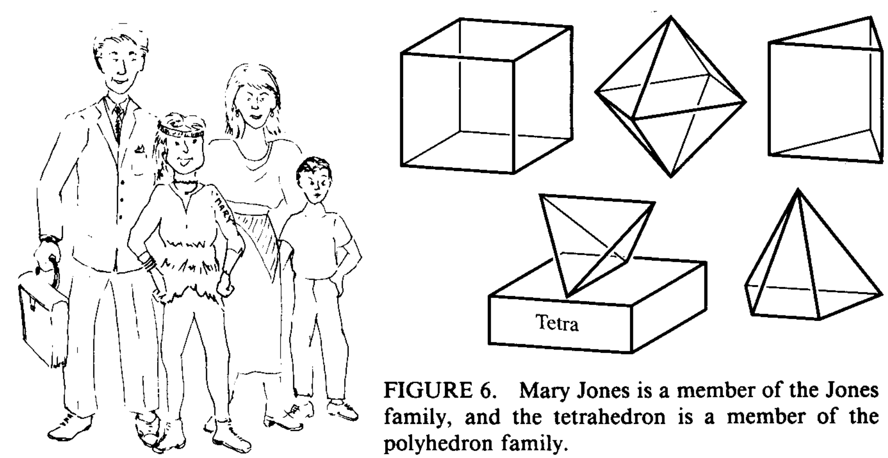

FIGURE 6. Mary Jones is a member of the Jones family, and the tetrahedron is a member of the polyhedron family.

"tetrahedron" to name a shape, we are at the same time locating it in its family tree and describing it in a meaningful way.

Although classification requires precision, there is no single "right" way to classify shapes. Shapes are classified into families and subfamilies in many different ways, depending on the properties that interest us. For example, the discovery that the orbits of the planets around the sun are ellipses, and not circles, revolutionized the study of astronomy; from this standpoint circles and ellipses are completely different. But one of the great achievements of the ancients was the discovery that both circles and ellipses are conic sections and in that sense are the same.

From the point of view of topology, the distinction between shapes that enclose regions, like balls, and shapes that have holes in them, like bagels, is fundamental; within these broad classes, all shapes are alike. But a football player would not be happy with a basketball as a substitute, nor would a basketball player be willing to make do with a baseball, because the individual kinds of balls have crucially different properties. As another example of cross-classification, architects know that it is important to build houses that are sturdy, not houses that might collapse. This concern transcends other ways that houses are commonly classified, such as large and small, single story or multistory, rectangular, or dome-like.

Classification skills develop gradually. Very young children learn to recognize a great many shapes without being formally taught. Their world is literally made of shapes: shapes that hold things, such as bowls and bags and baskets; shapes to play with, such as balls and puzzles and blocks; shapes to use, such as chairs and spoons and beds. Thousands of shapes are part of children's lives. Later, in school, children learn

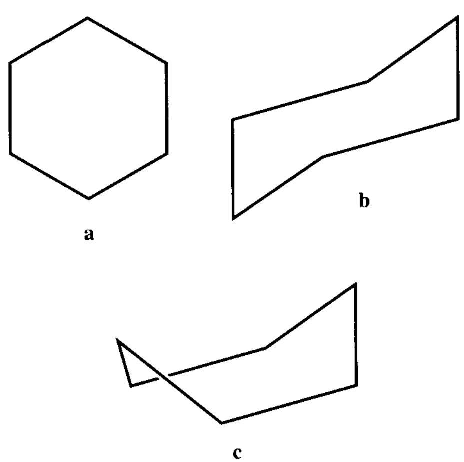

FIGURE 7. Three hexagons that are important in chemistry. The planar hexagon (a) occurs in benzene (see also Figure 30 below). The hexagons in (b) and (c) are intended to be nonplanar; both are conformations of cyclohexane. Hexagons made out of flexible straws can easily assume any of these shapes.

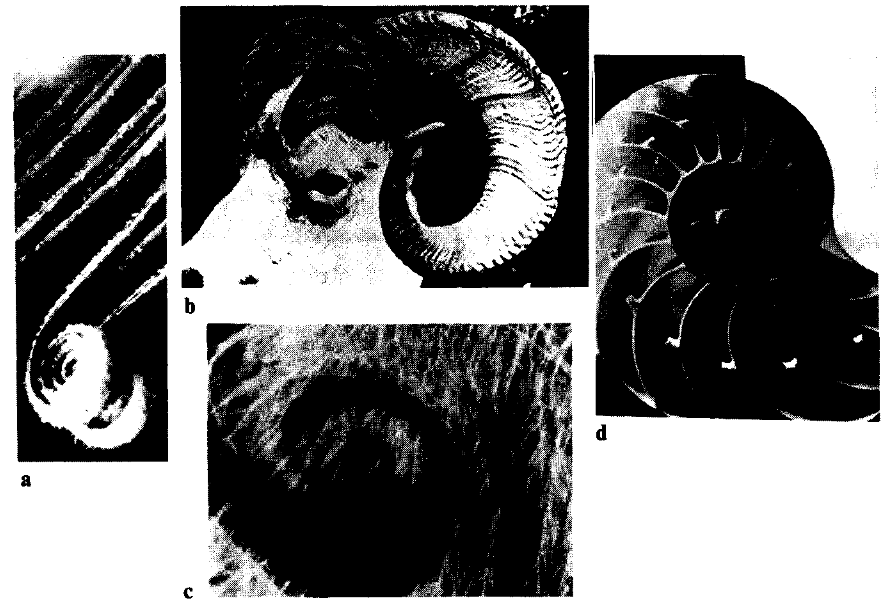

FIGURE 8. Four natural spirals: (a) leaves of the sago palm, (b) horns of a mountain sheep, (c) glycerin mixed with food coloring and ink, (d) the chambered nautilus. The common shape suggests a common creative mechanism, despite the striking differences in material, scale, and natural forces.

names for some of them, such as circles, spheres, polygons, and some simple polyhedra.

Alas, in our schools identification and classification of shapes usually stop just at the point where they can begin to be really interesting—where they begin to explore structures in three-dimensional space. How many people realize that even polygons that are not flat can be interesting and important? Many molecules have polygonal shapes, but often these polygons are crumpled and their conformations are the key to their chemical properties (Figure 7). Besides finite polygons and polygons whose edges don't cross, there are zig-zag, star, and helical polygons. By broadening the definition of polygon to include any closed loop, we may also study knots. In addition to their obvious practical importance for tying things, knots enter into the design of networks such as cloverleafs and are helpful in understanding the structure of some biological molecules. Soap bubbles, soap films, and froths are also endless sources of fascinating geometrical principles.

The study of polyhedra can be extended from simple shapes that are easy to construct to others, such as star polyhedra, that are more complex. Equally important are patterns, such as tilings of the plane, that are beautiful as well as useful. The helix and the spiral are fundamental to biology and astronomy as well as to mathematics. But even today, when "double helix" has become almost a household phrase, few people realize that there is a fundamental difference between a helix, which twists around an axis at a constant distance from it, and a spiral (Figure 8). Most so-called spiral staircases are really helical, for obvious practical reasons. Imagine what we would be like if our DNA wound itself in spirals, or what the universe would be like if galactic spirals were helices!

Senechal, Marjorie (1990). Shape. In *On the Shoulders of Giants: New Approaches to Numeracy.* Edited by Lynn Arthur Steen (pp. 139-148). Washington, D.C.: National Academy Press.