

Pattern

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“He just saw further than the rest of us.” The subject of this remark, cyberneticist Norbert Wiener, is one of many exceptional scientists who broke the bonds of tradition to create entirely new domains for mathematicians to explore. Seeing and revealing hidden patterns are what mathematicians do best. Each major discovery opens new areas rich with potential for further exploration. In the last century alone, the number of mathematical disciplines has grown at an exponential rate; examples include the ideas of Georg Cantor on transfinite sets, Sonja Kovalevsky on differential equations, Alan Turing on computability, Emmy Noether on abstract algebra, and, most recently, Benoit Mandelbrot on fractals.

To the public these new domains of mathematics are *terra incognita*. Mathematics, in the common lay view, is a static discipline based on formulas taught in the school subjects of arithmetic, geometry, algebra, and calculus. But outside public view, mathematics continues to grow at a rapid rate, spreading into new fields and spawning new applications. The guide to this growth is not calculation and formulas but an open-ended search for pattern.

Mathematics has traditionally been described as the science of number and shape. The school emphasis on arithmetic and geometry is deeply rooted in this centuries-old perspective. But as the territory explored by mathematicians has expanded—into group theory and statistics, into optimization and control theory—the historic boundaries of mathematics have all but disappeared. So have the boundaries of its

applications: no longer just the language of physics and engineering, mathematics is now an essential tool for banking, manufacturing, social science, and medicine. When viewed in this broader context, we see that mathematics is not just about number and shape but about pattern and order of all sorts. Number and shape—arithmetic and geometry—are but two of many media in which mathematicians work. Active mathematicians seek patterns wherever they arise.

Thanks to computer graphics, much of the mathematician's search for patterns is now guided by what one can really see with the eye, whereas nineteenth-century mathematical giants like Gauss and Poincaré had to depend more on seeing with their mind's eye. "I see" has always had two distinct meanings: to perceive with the eye and to understand with the mind. For centuries the mind has dominated the eye in the hierarchy of mathematical practice; today the balance is being restored as mathematicians find new ways to see patterns, both with the eye and with the mind.

Change in the practice of mathematics forces re-examination of mathematics education. Not just computers, but also new applications and new theories have expanded significantly the role of mathematics in science, business, and technology. Students who will live and work using computers as a routine tool need to learn a different mathematics than their forefathers. Standard school practice, rooted in traditions that are several centuries old, simply cannot prepare students adequately for the mathematical needs of the twenty-first century.

Shortcomings in the present record of mathematical education also provide strong forces for change. Indeed, since new developments build on fundamental principles, it is plausible, as many observers often suggest, that one should focus first on restoring strength to time-honored fundamentals before embarking on reforms based on changes in the contemporary practice of mathematics. Public support for strong basic curricula reinforces the wisdom of the past—that traditional school mathematics, if carefully taught and well learned, provides sound preparation both for the world of work and for advanced study in mathematically based fields.

The key issue for mathematics education is not *whether* to teach fundamentals but *which* fundamentals to teach and *how* to teach them. Changes in the practice of mathematics do alter the balance of priorities among the many topics that are important for numeracy. Changes in society, in technology, in schools—among others—will have great impact on what will be possible in school mathematics in the next century. All of these changes will affect the fundamentals of school mathematics.

To develop effective new mathematics curricula, one must attempt to foresee the mathematical needs of tomorrow's students. It is the

present and future practice of mathematics—at work, in science, in research—that should shape education in mathematics. To prepare effective mathematics curricula for the future, we must look to patterns in the mathematics of today to project, as best we can, just what is and what is not truly fundamental.

FUNDAMENTAL MATHEMATICS

School tradition has it that arithmetic, measurement, algebra, and a smattering of geometry represent the fundamentals of mathematics. But there is much more to the root system of mathematics—deep ideas that nourish the growing branches of mathematics. One can think of specific mathematical structures:

- Numbers
- Algorithms
- Ratios
- Shapes
- Functions
- Data

or attributes:

- Linear
- Periodic
- Symmetric
- Continuous
- Random
- Maximum
- Approximate
- Smooth

or actions:

- Represent
- Control
- Prove
- Discover
- Apply
- Model
- Experiment
- Classify
- Visualize
- Compute

or abstractions:

- Symbols
- Infinity
- Optimization
- Logic
- Equivalence
- Change
- Similarity
- Recursion

or attitudes:

- Wonder
- Meaning
- Beauty
- Reality

or behaviors:

- Motion
- Chaos
- Resonance
- Iteration
- Stability
- Convergence
- Bifurcation
- Oscillation

or dichotomies:

- Discrete vs. continuous
- Finite vs. infinite
- Algorithmic vs. existential
- Stochastic vs. deterministic
- Exact vs. approximate

These diverse perspectives illustrate the complexity of structures that support mathematics. From each perspective one can identify various strands that have within them the power to develop a significant mathematical idea from informal intuitions of early childhood all the way through school and college and on into scientific or mathematical research. A sound education in the mathematical sciences requires encounter with virtually all of these very different perspectives and ideas.

Traditional school mathematics picks very few strands (e.g., arithmetic, geometry, algebra) and arranges them horizontally to form the curriculum: first arithmetic, then simple algebra, then geometry, then more algebra, and finally—as if it were the epitome of mathematical knowledge—calculus. This layer-cake approach to mathematics education effectively prevents informal development of intuition along the multiple roots of mathematics. Moreover, it reinforces the tendency to design each course primarily to meet the prerequisites of the next course, making the study of mathematics largely an exercise in delayed gratification. To help students see clearly into their own mathematical futures, we need to construct curricula with greater vertical continuity, to connect the roots of mathematics to the branches of mathematics in the educational experience of children.

School mathematics is often viewed as a pipeline for human resources that flows from childhood experiences to scientific careers. The layers in the mathematics curriculum correspond to increasingly constricted sections of pipe through which all students must pass if they are to progress in their mathematical and scientific education. Any impediment to learning, of which there are many, restricts the flow in the entire pipeline. Like cholesterol in the blood, mathematics can clog the educational arteries of the nation.

In contrast, if mathematics curricula featured multiple parallel strands, each grounded in appropriate childhood experiences, the flow of human resources would more resemble the movement of nutrients in the roots of a mighty tree—or the rushing flow of water from a vast watershed—than the increasingly constricted confines of a narrowing artery or pipeline. Different aspects of mathematical experience will attract children of different interests and talents, each nurtured by challenging ideas that stimulate imagination and promote exploration. The collective

effect will be to develop among children diverse mathematical insight in many different roots of mathematics.

FIVE SAMPLES

This volume offers five examples of the developmental power of deep mathematical ideas: dimension, quantity, uncertainty, shape, and change. Each chapter explores a rich variety of patterns that can be introduced to children at various stages of school, especially at the youngest ages when unfettered curiosity remains high. Those who develop curricula will find in these essays many valuable new options for school mathematics. Those who help determine education policy will see in these essays examples of new standards for excellence. And everyone who is a parent will find in these essays numerous examples of important and effective mathematics that could excite the imagination of their children.

Each chapter is written by a distinguished scholar who explains in everyday language how fundamental ideas with deep roots in the mathematical sciences could blossom in schools of the future. Although not constrained by particular details of present curricula, each essay is faithful to the development of mathematical ideas from childhood to adulthood. In expressing these very different strands of mathematical thought, the authors illustrate ideals of how mathematical ideas should be developed in children.

In contrast to much present school mathematics, these strands are alive with action: pouring water to compare volumes, playing with pendulums to explore dynamics, counting candy colors to grasp variation, building kaleidoscopes to explore symmetry. Much mathematics can be learned informally by such activities long before children reach the point of understanding algebraic formulas. Early experiences with such patterns as volume, similarity, size, and randomness prepare students both for scientific investigations and for more formal and logically precise mathematics. Then when a careful demonstration emerges in class some years later, a student who has benefited from substantial early informal mathematical experiences can say with honest pleasure “Now I see why that’s true.”

CONNECTIONS

The essays in this volume are written by five different authors on five distinct topics. Despite differences in topic, style, and approach, these essays have in common the lineage of mathematics: each is connected in myriad ways to the family of mathematical sciences. Thus it should